Flows and Decompositions of Games: Harmonic and Potential Games

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Potential Games: A Special Class

Potential games are a special class of games that admit tractable static and dynamic analysis.

- A pure strategy Nash equilibrium always exists.
- Natural learning dynamics converge to a pure Nash equilibrium.
- [Monderer and Shapley 96], [Young 98], [Marden, Arslan, Shamma 07, 09]

Only a few games in economics, social sciences, or engineering are potential games

- e.g., congestion games.

Motivation of Our Research

- What makes potential games “special”?
- What is the underlying preference structure that leads to the desirable properties of potential games?
- Are there other classes of games with distinguished static and dynamic properties?
- How do we determine whether a game is “close" to a potential game?

Main Contributions

We obtain a direct sum decomposition of games:

- We provide a representation of finite games as flows on graphs.
- These flows admit an orthogonal decomposition, which leads to a decomposition of the space of games to potential ($\mathcal{P}$), harmonic ($\mathcal{H}$) and nonstrategic ($\mathcal{N}$) components:

$$\underbrace{\mathcal{P} \quad \oplus \quad \overbrace{\mathcal{N}}^{}}_{\text{Potential games}} \!\!\!\!\!\!\!\! \overbrace{\phantom{\mathcal{N}} \quad \oplus \quad \mathcal{H}}^{\text{Harmonic games}}.$$

- Closed form expressions for each of these components.

This decomposition allows us to identify classes of games with distinct properties.

- For instance, potential games always have pure equilibria, whereas harmonic games generically do not.

Main Contributions – II

Decomposition framework can be used to find close potential games to a given game.

Properties of potential games can be extended to near-potential games:

- Approximate equilibria of a game can be characterized in terms of the equilibria of a close potential game.
- Convergence properties of dynamics can be studied using the relation of the game to a close potential game.

Outline

1. Decomposition of games

2. Properties of potential and harmonic games

3. Equilibria and dynamics in near-potential games

Potential Games and Nash Equilibrium

- Finite games in strategic form: $\mathcal{G} = \langle \mathcal{M}, \{A^m\}_{m\in\mathcal{M}}, \{u^m\}_{m\in\mathcal{M}} \rangle$.
- $A = \Pi_{m\in\mathcal{M}} A^m$ denotes the set of strategy profiles, and $u^m : A \to \mathbb{R}$.
- A unilateral deviation by player m: $(p^m, \mathbf{p}^{-m}) \to (q^m, \mathbf{p}^{-m})$.
- $\mathcal{G}$ is an exact potential game if there exists a function $\phi : A \to \mathbb{R}$, such that for all $m \in \mathcal{M}$, $p^m,\ q^m \in A^m$, and $\mathbf{p}^{-m} \in A^{-m}$:

$$u^m(q^m, \mathbf{p}^{-m}) - u^m(p^m, \mathbf{p}^{-m}) = \phi(q^m, \mathbf{p}^{-m}) - \phi(p^m, \mathbf{p}^{-m}).$$

	O	F
O	3, 2	0, 0
F	0, 0	2, 3

Table: Battle of the Sexes

	O	F
O	3	1
F	0	3

Table: Potential function

- A global maximum $\mathbf{p}$ of the potential is a pure Nash equilibrium.

$$u^m(p^m, \mathbf{p}^{-m}) - u^m(q^m, \mathbf{p}^{-m}) \geq 0 \qquad \text{for all } m \in \mathcal{M},\ q^m \in A^m.$$

- Natural learning dynamics converge to a NE.

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Potential Games and Cycles

Is matching pennies a potential game?

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Table: Matching Pennies

Consider a cycle of unilateral deviations:

$$\begin{tikzcd}
(H,H) \arrow[r, "2"] & (H,T) \arrow[d, "2"] \\
(T,H) \arrow[u, "2"] & (T,T) \arrow[l, "2"]
\end{tikzcd}$$

Figure: Utility improvements.

Since the utility of the deviating player improves at each step, this cannot be a potential game:

- At each deviation payoff increases by 2,
- When the cycle is completed the potential would need to increase by 8.
- Monderer and Shapley 96: $\mathcal{G}$ is a potential game $\leftrightarrow$ total utility change over a unilateral deviation cycle equals to zero.

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Flows on Graphs

What is the global structure of these cycles?

Conceptually similar to structure of (continuous) vector fields.

