Stein's Method and Characteristic Functions

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We consider a method to obtain limit theorems for various functions of independent and weakly dependent random variables, based on characterization of corresponding limit distributions by means of differential equations for their characteristic functions. This approach was first realized by the author in 1976 in his proof of an estimate for the rate of convergence in the Central Limit Theorem for the sum of weakly dependent random variables, [2], [3], being in its turn motivated by the famous Stein paper of 1972, [1]. In the talk we discuss applications of our approach to the Central Limit Theorem for different models including quadratic forms and linear statistics of eigenvalues of random matrices. We also consider some non-central limit theorems. See [4], [5], [6], [7], [8].

References

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