Group Actions and Countable Dense Homogeneous Spaces

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Basic Definition

All spaces under discussion are separable and metrizable.

Definition

A space X is *countable dense homogeneous* (CDH), if given any two countable dense subsets D and E of X, there is a homeomorphism f of X such that $f(D) = E$.

Definition

A space X is ω *absorbing* (ωA), if it has the following absorption property: for every countable dense subset D of X and every x of X, there is a homeomorphism of X such that $f(D \bigcup \{x\}) \subseteq D$.

Definition

A space X is *weakly countable dense homogeneous* $(wCDH)$, provided that for all finite subset $F \subseteq X$, and D, $E \subseteq X \setminus F$ countable dense in X, there is a homeomorphism f of X, that restricts to the identity on F and $f(D) \subset E$

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A space is *strongly countable dense homogeneous* $(SCDH)$ if the homeomorphism moving one countable dense set onto the other can be chosen in such a way that it is limited by a given open cover of the space.

The notions of $wCDH$, $SCDH$ and ωA were introduced by van Mill.

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A space X has the *separation property* (SP), provided that for any subsets A of X with A countable and B meager, there is a homeomorphism f of X such that $f(A) \cap B = \emptyset$.

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Remark

The notion of SP also was introduced by van Mill. It is easy to see that every CDH Baire space has SP. Note that not every CDH space is Baire space, in general, the separation property need not be weaker than the property of CDH.

Definition

A *continuum* is a compact connected space. A continuum M is called*indecomposable* if it is not the union of two proper sub-continua, and a continuum which is not indecomposable continuum is called *decomposable*.

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A continuum is *hereditarily indecomposable* if it contains no decomposable subcontinuum. It is known that every nondegenerate locally connected continuum is decomposable.

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A space X is called *strongly locally homogeneous*(SLH) if it has an open base $\mathcal{B}$ such that for all $B \in \mathcal{B}$ and x, $y \in B$ there is a homeomorphism f of X which is supported on B(that is, f is the identity outside B) and moves x to y.

Definition

A space X can be separated by its subset H if $X \backslash H$ is disconnected.

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The Known Results

The first result about $wCDH$ is that if the homeomorphism needed in the countable dense homogeneous property come from an action of a group, then the underlying space is $wCDH$ (see [11]).

van Mill showed that in a Polish space,the following statements are equivalent (see [14]):

- X is $SCDH$ space.
- For every open cover $\mathcal{U}$ of X, every finite subset F of X and every $x \in X \setminus F$, there is a neighborhood V of x such that for all $y \in V$ there is a homeomorphism $f \in \mathcal{H}$ that is limited by U, restricts to the identity on F, and sends x to y

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What will we consider?

We consider the following questions raised by van Mill (see [10] and [14]):

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Is every $wCDH$ space CDH ?

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Is every $\omega - A$ space $wCDH$?

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Theorem (Effros Theorem (Version A))

Suppose that a Polish group G act transitively on a metrizable space X, then the following statements are equivalent:

- *G acts micro-transitively on X.*
- *X is polish.*
- *X is of the second category.*

The following stronger version of the Effros Theorem was proved by van Mill:

Theorem (Effros Theorem (Version B))

Suppose that an analytic group G acts transitively on a metrizable space X. If X is of the second category, then G acts micro-transitively on X.

Theorem (Effros Theorem (Version A))

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Basic Tools

Theorem (The Inductive Convergence Criterion)

Let (X, ρ) be a compact metric space, and for each n let h_n be homeomorphism of X. If for each n, $\tilde{\rho}(h_n, h_{n+1})<2^{-n}$, and $\tilde{\rho}(h_{n+1}, h_n)<3^{-n}$min{ min $\{\rho(h_i$(x), $h_i(y))$: ρ(x, y)$\geq n^{-1}\}$: 1$\leq$i$\leq n\}$. Then h$=\lim_n h_n$ is a homeomorphism of X, and be denoted by $\lim_n h_n \circ h_{n-1} \ldots \circ h_1$, which is called the infinite left product of the sequence(h_n).

Basic Tools

Lemma

(see [10]) Let X be a space without isolated points. Assume that the group G makes X $wCDH$, then for every finite subset F of X, every G_F-invariant subset of $X \backslash F$ is open.

Lemma

(see [5]) Let X is a separable metrizable space without isolated points. Then X is meager in itself if and only if it contains a countable dense subset which is G_δ in X.

