COHOMOLOGY OF REGULAR HESSENBERG VARIETIES AND REPRESENTATIONS OF SYMMETRIC GROUPS

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Hessenberg varieties $Hess(X,h)$ are subvarieties of the flag variety $Flag(\mathbb{C}^n)$ defined by two parameters:

(1) a linear operator X on $\mathbb{C}^n$; and
(2) a Hessenberg function $h\colon [n] \to [n]$.

Their geometry and (equivariant) topology have been studied extensively since the late 1980s (see [2], [3] for example). This subject lies at the intersection of, and makes connections between, many research areas such as: geometric representation theory, combinatorics, and algebraic geometry and topology. Hessenberg varieties also arise in the study of the quantum cohomology of the flag variety (see the references in the research announcement [1]).

The Hessenberg variety $Hess(X,h)$ is called *regular nilpotent* when X is nilpotent and of full rank and *regular semisimple* when X is semisimple with distinct eigenvalues. The class of regular nilpotent Hessenberg varieties contains Peterson varieties while the class of regular semisimple Hessenberg varieties contains the toric varieties associated with Weyl chambers of type A.

In this talk, we will discuss the cohomology ring of a regular nilpotent Hessenberg variety and its relation to the cohomology ring of a regular semisimple Hessenberg variety (with a common h) in terms of representations of the symmetric group $\mathfrak{S}_n$. If time permits, I will explain its relation to the chromatic symmetric function of a graph associated with h (Shareshian-Wachs conjecture). This is a joint work with Hiraku Abe, Megumi Harada and Tatsuya Horiguchi.

References

[1] H. Abe, M. Harada, T. Horiguchi and M. Masuda, *The equivariant cohomology rings of regular nilpotent Hessenberg varieties in Lie type A: a research announcement*, Morfismos, vol. 18, no.2, 2014, pp. 51-65.

[2] M. Brion and J. Carrell, *The equivariant cohomology ring of regular varieties*, Michigan Math. J. 52 (2004), 189–203.

[3] F. De Mari, C. Procesi and A. Shayman, *Hessenberg varieties*, Trans. Amer. Math. Soc. 332 (1992), 529–534.