Abstract

Title: The inclusion of the configuration space into the product for the sphere and the projective plane

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Given a space X we denote by $F_n(X)$ the nth configuration space of X, which is the subset of consisting of the elements $(x_1, \ldots, x_n)$ of the Cartesian product $X \times \cdots \times X$ (n copies) for which $x_i \neq x_j$ for $i \neq j$. In order to understand better $F_n(X)$, we compare certain properties of the two spaces $F_n(X)$ and $X \times \cdots \times X$ (n copies), such as their fundamental group and their homotopy type. In the latter case, this corresponds to determining the homotopy fibre of the inclusion, i.e. a space $F(\iota)$ such that

$$F(\iota) \to F_n(X) \to X \times \cdots \times X$$

looks a fibration. The study of these questions is especially interesting when X is a surface. The two problems are related and they may be equivalent or not, depending on X. In this work, we study the cases where X is either the sphere or the projective plane. The groups are determined by means of a presentation, and a few of their properties are explored. Concerning the homotopy fibre of the inclusion, the result is given in terms of the 2-sphere, loop spaces and equivariant configuration spaces, a concept introduced in [FX]. The results above contain a solution of a problem for X either S^2 or RP^2, which in the case of X a closed surface different from S^2 and RP^2 it was proposed by [Bi] and solved in [Gol]. One type of result that we get, is illustrated in the following statement: *Theorem: Let* $n \geq 2$. *The homotopy fibre* I_ι *of the inclusion map* $\iota : F_n(S^2) \to \Pi_1^n S^2$ *has the homotopy type of* $F_{n-1}(D^2) \times \Omega(\Pi_1^{n-1} S^2)$*, or equivalently of* $K(P_{n-1}, 1) \times \Omega(\Pi_1^{n-1} S^2)$*, where* $\Omega(\Pi_1^{n-1} S^2)$*, denotes the loop space of* $\Pi_1^{n-1} S^2$*.*

References

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