Multimarket Contact under Private Monitoring: A Belief-Free Approach

Tadashi Sekiguchi
Institute of Economic Research, Kyoto University

December 2, 2015

Foreword

- work in progress, very preliminary, and incomplete in many respects

- my current purpose is just to explain what I am going to do, rather than to sell what I have done so far

- hope it sounds interesting

Repeated Prisoners' Dilemma (PD)

		player 2	
		C	D
player 1	C	$1, 1$	$-l, 1+g$
	D	$1+g, -l$	$0, 0$

- we assume $g > 0$, $l > 0$, and $g - l < 1$

  ♠ defection is dominant, but mutual cooperation uniquely maximizes total payoffs

- perpetual play of mutual cooperation is an equilibrium outcome if

$$\delta \geq \frac{g}{g+1}$$

  ♠ the equilibrium payoff (average, discounted total stage payoffs) is 1

A Variation: Imperfect Public Monitoring

		player 2	
		C	D
player 1	C	$1, 1$	$-l, 1+g$
	D	$1+g, -l$	$0, 0$

- let us now assume that the other player's action is unobservable
- there is an imperfect signal of the actions, success (G) or failure (B)
- player i's stage payoff depends on his action $a_i \in \{C, D\}$ and the signal $y \in \{G, B\}$, denoted by $v_i(a_i, y)$
- we assume

$$\text{Prob}(B|C,C) < \text{Prob}(B|C,D) = \text{Prob}(B|D,C)$$

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{player 2}} \\
 & & C & D \\
\cline{3-4}
\text{player 1} & C & 1,1 & -l, 1+g \\
\cline{3-4}
 & D & 1+g, -l & 0,0 \\
\cline{3-4}
\end{array}
$$

- here, the payoff matrix expresses the *expected payoffs* given an action pair (a_1, a_2)

♠ it thus holds that

$$u_i(a_1, a_2) = \text{Prob}(G|a_1, a_2)v_i(a_i, G) + \text{Prob}(B|a_1, a_2)v_i(a_i, B)$$

♠ you can't say, for example, a cooperator can tell the other guy's action, depending on whether his payoff is 1 or $-l$

Multimarket Contact

	C	D
C	$1, 1$	$-l, 1+g$
D	$1+g, -l$	$0, 0$

$\ldots$

	C	D
C	$1, 1$	$-l, 1+g$
D	$1+g, -l$	$0, 0$

- there are M games of this kind, simultaneously played in each period

- the signals are assumed to be independent across the M games

- can an equilibrium *per-game* payoff of the M repeated games exceed that of a single repeated game?

    ♠ easy to attain the same value (playing a single-game equilibrium independently)

Multimarket Contact in Its True Sense

- in this paper, multimarket contact is M repeated prisoners' dilemma, but this is of course not its true meaning

- the original question is: does the very fact that oligopolistic firms simultaneously compete over multiple markets facilitate collusion in itself?

- multimarket contact is abundant in reality

  ♠ nationwide firms compete over local markets (airlines)

  ♠ conglomerates compete over several industries

- important implications on evaluation of mergers and interpretation of concentration indexes

Some Existing Debates

- a famous claim: multimarket contact facilitates collusion

  ♠ Corwin Edwards, in his 1957 testimony before the U.S. Senate Anti-Trust and Monopoly Subcommittee:

  > The multiplicity of their contacts may blunt the edge of their competition. A prospect of advantage from vigorous competition in one market may be weighted against the danger of retaliatory forays by the competitor in other markets.

  ♠ cited by, for instance, Scherer's textbook on IO and authors on multimarket contact

- one theoretical rebuttal: the "irrelevance result" by Bernheim and Whinston (1990)

- their point: why not vigorously compete in *all* markets?

