Dynamics Characteristic of Solitons for Davey-Stewartson System

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Abstract: We study the Cauchy problem of the Davey-Stewartson system:

$$iu_t + \triangle u + |u|^{p-1}u + E(|u|^2)u = 0, \quad t \in [0;T), \quad x \in R^N \qquad (GDS)$$

where $1 < p < \frac{N+2}{(N-2)^+}$, which appears as mathematical models for the evolution of shallow-water waves having predominant direction of travel. We shall investigate the variational characteristic of various solitons to (GDS) and use them to study the dynamical properties the Cauchy problem of (GDS). First, when $N = 2, 3$, by constructing a type of cross-constrained variational problem and establishing so called cross-constraint manifolds of the evolution flow, we derive a sharp thresholds for blow-up and global existence of the solutions to the Cauchy problem for (GDS) provided $1 + \frac{4}{N} < p < \frac{N+2}{(N-2)^+}$. Especially, for $N = 2$ and $N = 3$, by using the profile decomposition of the bounded sequences in H^1, we get the generalized Gagliardo-Nirenberg inequalities and the sharp thresholds of the existence of blow-up solutions of (GDS) for $3 < p < +\infty$ and $1 + \frac{4}{3} \le p < 5$, respectively. Then, we study the dynamical properties of blow-up solutions. We obtain the concentration of blow-up solutions of (GDS) for $1 < p \le 3$ and the limiting profile of blow-up solutions of (GDS) for $N = 2$ and $p = 3$. Finally, we study the stability of standing waves of (GDS).