We need the combinatorial analogue (flows), a well-developed theory from algebraic topology.

Consider an undirected graph $G = (A, E)$, with set of nodes A, and set of edges E.

Edge flows: $X : A \times A \to \mathbb{R}$ such that $X(\mathbf{p}, \mathbf{q}) = -X(\mathbf{q}, \mathbf{p})$ if $(\mathbf{p}, \mathbf{q}) \in E$, and 0 otherwise.

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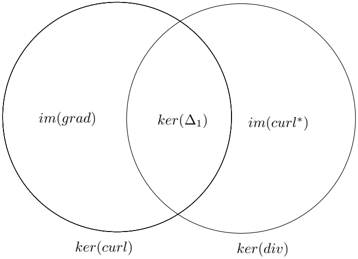
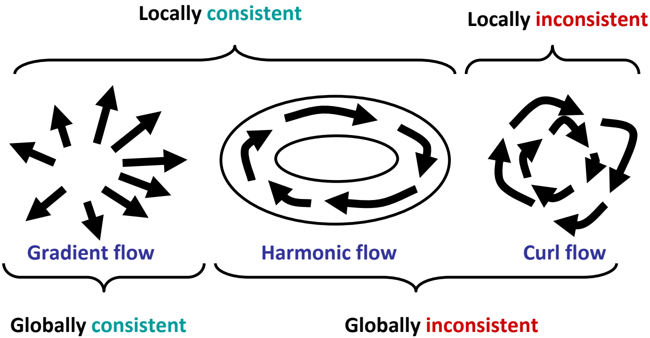
Helmholtz (Hodge) Decomposition

The Helmholtz Decomposition allows for orthogonal decomposition of the space of edge flows into these three flow components:

- Gradient flow: globally consistent component.
- Harmonic flow: locally consistent, but globally inconsistent. component
- Curl flow: locally inconsistent component.



Helmholtz (Hodge) Decomposition



Flow Representations of Games

- Let E^m denote the set of pairs of strategy profiles that differ in the strategy of only player m, and $E = \cup_m E^m$.
- The game graph is defined as the undirected graph $G = (A, E)$.

- For all $m \in \mathcal{M}$, and $f \in C_0$ we define a difference operator D_m:

$$(D_m f)(\mathbf{p}, \mathbf{q}) = \begin{cases} f(\mathbf{q}) - f(\mathbf{p}) & \text{if } (\mathbf{p}, \mathbf{q}) \in E^m \\ 0 & \text{otherwise.} \end{cases}$$

- The flow generated by a game is given by $X = \sum_{m \in \mathcal{M}} D_m u^m$.

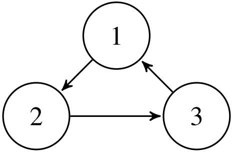
	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Table: Matching Pennies

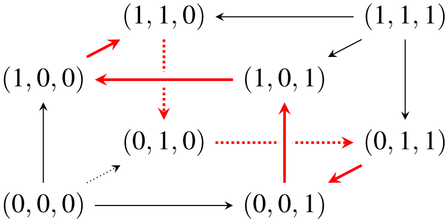
$$\begin{tikzcd}
(H,H) \arrow[r, "2"] & (H,T) \arrow[d, dashed, "2"] \\
(T,H) \arrow[u, dashed, "2"] & (T,T) \arrow[l, "2"']
\end{tikzcd}$$

Figure: Flow Representation.

Game Flows: 3-Player Example



- $E^m = \{0, 1\}$ for all $m \in \mathcal{M}$, and payoff of player i be -1 if its strategy is the same with its successor, 0 otherwise.
- This game is neither an exact nor an ordinal potential game.



Strategically Equivalent Games

Consider the following two games:

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

	H	T
H	2, -1	-1, 1
T	0, 1	1, -1

These games have the same utility change due to unilateral deviations, and therefore yield the same flows.

They are also strategically equivalent (same equilibrium and dynamic properties).

The nonstrategic component of player m's utility belongs to kernel of D_m.

- $\Pi_m = D_m^\dagger D_m$ is a projection operator which eliminates the nonstrategic component,
- Nonstrategic component of player m's utility: $u^m - \Pi_m u^m$.