  ♠ Edward's deviation, where a firm price-cuts only in one market, is not an optimal deviation

- if all markets are identical, then multimarket contact simply multiplies both the deviation gain and future punishments by the same size

  ♠ therefore the *per market* profit should not change

- Bernheim and Whinston assumes perfect monitoring

Support for Edwards' Argument

- the irrelevance result does not extend to imperfect public monitoring

- Matsushima (2001): for any discount factor where full collusion is sustainable in its perfect monitoring counterpart (except for the threshold level), the per-market profit under full collusion is approximately sustained if the number of markets is sufficiently large

  ♠ conglomerates can collude as if monitoring were perfect

- Kobayashi and Ohta (2012): the most collusive per-market PPE payoff when the discount factor is arbitrarily close to 1 is increasing in the number of markets

  ♠ **a free-lunch result**: the increase in the *total* payoffs equals the level when the added market has perfect monitoring

- Sekiguchi (in preparation): given a number of markets and a discount factor where non-zero collusion is sustainable, adding one more market never decreases the per-market payoff under the most collusive PPE

  ♠ a generic result: for almost-all those situations, the per-market payoff increases

- adding two or more markets under those situations always increases the per-market payoff

  ♠ cover the case of moderate discounting, where the free-lunch result does not hold

  ♠ I am just eating ordinarily-priced lunch

Today's Talk: Multimarket Contact under Private Monitoring

- based on Iwasaki, Sekiguchi, Yamamoto and Yokoo (in prepare-tion)

- the same issue, for the framework of imperfect private monitoring

- the other player's action is unobservable

- this time, there is an imperfect and private signal of the other player's action, which is either g or b

  ♠ the other player's signal is unobservable

- player i's payoff depends on his action $a_i \in \{C, D\}$ and his signal $y_i \in \{G, B\}$, denoted by $v_i(a_i, y_i)$

		player 2	
		C	D
player 1	C	$1, 1$	$-l, 1+g$
	D	$1+g, -l$	$0, 0$

- let $p_i(y_i|a_1, a_2)$ be the probability with which player i's signal is y_i under an action pair $a = (a_1, a_2) \in \{C, D\}^2$

  ♠ the marginal distribution of the signal pairs

- again, the payoff matrix expresses the expected payoffs

$$u_i(a_1, a_2) = p_i(g|a_1, a_2)v_i(a_i, g) + p_i(b|a_1, a_2)v_i(a_i, b)$$

- we assume a single parameter $p \in (1/2, 1)$ describes the monitoring structure: for any i, any y_i and any a

$$p_i(y_i|a) = \begin{cases} p & \text{if } (a_j, y_i) \in \Big\{(C, g), (D, b)\Big\} \\ 1 - p & \text{otherwise} \end{cases}$$

- consider an infinitely repeated game where M PDs with imperfect private monitoring are simultaneously played

- the information player i gets in one period is an element of $I_i \equiv \{C, D\}^M \times \{g, b\}^M$

- player i's private history at period t ($t \geq 0$) is an element of $H_i^t \equiv (I_i)^t$

  ♠ H_i^0 is a singleton

- let $H_i = \cup_{t \geq 0} H_i^t$, which is the set of player i's histories

- player i's strategy is a mapping from H_i to the set of probability distributions over $\{C, D\}^M$

- a player's payoff of a strategy pair is the average, δ-discounted sum of the stage-game payoffs

Two-State Pure-Action Automaton Strategy

- we will limit attention to a particular equilibrium concept by a particular class of strategies

  ♠ not limiting the strategy space

- the strategy is characterized by the following parameter set

  (i) two elements of $\{C, D\}^M$: denoted by a_R and a_P

  (ii) two functions from $\{C, D\}^M \times \{g, b\}^M$ to $[0, 1]$: denoted by ϵ^R and ϵ^P

