Embedding theorems for quasitoric manifolds.

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The talk is based on joint work with Andrey Kustarev.

The central subject are theorems on equivariant embeddings of quasitoric manifolds in the terms of combinatorial data.

Our initial task was to improve the following classical results in the case of quasitoric manifolds:

Theorem 0.1 (Mostov-Palais)**.** *Let M be a compact smooth manifold with a smooth action of compact Lie group G. Then there exists a smooth embedding $M \to \mathbb{R}^N$ equivariant with respect to a linear representation $G \to GL(N, \mathbb{R})$.*

Theorem 0.2 (Kodaira)**.** *Let M be a compact complex manifold with a positive holomorphic linear bundle. (For example, M possesses a rational Kähler form.) Then there exists a complex-analytic embedding $M \to \mathbb{C}P^N$.*

Theorem 0.3 (Gromov-Tishler)**.** *Let M be a compact symplectic manifold with an integral symplectic form ω. Then there exists a symplectic embedding of M to $\mathbb{C}P^N$ with the standard symplectic form.*

Using the combinatorial data we describe the corresponding structures on the underlying quasitoric manifolds and obtain our results. For example:

Theorem 0.4. *Let M be a smooth quasitoric manifold. Then the moment map $\pi\colon M \to P \subset \mathbb{R}^n$ can be extended to a real-algebraic embedding $M \subset \mathbb{R}^n \times \mathbb{C}^q$ equivariant with respect to the representation $\mathbb{T}^n \to \mathbb{T}^q$.*

Here the number q does not exceed the number $f_1(P)$ of edges of P. We show that in many cases $q < f_1(P)$.

In the case of 3-dimension Stasheff polytope K_5 we describe explicitly the 6-dimension quasitoric manifold with equivariant embedding $M \subset \mathbb{R}^3 \times \mathbb{C}^6$. Note that $f_1(K_5) = 21$.

We construct the projective map $M \to \mathbb{C}P^{q-1}$ for any quasitoric manifold M and describe the conditions on combinatorial data when this map is embedding.