

Orbit type filtrations of torus manifolds and combinatorics of simplicial posets

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Let X be a closed $2n$ -manifold on which the compact n -torus T acts in a locally standard manner. The orbit space X/T is a manifold with corners; let S be the simplicial poset dual to X/T . Consider a filtration of X by torus invariant subsets X_i , the union of all i -dimensional orbits, and take the homological spectral sequence associated with this filtration. In the classical case, i.e. when X is a toric or quasitoric manifold, this spectral sequence collapses at the second page, all entries of $E_{p,q}^\infty$ are concentrated at the diagonal $p = q$, and their ranks are h -numbers of S . The same holds when all faces of the orbit space are acyclic. In more general situations the spectral sequence does not collapse at a second page. Nevertheless sometimes it can be described in full.

In combinatorial commutative algebra there are notions of h' - and h'' -numbers of simplicial manifolds which generalize h -numbers of spheres. We obtain these numbers as the ranks of certain terms of the spectral sequence. In particular, this gives a topological proof that h'' -numbers of Buchsbaum posets are nonnegative, which was previously proved algebraically by Novik and Swartz.