Ground State Patterns and Phase Transitions for Spin-1 Bose-Einstein Condensates, Numerics and Analysis

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Abstract

The ultra-cold non-interacting Boson gases form so-called Bose-Einstein condensates (BECs). It is desirable to have a rigorous mathematical theory for such quantum phenomenon. In this talk, I will report our numerical and analytical results on the ground state patterns and their phase transitions of spin-1 BECs in a uniform magnetic field. The macroscopic model for spin-1 BECs is a generalized Gross-Pitaevskii equation, which is a nonlinear Schrödingier system.

For numerical studies, I will present a complete investigation of the ground state patterns and phase diagrams of the spin-1 BECs. Two types of phase transitions are found. The first type is a transition from a two-component (2C) state to a three-component (3C) state. The second type is a symmetry breaking in the 3C state. Then, a phase separation of the spin components follows. In the semi-classical regime, It is found that these two phase transition curves are gradually merged.

For analytic studies, we provide a rigorous proof for the phase transition from 2C to 3C for antiferromagnetic systems. The key is a mass-redistribution lemma, that is, a redistribution of the mass densities between different components will decrease the kinetic energy. In the semi-classical regime, the Thomas-Fermi approximation and the Γ-convergence theory are adopted to study the ground state patterns and their phase transitions. A complete semi-classical theory will be reported.

References

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