Redefining Potential Games

- Note that a game is an exact potential game if and only if for all $m \in \mathcal{M}$:

$$D_m u^m = D_m \phi.$$

- $\delta_0 = \sum_{m \in \mathcal{M}} D_m$ is a combinatorial gradient operator.
- Image spaces of operators $\{D_m\}_{m \in \mathcal{M}}$ are orthogonal.
- A game is an exact potential game if and only if

$$\sum_{m \in \mathcal{M}} D_m u^m = \sum_{m \in \mathcal{M}} D_m \phi = \delta_0 \phi.$$

Exact Potential Games - Alternative Definition

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Decomposition: Potential, Harmonic, and Nonstrategic

Decomposition of the game flows induces a partition of the space of games:

- When going from utilities to flows, the nonstrategic component is removed.
- Since we start from utilities (not preferences), always locally consistent.
- Therefore, two flow components: gradient and harmonic
    - potential component corresponds to gradient flow.
    - harmonic component corresponds to harmonic flow.

Thus, the space of games has a canonical direct sum decomposition:

$$G = \mathcal{P} \oplus \mathcal{N} \oplus \mathcal{H}.$$

Closed form expressions for decomposition

Theorem (Decomposition)

1. *The Potential Component:* $u_P^m = \Pi_m \phi$,
2. *The Harmonic Component:* $u_H^m = \Pi_m u^m - \Pi_m \phi$,
3. *The Nonstrategic Component:* $u_N^m = (I - \Pi_m) u^m$,

where $\phi = \left(\sum_m D_m\right)^\dagger \sum_{m \in \mathcal{M}} D_m u^m$ *and* $\Pi_m = D_m^\dagger D_m$.

$$\begin{tikzcd}
\mathcal{G}_{\mathcal{M},E} \arrow[dr, "\{D^m\}"] & & \\
\phi \arrow[r, "\delta_0"'] & C_1 \arrow[r, "\delta_1"'] & C_2
\end{tikzcd}$$

Example

	A	B
A	4, 2	1, -2
B	-4, 3	-1, -3

Table: A two player game

$$
\begin{tikzcd}
(A,A) & (A,B) \arrow[l, "4"'] \\
(B,A) \arrow[u, "8"'] & (B,B) \arrow[l, "6"'] \arrow[u, "2"']
\end{tikzcd}
$$

Figure: Flow Representation

	A	B
A	3, 3	2, -3
B	-3, 2	-2, -2

Table: Potential Component

$$
\begin{tikzcd}
(A,A) & (A,B) \arrow[l, "6"'] \\
(B,A) \arrow[u, "6"'] & (B,B) \arrow[l, "4"'] \arrow[u, "4"']
\end{tikzcd}
$$

Figure: Flow Representation

	A	B
A	1, -1	-1, 1
B	-1, 1	1, -1

Table: Harmonic Component

$$
\begin{tikzcd}
(A,A) \arrow[r, "2"] & (A,B) \arrow[d, "2"] \\
(B,A) \arrow[u, "2"'] & (B,B) \arrow[l, "2"']
\end{tikzcd}
$$

Figure: Flow Representation

Properties of the Decomposition

Using the decomposition we identify classes of games with distinct properties:

- Set of potential games: $\mathcal{P} \oplus \mathcal{N}$.
- Set of harmonic games: $\mathcal{H} \oplus \mathcal{N}$.

Potential and harmonic games have very different equilibrium properties:

Theorem

Potential games always have a pure Nash equilibrium

Theorem

Harmonic games generically do not have pure Nash equilibria.

Properties of the Decomposition

Other interesting properties of harmonic games:

- A mixed NE: each player chooses a strategy uniformly at random.
- Players never strictly prefer their equilibrium strategies.
- Two players: set of mixed Nash equilibria = set of correlated equilibria
- Two players and equal number of strategies: Uniformly mixed strategy is the unique mixed NE.

The Nonstrategic Component

The nonstrategic component does not affect equilibrium sets.

However, payoff related properties, such as efficiency, are determined by the nonstrategic component.

A strategy profile $\mathbf{p}$ is Pareto optimal if there does not exist any other strategy profile $\mathbf{q}$ such that,

$$u^m(\mathbf{q}) \geq u^m(\mathbf{p}), \qquad \text{for all } m \in \mathcal{M}$$
$$u^k(\mathbf{q}) > u^k(\mathbf{p}), \qquad \text{for some } k \in \mathcal{M}.$$

Theorem

Given a game $\mathcal{G}$, there exists a game $\hat{\mathcal{G}}$ with the same potential and harmonic components such that the set of pure Nash equilibria of $\hat{\mathcal{G}}$ coincides with the set of Pareto optimal strategy profiles.

