ON REPRESENTATIONS OF VIRTUAL BRAID GROUP BY AUTOMORPHISMS OF SOME GROUPS

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In my talk I shall speak on some results which was found jointly with M. V. Meshchadim and Yu. A. Mikhal'chishina.

We introduce some representation ψ of the virtual braid group VB_n into the automorphism group $Aut(F_{n,2n+1})$ of a free product $F_{n,2n+1} = F_n * \mathbb{Z}^{2n+1}$, where F_n is a free group and $\mathbb{Z}^{2n+1}$ is a free abelian group. This representation generalizes some other representations. In particular, the representation $\varphi_0 : VB_n \longrightarrow Aut(F_n)$ defined in [7]; the representation $\varphi_1 : VB_n \longrightarrow Aut(F_{n+1})$ defined in [5], [1] (see also, [3]); the representation $\varphi_2 : VB_n \longrightarrow Aut(F_{n,n+1})$ defined in [6]; the representation $\varphi_3 : VB_n \longrightarrow Aut(F_{n,2})$ defined in [4]. The question about faithfulness of the representations $\varphi_1, \varphi_2, \varphi_3$ was opened (the representation φ_0 is in fact a representation of the welded braid group and has non-trivial kernel for $n \geq 3$). Recently O. Chterental [2] proved that for $n > 3$ the representation φ_1 has non-trivial kernel. Using the same approach we prove

Proposition 1. *The representations φ_2 and φ_3 have non-trivial kernel for $n > 3$.*

Using any of the representation $\psi, \varphi_0, \varphi_1, \varphi_2, \varphi_3$ one can defines a group $G_\psi(L)$, $G_{\varphi_0}(L)$, $G_{\varphi_1}(L)$, $G_{\varphi_2}(L)$, $G_{\varphi_3}(L)$ of a virtual link L. A connection between these groups gives

Theorem. *The groups $G_{\varphi_0}(L)$, $G_{\varphi_1}(L)$, $G_{\varphi_2}(L)$, $G_{\varphi_3}(L)$ are homomorphic images of the group $G_\psi(L)$. If L is a virtual knot, then we have isomorphisms $G_\psi(L) \cong G_{\varphi_1}(L) \cong G_{\varphi_2}(L) \cong G_{\varphi_3}(L)$.*

A connection between the group $G_\psi(L)$ and the group of classical link L gives

Proposition 2. *If L is a classical link, then $G_\psi(L) \cong \pi_1(S^3 \setminus L) * A$ for some free abelian group A.*

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