  ♠ called state transition functions

- it corresponds to the following automaton strategy

(i) the player is always at either state R or state P

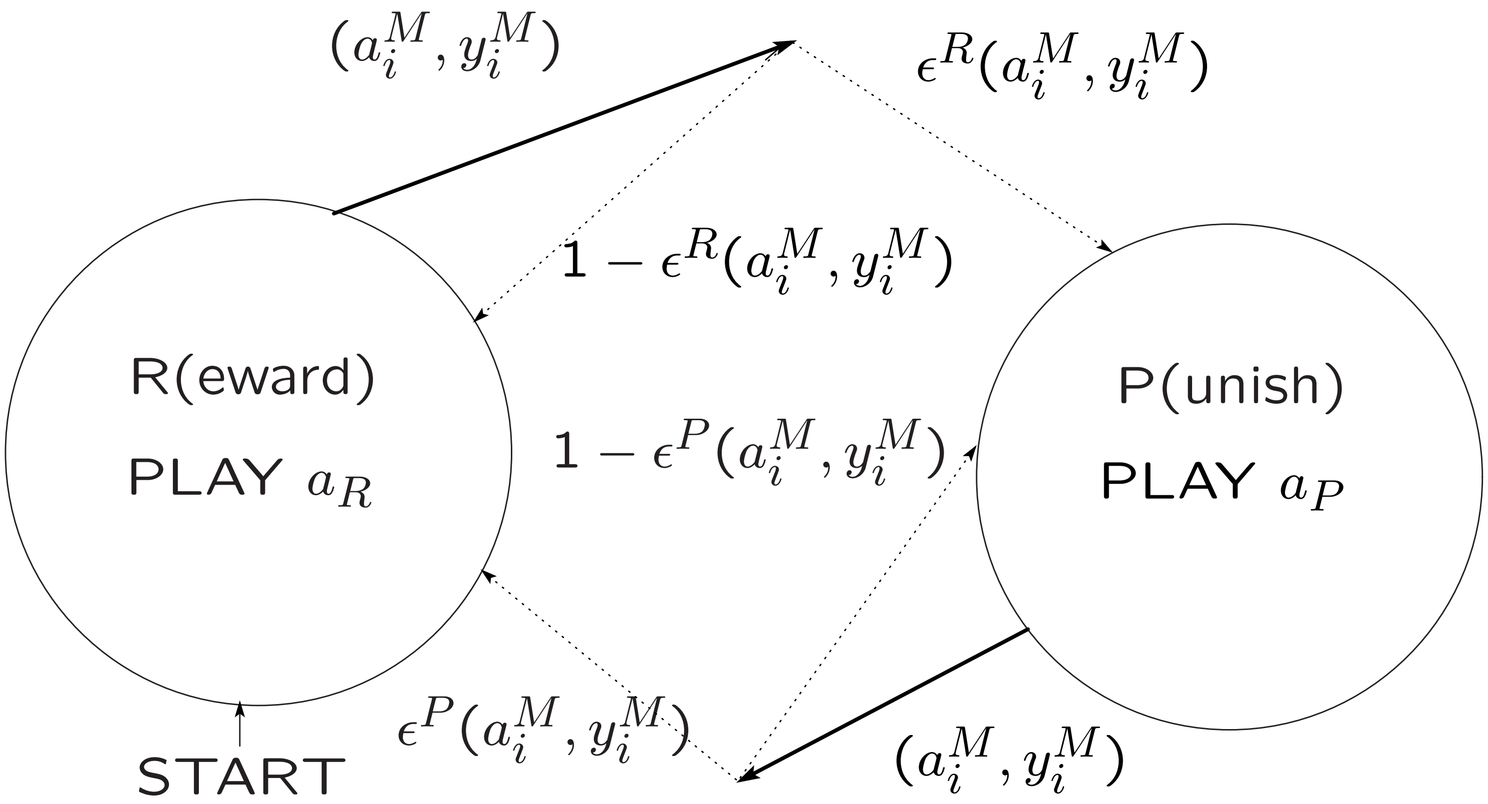
♠ the initial state is R

(ii) the player plays a_R at state R, and plays a_P at state P

(iii) if the player is at state R, and if the current outcome is $(a_i^M, y_i^M) \in \{C, D\}^M \times \{g, b\}^M$, the state in the next period is P with probability $\epsilon^R(a_i^M, y_i^M)$

(iv) if the player is at state P, and if the current outcome is $(a_i^M, y_i^M) \in \{C, D\}^M \times \{g, b\}^M$, the state in the next period is R with probability $\epsilon^P(a_i^M, y_i^M)$

- like this:



- since the state transition is stochastic, it is not a pure strategy in general

Belief-Free Equilibrium

- Ely and Valimaki (2002), Ely, Horner and Olszewski (2005)

Definition 1 *A two-state pure-action automaton strategy pair is a belief-free equilibrium if any player's continuation strategy at any state is optimal against the other player's continuation strategy at any state.*

- a belief-free equilibrium is a sequential equilibrium

- even if a player were to learn the other player's state, his continuation strategy continues to be optimal