Near-Potential Games: Equilibria and Dynamics

Decomposition allows for approximations with a potential game.

- Component in $\mathcal{P} \oplus \mathcal{N}$, is the best approximation with a potential game.

Near-potential game: A game with a "small" harmonic component $\mathcal{G} - \hat{\mathcal{G}}$.

Properties of $\mathcal{G}$ can be studied using those of the potential game $\hat{\mathcal{G}}$:

- Any equilibrium of $\mathcal{G}$ is an ϵ-equilibrium of $\hat{\mathcal{G}}$.
- Natural learning dynamics (e.g., fictitious play, better/best response) converge to an ϵ-equilibrium set.
- where $\epsilon \propto \mathcal{G} - \hat{\mathcal{G}}$

Static and dynamic properties of a game can be (approximately) characterized using its potential + nonstrategic components.

Approach for Analyzing Dynamics

Assume that $\mathcal{G}$ is "close" to a potential game $\hat{\mathcal{G}}$. Then, the dynamics in $\mathcal{G}$ can be analyzed by:

- Characterizing the outcome of dynamics in $\hat{\mathcal{G}}$.
- Using this characterization and the fact that the two games are close, to approximately characterize the outcome of dynamics in $\mathcal{G}$.

The potential function of $\hat{\mathcal{G}}$ provides a natural Lyapunov function for analyzing both games.

- The difference between the potential component and the original game is a perturbation on this system.

Discrete Time Better Response Dynamics

Discrete time better response dynamics:

- At each time step t a single player is chosen at random, using a probability distribution with full support over the set of players.
- Say player m is chosen for update and $\mathbf{r} \in E$ is the current strategy profile.
- Player m does not modify its strategy if $u^m(\mathbf{r}) = \max_{q^m} u^m(q^m, \mathbf{r}^{-m})$, and otherwise it updates its strategy to a strategy in $\{q^m | u^m(q^m, \mathbf{r}^{-m}) > u^m(\mathbf{r})\}$, chosen uniformly at random.

Convergence in potential games follows by observing that with each update the potential strictly increases.

Discrete Time Better Response Dynamics – 2

Theorem

Consider some game $\mathcal{G}$ and let $\hat{\mathcal{G}}$ be a close potential game, such that $d(\mathcal{G}, \hat{\mathcal{G}}) = \delta$. In $\mathcal{G}$, the trajectory of the better-response dynamics is contained in the ϵ-equilibrium set, after finite time with probability 1, where $\epsilon = \delta|E|$.

- The update process can be modeled with a Markov chain.
- With probability 1, convergence to a recurrent class in finite time.
- Recurrent classes can be characterized with better response cycles.

Logit-Response Dynamics

Logit-Response Dynamics:

- At each step t, a single player is chosen at random for updating its strategy.
- Say player m is chosen for update and $\mathbf{r} \in E$ is the strategy profile.
- Logit response dynamics is the update process, where this player chooses $q^m \in E^m$ with probability,

$$P_\tau^m(q^m|\mathbf{r}) = \frac{e^{\frac{1}{\tau}u^m(q^m,\mathbf{r}^{-m})}}{\sum_{p^m \in E^m} e^{\frac{1}{\tau}u^m(p^m,\mathbf{r}^{-m})}}.$$

If $\tau \to 0$, players choose their best responses at each update.

Logit-Response Dynamics – 2

- Logit-response dynamics can be modeled with an irreducible Markov-chain with transition probabilities:

$$P_\tau(\mathbf{p} \to \mathbf{q}) = \begin{cases} \alpha_m P_\tau^m(q^m|\mathbf{p}) & \text{if } \mathbf{q}^{-m} = \mathbf{p}^{-m} \\ 0 & \text{otherwise.} \end{cases}$$

- Let μ_τ be the stationary distribution. We refer to a strategy profile $\mathbf{q}$ such that $\lim_{\tau\to 0} \mu_\tau(\mathbf{q}) > 0$ as a stochastically stable strategy profile.
- In potential games, the stochastically stable states are the maximizers of the potential function.

