Title: The Topology and Geometry of Moment-angle Manifolds

Abstract: Let K_1 and K_2 be two simplicial (n-1)-spheres, Z_{K_1} and Z_{K_2} be the corresponding moment-angle manifolds. We prove that the moment-angle manifold Z corresponding to $K_1 \# K_2$ is homeomorphic to $\mathcal{G}^{m_2-n}(Z_{K_1}) \# \mathcal{G}^{m_1-n}(Z_{K_2}) \# \overset{m_1+m_2-2n-1}{\#} (\binom{m_1+m_2-2n}{j+1} - \binom{m_1-n}{j+1} - \binom{m_2-n}{j+1}) S^{j+2} \times S^{m_1+m_2-j-2}$. The operation \mathcal{G} on an manifold M^k is defined as: $\mathcal{G}(M^k) = \partial[(M^k - int(D^k)) \times D^2]$. If K_1 and K_2 are duals of two n-simple polytopes, the homeomorphism can be diffeomorphism. Besides, we consider the question of which moment-angle manifolds can be endowed with metrics of positive Ricci curvature. We will give some preliminary results and some conjectures on this problem.

Remarks: According to our results, while studying the topology of moment-angle manifolds corresponding to the simplicial spheres, we could only consider the indecomposable ones. S-ince now, we don't know many examples of manifolds with positive Ricci curvature(Biquotient, connected sums of $S^{n_i} \times S^{m_i}$, Fano varieties, etc). We hope that there exist many moment-angle manifolds that can be endowed with metrics of positive Ricci curvature. We prove that moment-angle manifolds corresponding to the toric Fano polytopes can be endowed with metrics of positive Ricci curvature. Besides, We will give several conjectures on this problem.