  ♠ make the task of computing beliefs irrelevant

The Case of $M = 1$: Ely-Valimaki Equilibrium

- hereafter, our solution concept is a belief-free equilibrium by two-state pure-action automaton strategies

- the following result is due to Ely and Valimaki (2002)

Proposition 1 *Suppose $M = 1$. If*

$$\delta\Big[2p - 1 - (1-p)(g+l) + \max\{g,l\}\Big] \geq \max\{g,l\} \tag{1}$$

holds, there exists a symmetric belief-free equilibrium which gives each player

$$1 - \frac{(1-p)g}{2p-1}. \tag{2}$$

Comments: Ely-Valimaki Equilibrium

- the equilibrium strategy has the following parameter set

$$a_R = C, \quad a_P = D, \quad \epsilon^R(\cdot, g) = \epsilon^P(\cdot, b) = 0.$$
$$\epsilon^R(\cdot, b) = \frac{(1-\delta)g}{\delta\Big\{2p - 1 - (1-p)(g+l)\Big\}},$$
$$\epsilon^P(\cdot, g) = \frac{(1-\delta)l}{\delta\Big\{2p - 1 - (1-p)(g+l)\Big\}}$$

- (1) may fail even when δ is arbitrarily large

  ♠ if $2p - 1 \leq (1-p)(g+l)$, no δ satisfies (1)

- if p is close to one 1 (nearly perfect monitoring), such an equilibrium exists and its payoff is nearly efficient

- the equilibrium payoff has a particular form:

$$1 - \frac{(1-p)g}{2p-1}$$

- the payoff of mutual cooperation minus the welfare loss due to imperfect observability

- the size of the loss is proportional to g, and inversely proportional to the likelihood ratio of moving from state R to P

$$\frac{1-p}{2p-1} = \frac{1}{\frac{p}{1-p} - 1},$$

$$\frac{p}{1-p} - 1 = \frac{p_1(b|\cdot, D)}{p_1(b|\cdot, C)} - 1 = \frac{p_1(b|\cdot, D) - p_1(b|\cdot, C)}{p_1(b|\cdot, C)}$$

- the private monitoring counterpart of the Abrue-Milgrom-Pearce value (for the most cooperative equilibrium payoff under public monitoring)

EV-Type Equilibrium with M Markets

- let us examine presence of multimarket contact effects

  ♠ is there any equilibrium per-market payoff greater than the EV equilibrium payoff?

- by the way, it is not so obvious whether the EV equilibrium payoff can be sustained

  ♠ if $M \geq 2$, independent play of the EV equilibrium strategy is not a two-state automaton strategy

- the following result affirmatively answer this question

Proposition 2 *For any M, if (1) holds, a symmetric belief-free equilibrium exists with its payoff*

$$M\left\{1 - \frac{(1-p)g}{2p-1}\right\}.$$

- the equilibrium strategy has the following parameter set

$$a_R = (C, C, \ldots, C), \quad a_P = (D, D, \ldots, D)$$
$$\epsilon^R(\cdot, y_i^M) = \frac{(1-\delta)gk}{M\delta\Big\{2p - 1 - (1-p)(g+l)\Big\}},$$
$$\epsilon^P(\cdot, y_i^M) = \frac{(1-\delta)l(M-k)}{M\delta\Big\{2p - 1 - (1-p)(g+l)\Big\}}$$

♠ k is the number of bad signals in y_i^M

- the actions shifts between all C to all D

♠ accordingly, the marginal transition probabilities of a bad signal are the benchmark levels divided by M

- what kind of modification improves the EV-type equilibrium payoff?

- among all two-state pure-action automaton strategies such that $a_R = (C, C, \dots, C)$, the above strategy has the following two features:

  (i) $a_P = (D, D, \dots, D)$

  (ii) the transition probability from R to P is proportional to the number of bad signals

- the next result indicates that part (ii) is essential

Proposition 3 *Any equilibrium payoff by a strategy satisfying* $a_R = (C, C, \dots, C)$ *and the condition* (ii) *above is at most*

$$M\left\{1 - \frac{(1-p)g}{2p-1}\right\}.$$

- if $M = 2$, the condition (i) is a constraint, too

Proposition 4 *Let $M = 2$. Then any belief-free equilibrium payoff with $a_R = (C, C)$ and $a_P = (D, D)$ is at most*

$$2\left\{1 - \frac{(1-p)g}{2p-1}\right\}.$$

- in order to verify multimarket contact effects under $M = 2$, we need to set $a_P = (C, D)$ and specify more complicated transition probabilities