Logit-Response Dynamics – 3

Theorem

Let $\mathcal{G}$ be a game and $\hat{\mathcal{G}}$ be a close potential game with potential ϕ, such that $d(\mathcal{G}, \hat{\mathcal{G}}) = \delta$. Then, the stochastically stable strategy profiles of $\mathcal{G}$ are

(i) *contained in $S = \{\mathbf{p} | \phi(\mathbf{p}) \geq \max_{\mathbf{q}} \phi(\mathbf{q}) - |E|\delta\}$,*

(ii) *ϵ-equilibria of $\mathcal{G}$, where $\epsilon = (|E| + 1)\delta$.*

- Proof relies on characterization using resistance trees [Young, 1993], [Marden and Shamma 2008].

- In near potential games, stochastically stable strategy profiles
  - approximately maximize the potential,
  - and are contained in an approximate equilibrium set.

Discrete Time Fictitious Play

In DT fictitious play, agents maintain empirical frequencies of other players' actions

- They assume (incorrectly) that others are playing randomly and independently according to empirical frequencies
- At each time instant t, every player m, chooses a strategy p_t^m such that

$$p_t^m \in \arg\max_{q^m \in E^m} u^m(q^m, \mu_t^{-m}).$$

- Empirical frequencies converge to a (mixed) equilibrium in potential games.
- Observe that the evolution of the empirical frequencies can be captured by the following equation:

$$\mu_{t+1} = \frac{t}{t+1}\mu_t + \frac{1}{t+1}I_t,$$

Discrete Time Fictitious Play – 2

Lemma

Consider a game $\mathcal{G}$ and let $\hat{\mathcal{G}}$ be a near-potential game with a potential function ϕ, such that $d(\mathcal{G}, \hat{\mathcal{G}}) = \delta$. Assume that at time $T > 0$, the empirical frequency vector μ_T is outside the ϵ equilibrium set of $\mathcal{G}$. Then

$$\phi(\mu_{T+1}) - \phi(\mu_T) \geq \frac{\epsilon - M\delta}{T+1} + O\left(\frac{1}{T^2}\right)$$

- Taylor expansion of ϕ around μ_T.
- ϕ is a multilinear function of its argument.
- Implication: if $\epsilon > M\delta$, for sufficiently large T, the potential function "increases" when evaluated at μ_T.

Discrete Time Fictitious Play – 3

Theorem

Consider a game $\mathcal{G}$ and let $\hat{\mathcal{G}}$ be a near-potential game, with potential function ϕ, such that $d(\mathcal{G}, \hat{\mathcal{G}}) = \delta$. In $\mathcal{G}$, the empirical frequency of fictitious play converges to the set

$$C = \{\mathbf{x} \in \Delta E | \phi(\mathbf{x}) \geq \min_{\mathbf{y} \in \mathcal{X}_{M\delta}} \phi(\mathbf{y})\}.$$

- Assume that $t \gg 0$ and $\mu_t \notin \mathcal{X}_{M\delta}$. Then

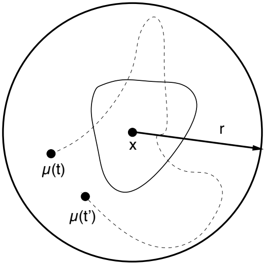
$$\phi(\mu_{t+1}) - \phi(\mu_t) \geq \frac{\epsilon' - M\delta}{2(t+1)} > 0.$$

- Thus $\mathcal{X}_{M\delta}$ is reached in finite time.

Discrete Time Fictitious Play – 4

Stronger version of the result can be obtained for games with finitely many equilibria:

- For small δ, $\mathcal{X}_{M\delta}$ has disjoint components.
- For sufficiently large t, μ_t can visit only one such component, containing equilibrium x.
- Using $\phi(\mu_{T+1}) - \phi(\mu_T) \geq \frac{\epsilon - M\delta}{T+1} + O\left(\frac{1}{T^2}\right)$, it is possible to construct a neighborhood r of x that contains the trajectory of μ_t for $t \geq t'$.



Summary

We provide a decomposition of games into potential, harmonic and nonstrategic components.

- Using the decomposition framework we identify classes of games with distinct properties.
- We establish interesting equilibrium properties of harmonic games.
- Approximation of a game with a potential game provides a systematic framework for approximate characterization of static and dynamic properties in arbitrary games.

Future directions:

- Dynamic properties of harmonic games.
- Designing incentive mechanisms for guaranteeing desirable limiting behavior in near-potential games.

Questions?