

COMBINATORICS OF MINIMALLY NON-GOLOD SIMPLICIAL COMPLEXES AND THEIR TORIC SPACES

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The notion of a Golod ring was introduced firstly by T. Gulliksen and G. Levin [7] for Noetherian local rings as a generalization of a class of local rings with rational Poincaré series studied by E. S. Golod [4] and is now a classical object of study in commutative algebra (homology of local rings). It appears in the graded case as a Stanley–Reisner ring (or face ring) $k[K]$ of a simplicial complex K on m vertices over a ring of integers or a field k having all Massey operations in $Tor_{k[v_1, \dots, v_m]}(k[K], k)$ trivial, and has already found a topological interpretation in toric topology. Due to V. M. Buchstaber and T. E. Panov theorem on the cohomology ring of a moment-angle complex $\mathcal{Z}_K$ [2] and the results of A. Berglund and M. Jollenbeck [1], it is just the case when the cup product and all higher Massey products in the ring $H^*(\mathcal{Z}_K; k)$ are trivial. For some special classes of simplicial complexes it was shown in [5], [6], [8], [9] that their face rings $k[K]$ are Golod ones and in all those cases (if integral homology groups of all induced subcomplexes K_I , $I \subset [m]$ in K are torsion free) the corresponding moment-angle complexes have homotopy types of wedges of spheres. We introduce a class of Golod complexes K (over any field) with all their induced subcomplexes having free homology groups, in which there is a $\mathcal{Z}_K$ that is not homotopy equivalent to any wedge of spheres although being a co-H-space.

In [1] the notion of a minimally non-Golod simplicial complex K (over k) was introduced, that is $k[K]$ is not Golod itself but deleting of any vertex from K turns the face ring into a Golod one. We consider mainly the case when K is a triangulated sphere or can be obtained from a minimal triangulation of a manifold by a sequence of certain combinatorial operations. For rationally minimally non-Golod sphere triangulations K we discuss the rational homotopy types, Poincaré series and formality of the moment-angle manifold $\mathcal{Z}_K$.

In the polytopal triangulation case, for many of the corresponding polytopes P (among them are the duals to vertex truncations of one or a product of two simplices as well as even dimensional neighbourly polytopes, combinatorially different from simplices) a description of diffeomorphism types of their moment-angle manifolds $\mathcal{Z}_P$ is well known in toric topology, see [3]. These manifolds are connected sums of sphere products with two spheres in each product. Their face rings were shown to be minimally non-Golod (over any field) in [10]. We introduce infinite families of minimally non-Golod complexes with all their induced subcomplexes having free homology groups but their moment-angle complexes being not homotopy equivalent to connected sums of products of spheres.

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