Braids and some other groups arising in geometry and topology

Vladimir Vershinin

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Problem session, discussion

Thompson groups

Question. (Matt Brin) Explore the group ring structure of the Thompson groups and their various submonoids

Question. Is F automatic?

Question. Let $\gamma(n)$ be the growth rate of F with respect to the $\{x_0, x_1\}$ generating set, and let:

$$\Gamma(t) = \sum_{n=0}^{\infty} \gamma(n) t^n$$

Is $\Gamma(t)$ a rational function?

Braids

Question 23 in Birman's book asks to determine a basis for Brunnian braid group over S^2. A connection between Brunnian braid groups over S^2 and the homotopy groups $\pi_*(S^2)$ is given by J.A. Berrick, F.R. Cohen, Y.L. Wong and J. Wu.

J.S. Birman, *Braids, links, and mapping class groups*, Annals of Math. Studies **82**, Princeton University Press, (1975).

J.A. Berrick, F.R. Cohen, Y.L. Wong, J. Wu, *Configurations, braids, and homotopy groups*, J. Amer. Math. Soc., **19** (2006), 265–326.

Brunnian words, Brunnian and other special braids

Brunnian words.

Let G be a group generated by a finite set X. An element $g \in G$ is called *Brunnian* if there exists a word $w = w(X)$ on X with $w = g$ such that, for each $x \in X$, $g = w$ becomes a trivial element in G by replacing all entries x in the word $w = w(X)$ to be 1.

Example. Let $X = \{x_1, \ldots, x_n\}$. Then $w = [[x_1, x_2], x_3, \ldots, x_n]$ is a Brunnian word in G. Any products of iterated commutators with their entries containing all elements from X are Brunnian words.

Problem (J. Wu)

Given a group G and a (finite) generating set X, find an algorithm for detecting a Brunnian word that can NOT be given as a product of iterated commutators with their entries containing all elements from X.

If G is a free group with X a basis, then all Brunnian words are given as products of iterated commutators with their entries containing all elements from X.

Most interesting Example. Let
$G = \langle x_0, x_1, \ldots, x_n \mid x_0 x_1 \cdots x_n = 1 \rangle$, where $X = \{x_0, x_1, \ldots, x_n\}$.
For $n = 2$, $[x_1, x_2]$ is Brunnian word in G. The solution of the question for this example may imply a combinatorial determination of homotopy groups of the 2-sphere [Jie Wu].

J. Wu, *On combinatorial descriptions of the homotopy groups of certain spaces*, Math. Proc. Camb. Phil. Soc., **130** (2001), no.3, 489–513.

2. Brunnian braids.

The Brunnian braids over general surfaces have been studied by V.G. Bardakov, R.Mikhailov, V.V. Vershinin and J. Wu.

Problem (J. Wu)

1. *Find a basis for Brunnian braid group.*
2. *Classifying Brunnian links obtained from Brunnian braids.*
3. *Classifying links obtained from Cohen braids, where the Cohen braids were introduced by V.G. Bardakov, V.V. Vershinin and J. Wu.*

V.G. Bardakov, R.Mikhailov, V.V. Vershinin, J. Wu, Brunnian Braids on Surfaces, Algebr. Geom. Topol., ***12*** *(2012), 1607–1648.*

V.G. Bardakov, V.V. Vershinin, J. Wu, On Cohen braids, Proc. Steklov Inst. Math., ***286*** *(2014), 16–32.*