

# Perturbed Singular Hankel Determinant Asymptotic Characterization: Based on Coulomb Fluid and Painlevé III

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## Abstract

The Hankel determinant,

$$D_n(t, \alpha, \beta) := \det \left( \int_0^1 x^{j+k} w(x; t, \alpha, \beta) dx \right)_{j,k=0}^{n-1},$$

originated from Pollacaek-Jacobi type weight,

$$w(x; t, \alpha, \beta) := x^\alpha (1-x)^\beta e^{-t/x}, \quad x \in [0, 1], \quad \alpha > 0, \quad \beta > 0, \quad t > 0,$$

In this work, our studies are based on a double scaling, where  $n$ , the dimension of the Hankel determinant tends to  $\infty$ , and  $t$  tends to 0, the scaled, the infinite dimensional—Hankel determinant, has an integral representation in terms of a particular  $P_{\text{III}}$ . Expansions of the Hankel determinant for small and large parameter are obtained, under the double scaling.