# TORIC TOPOLOGY OF FULLERENES

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The talk is based on the joint work with Victor Buchstaber.

Let $P$ be a simple convex 3-polytope. A *$k$-belt* is a cyclic sequence $(F_1, \dots, F_k)$ of 2-faces, such that $F_{i_1} \cap \dots \cap F_{i_r} \neq \varnothing$ if and only if $\{i_1, \dots, i_r\} \in \{\{1,2\}, \dots, \{k-1,k\}, \{k,1\}\}$. $P$ is a *flag* polytope if it is not a simplex and does not contain 3-belts.

**Theorem 1.** [Bu-Er] A simple 3-polytope $P$ is flag if and only if it is combinatorially equivalent to a polytope obtained from the cube by a sequence of edge truncations and truncations along two incident edges lying in a $k$-gonal face with $k \geqslant 6$.

A *fullerene* (see [DeSS13]) is a simple convex 3-polytope with all facets being pentagons and hexagons.

**Theorem 2.** Any fullerene $P$ is a flag polytope [Bu-Er]. It contains no 4-belts and has $12+k$ five-belts, where 12 belts surround pentagons and $k$ belts consist of hexagons with any hexagon intersecting neighbours by opposite edges. Moreover, if $k > 0$ then $P$ consists of two dodecahedral caps and $k$ hexagonal 5-belts between them.

Toric topology associates to each simple $n$-polytope $P$ with facets $F_1, \dots, F_m$ an $(m+n)$-dimensional *moment-angle manifold* $\mathcal{Z}_P$ with canonical action of the torus $T^m = (S^1)^m$. This gives a tool to study the combinatorics of simple polytopes in terms of the algebraic topology of moment-angle manifolds an visa versa. V. Buchstaber and T.Panov proved (see [Bu-Pa]) that $\mathrm{Tor}_{\mathbb{Q}[v_1,\dots,v_m]}(\mathbb{Q}[P], \mathbb{Q}) \simeq H^*(\mathcal{Z}_P, \mathbb{Q})$, where $\mathbb{Q}[P]$ is the *Stanley-Reisner ring* $\mathbb{Q}[P] = \mathbb{Q}[v_1, \dots, v_m]/(v_{i_1} \dots v_{i_k} = 0\colon F_{i_1} \cap \dots \cap F_{i_k} = \varnothing)$.

**Corollary.** For a fullerene $P$ we have $\beta^{-1,6} = \beta^{-2,8} = 0$, $\beta^{-3,10} = 12 + k$, $k \geqslant 0$. Moreover, if $k > 0$ then $P$ has the form described in Theorem 2. The product map $H^3(\mathcal{Z}_P) \otimes H^3(\mathcal{Z}_P) \to H^6(\mathcal{Z}_P)$ is trivial.

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## References

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