Multimarket Contact Effects when $M = 2$

Proposition 5 *Let $M = 2$. If*

$$\delta \geq \frac{\max\left\{g, (1-p)(g+l)\right\}}{p\max\left\{g, (1-p)(g+l)\right\} + (1-p)(2p-1)} \tag{3}$$

holds, there exists a symmetric belief-free equilibrium whose payoff is

$$2 - \frac{(1-p)g}{2p-1}.$$

- a free-lunch result

- if (3) holds, the equilibrium condition for the EV equilibrium also holds

- (3) may fail even if δ is very large

  ♠ if $2p - 1 \leq \max\left\{g, (1-p)(g+l)\right\}$, no δ satisfies (3)

- even nearly perfect monitoring, (3) may fail

  ♠ if $g \geq 1$, no p and δ satisfy (3)

- the equilibrium strategy is such that $a_R = (C, C)$ and $a_P = (C, D)$

  ♠ the transition probabilities depend on $\max\left\{g, (1-p)(g+l)\right\}$

- if max $\Big\{g, (1-p)(g+l)\Big\} = g$ (namely, $pg \geq (1-p)l$), we set

$$\epsilon^R(\cdot, gg) = \epsilon^R(\cdot, gb) = \epsilon^R(\cdot, bg) = 0,$$
$$\epsilon^R(\cdot, bb) = \frac{(1-\delta)g}{\delta(1-p)(2p-1-g)}$$

and

$$\epsilon^P(\cdot, gg) = \frac{(1-\delta)(g+l)}{\delta(2p-1-g)},$$
$$\epsilon^P(\cdot, gb) = \frac{(1-\delta)\Big\{pg-(1-p)l\Big\}}{\delta p(2p-1-g)},$$
$$\epsilon^P(\cdot, bg) = \epsilon^P(\cdot, bb) = 0$$

- the state shifts form R to P, only when all signals are bad

♠ the most collusive equilibrium under public monitoring has the same structure (Kobayashi-Ohta)

- if $\max\Big\{g,(1-p)(g+l)\Big\}=(1-p)(g+l)$ (namely, $pg \le (1-p)l$), we set

$$\epsilon^R(\cdot,gg)=\epsilon^R(\cdot,gb)=\epsilon^R(\cdot,bg)=0,$$
$$\epsilon^R(\cdot,bb)=\frac{(1-\delta)g}{\delta(1-p)\Big\{2p-1-(1-p)(g+l)\Big\}}$$

and

$$\epsilon^P(\cdot,gg)=\frac{(1-\delta)(g+l)}{\delta\Big\{2p-1-(1-p)(g+l)\Big\}},$$
$$\epsilon^P(\cdot,bg)=\frac{(1-\delta)\Big\{(1-p)l-pg\Big\}}{\delta(1-p)\Big\{2p-1-(1-p)(g+l)\Big\}},$$
$$\epsilon^P(\cdot,gb)=\epsilon^P(\cdot,bb)=0$$

- again, the same structure as the Kobayashi-Ohta equilibrium

- the two cases differ only in the transition probabilities from P to R

Remarks

- when the other player is at state R under the equilibrium, any one-shot deviation where a player selects D in one PD attains the same payoff as conforming to his strategy

- the one-shot deviation where he selects D in both PDs is sub-optimal

  ♠ given that he is punished only when the other player's signal is all bad, defection in one PD makes him one step closer to the punishment, which makes him reluctant to defect in the other, too

- thus we virtually eliminate the full defection from the incentive constraints, which helps to support a higher equilibrium payoff

- in order to attain the higher value, however, we need to carefully design the transition probabilities from P to R, too

  ♠ otherwise, the punishment is not severe enough to provide incentives to cooperate

- when the other player is at state P under the equilibrium, the only suboptimal one-shot deviation is to defect in the first PD and to cooperate in the second

  ♠ the player is indifferent among all other one-shot deviations

- thus we virtually eliminate this deviation from the incentive constraints, which helps to design an effective punishment