Lemma

(see[12]) If X contains a dense polish subspace and has the SP, then X is completely Baire.

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Our results

Lemma 1

Every analytic $wCDH$ is completely Baire space.

Theorem 1

If there is an analytic group G makes X $wCDH$, then X is CDH.

Theorem 2

Every locally compact $wCDH$ space is CDH.

Theorem 3

If a analytic group G makes X $wCDH$, then X is polish.

Theorem 4

If a group G makes X ωA, then for each finite subset F of X, G_F makes $X \setminus F$ ωA.

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corollary 1

Every locally compact ωA space is a $wCDH$ space.

corollary 2

Every locally compact CDH space is $SCDH$.

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corollary 3

If X is a locally compact space, then the following statements are equivalent:

- X is a CDH space.
- X is a $wCDH$ space.
- X is an ωA space.
- X is a $SCDH$ space

Theorem 5

If CDH continuum X hereditarily $CDH$with respect to subcontinuum, then X hereditarily CDH with respect to open subset.

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For a filter F on ω, it has SP iff it is non-meager.

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Outline for the proof of lemma 1

The proof of Lemma 1 is inspired by Hru$\check{s}\acute{a}$k and Zamora Avil$\acute{e}$s (see [6]), and van Mill (see [12]). we just need to show the following Claims:

Claim 1

Every nonempty open subset of X is uncountable.

Claim 2

Analytic $wCDH$ space is Baire.

Claim 3

Analytic $wCDH$ space has SP.

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Outline for the proof of Theorem 1

The following Claim is essential in the proof of Theorem 1

Claim

For each finite subset $F \subseteq X$, each point $x \in X \backslash F$, each $\varepsilon > 0$, there is an open neighborhood P of x in $X \backslash F$ such that for each $a \in P$, there is h $\in G_F$ such that the following conditions are satisfied:

- $h(x) = a$.
- $\hat{\rho}(h, id_X) < 1/2\varepsilon$.

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Outline for the proof of Theorem 1

Let $A = \{ a_0, a_1, \ldots \}$, $B = \{b_0, b_1, \ldots \}$ be faithfully indexed dense subsets of X, we need to construct a sequence $(h_n)_n$ of homeomorphism of X such that the following conditions are satisfied:

- $h_n \circ \ldots \circ h_1(a_i) = h_{2i} \circ \ldots \circ h_1(a_i) \in$ B for each i and $n \geq 2i$.
- $(h_n \circ \ldots \circ h_1)^{-1}(b_i) = (h_{2i+1} \circ \ldots \circ h_1)^{-1}(b_i) \in A$ for each i and each $n \geq 2i + 1$.

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Outline for the proof of Theorem 3

Let $F = \emptyset$, $\mathcal{U} = \{G_F x\colon x \in X\}$ is open cover of X with each element is analytic, then there is a countable subcover $\mathcal{V}$ such that $\bigcup \mathcal{U} = \bigcup \mathcal{V}$. Therefore X is analytic $wCDH$ space and so it is a Baire space by Lemma 1. By the Claim 3 in the proof of Lemma 3.1 we have that X has SP. However, in [12], it states that if the homeomorphism needed in the separation property come from an action of an analytic group, then the underlying space is polish. Thus we completed the proof.

Outline for the proof of Theorem 4

We will complete the proof by the following Claim.

Claim

If group G makes X ωA, then for each finite subset F of X, each countable dense D of X, there is a homeomorphism h of X such that $h(F \bigcup D) \subset D$.

Let $F = \{ a_1, a_2, \ldots, a_n \}$. $\exists$ $f_1 \in \mathcal{H}(X)$ such that $f_1(\{a_1\} \bigcup D) \subseteq D$; $\exists$ $f_2 \in \mathcal{H}(X)$ such that $f_2(\{a_2\} \bigcup (\{a_1\} \bigcup D)) \subseteq D \bigcup \{a_1\}$; $\ldots$; $\exists$ $f_n \in \mathcal{H}(X)$ such that $f_n(\{a_n\} \bigcup (\{a_{n-1}\} \bigcup \ldots \bigcup \{a_1\} \bigcup D)) \subseteq \{a_{n-1}\} \bigcup \ldots \bigcup \{a_1\} \bigcup D$. Let h $= f_1 \circ f_2 \circ \ldots \circ f_n$ and h is what we needed.

