REVERSE MATHEMATICS, WELL-QUASI-ORDERS, AND NOETHERIAN SPACES

PAUL SHAFER

Abstract. A quasi-order Q induces various quasi-orders on $\mathcal{P}(Q)$, but if Q is a well-quasi-order, these induced quasi-orders need not necessarily be well-quasi-orders. Goubault-Larrecq [2] proposes that *Noetherian spaces*, which are topological spaces in which every descending sequence of closed sets stabilizes, can be seen as topological generalizations of well-quasi-orders. He proves that if Q is a well-quasi-order, then two of the natural topologies induced on $\mathcal{P}(Q)$ by the quasi-orders induced on $\mathcal{P}(Q)$ by Q are Noetherian. We analyze the logical strengths of these theorems in the style of reverse mathematics, proving that they are equivalent to ACA_0 over RCA_0. This analysis employs Dorais's framework of countable second-countable spaces in second-order arithmetic [1] when working with topologies on $\mathcal{P}_{\mathrm{f}}(Q)$ (the finite subsets of Q), and it employs a coding similar to the familiar coding of complete separable metric spaces in second-order arithmetic when working with the full $\mathcal{P}(Q)$. Our results may also be seen as strengthening some of the results from [3] concerning the reverse mathematics of well-quasi-orders. This is joint work with Emanuele Frittaion, Alberto Marcone, Matt Hendtlass, and Jeroen Van der Meeren.

References

[1] François G. Dorais, *Reverse mathematics of compact countable second-countable spaces*, preprint.

[2] Jean Goubault-Larrecq, *On Noetherian Spaces*, ***Proceedings of the 22nd Annual IEEE Symposium on Logic in Computer Science (LICS'07)*** (Wrocław, Poland), IEEE Computer Society Press, Washington, DC, USA, 2007, pp. 453–462.

[3] Alberto Marcone, *Wqo and bqo theory in subsystems of second order arithmetic*, ***Reverse mathematics 2001*** (Stephen G. Simpson, editor), Association for Symbolic Logic, La Jolla, CA, USA, 2005, pp. 303–330.

Department of Mathematics, Ghent University, Krijgslaan 281 S22, B-9000 Ghent, Belgium
E-mail address: paul.shafer@ugent.be
URL: http://cage.ugent.be/~pshafer/