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Outline for the proof of Theorem 4

For each point $x \in X \backslash F$, each $D \subseteq X \backslash F$ countable dense in X, we just need to show that there exists a $h \in G_F$ such that $h(\{x\} \bigcup D) \subseteq D$. The proof is similar to the proof of Proposition 4.1 in [11] and can be found in [10]. We need to construct a sequence of homeomorphisms $\{h_\alpha\colon\ \alpha < \omega_1\}$ such that

$$(\dagger)\ h_\alpha(F \bigcup \{x\} \bigcup D) \subseteq \bigcup_{\beta<\alpha} D_\beta.$$

Where $D_i = D$ for each $i < \omega_1$. Let h_0 be an element in G such that $h_0(F \bigcup \{x\} \bigcup D) \subseteq D$.
Suppose $\{h_\beta\colon\ \beta < \alpha\} \subseteq G$ have been constructed for some $\alpha < \omega_1$ such that $h_\beta(F \bigcup \{x\} \bigcup D) \subseteq \bigcup_{i<\beta} D_i$ Then pick $h_\gamma \in G$ $(\beta < \gamma)$ such that $h_\gamma(h_\beta(F \bigcup \{x\} \bigcup D)\ ) \subseteq h_\beta(F \bigcup \{x\} \bigcup D)$ $\subseteq \bigcup_{i<\beta} D_i$. Let $h_\alpha = h_\gamma \circ h_\beta$ $(\gamma < \alpha)$. For $0 \leq \alpha < \omega_1$, let T_α be a nonempty finite subset of $[1, \omega_1)$ such that $h_\alpha(F) \subseteq \bigcup_{\beta \in T_\alpha} h_\beta(D)$.

Outline for the proof of Theorem 4

By the Pressing Down Lemma, for the function

$$T\colon [1, \omega_1) \to [\omega_1]^{<\omega}$$

defined by $T(\alpha) = T_\alpha$, the fiber $B = T^{-1}(A)$ is uncountable for some $A \in [\omega_1]^{<\omega}$. Then $h_\alpha(F) \subseteq \bigcup_{\beta \in A} D_\beta$ for every $\alpha \in \beta$. Since $\bigcup_{\beta \in A} D_\beta$ is countable and B is uncountable, we may assume that $h_\alpha | \ F = h_\beta | F$ for all α, $\beta \in B$ and that $h_\beta(\{x\} \bigcup D) \subseteq D$ by (†).

In [14], van Mill showed that every compact CDH space is $SCDH$, and asked that whether every locally compact CDH is $SCDH$ or not. Note that in the proof of van Mill, the essential step is to showed that for each finite $F \subseteq X$, each $x \in X \setminus F$, and each $\varepsilon > 0$, there is an open neighborhood P of x in X such that for each there exist an element $h \in \mathcal{H}_F$ such that $h(x) = a$ and h move no point more than ε. Clearly, this is just the Claim in the proof of Theorem 1 in this paper, then we can easily prove corollary 2.

Outline for the proof of Theorem 5

The following Lemmas are used in our proof.

Lemma(see [3])

If X is a continuum which contains no non-degenerate, hereditarily indecomposable subcontinuum, then dim$X \leq 1$.

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Lemma(see [13])

Let X be a continuum is hereditarily indecomposable if and only if for any two subcontinuums D and E, if $D \bigcap E \neq \emptyset$, then $D \subseteq E$ or $E \subseteq D$.

Lemma

Let X be a locally compact CDH space, then each open subset A of X with $F = \overline{A} \setminus A$ finite is CDH.

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Outline for the proof of Theorem 6

Recall the following definition:

Definition

$\mathcal{F}$ is called a *filter* if it is closed under taking supersets and finite intersections.

Remark

van Mill showed that if G is a Barie group acting on space X which is not meager in itself, then for arbitrary subsets A, B of X with A being countable and B being meager, the set $\{g\colon\ g \in G$ and $g(A) \cap B = \emptyset\}$ is dense in G.

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van Mill showed that if G is a Barie group acting on space X which is not meager in itself, then for arbitrary subsets A, B of X with A being countable and B being meager, the set $\{g\colon\ g\in G$ and $g(A) \cap B = \emptyset\}$ is dense in G.

Outline for the proof of Corollary 4 and 5

Recall the following result of D. Repovš, L. Zdomskyy, and S. Zhang.

There is a non-meager non-CDH filter in ZFC.

we also recall the following result of K. Kunen, A. Medini, and L. Zdomskyy.

A filter F is CDH iff F is a P-filter with SP.

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Thank you!