TELECOM ParisTech

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The Stein-Dirichlet-Malliavin method

L. Decreusefond

New directions in Stein's method

The middle-class gentleman, Molière, 1670

MONSIEUR JOURDAIN: *By my faith! For more than forty years I have been speaking prose without knowing anything about it.*

What is Stein's method ?

Goal

Evaluate

$$\sup_{F\in\mathcal{F}} \int F \mathrm{d}\mathbf{P} - \int F \mathrm{d}\mathbf{Q}$$

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Steps

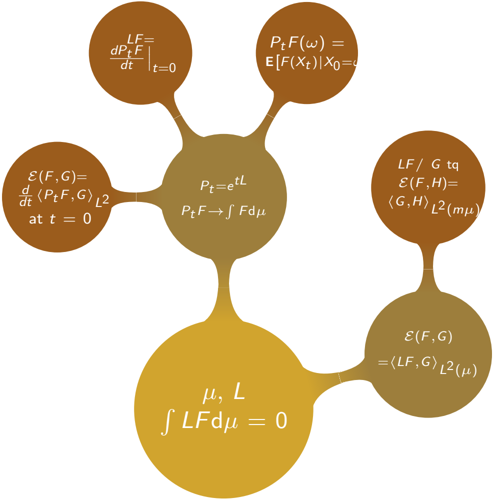
- Identify $\mathbf{P}$ as a solution of the functional problem

$$\int LF \mathrm{d}\mu = 0 \iff \mu = \mathbf{P}$$

- Solve "Stein's equation" : $\exists ? F, LH = F$ for $F \in \mathcal{F}$
- Regularity properties of H_F
- Size-biased coupling, exchangeable pair, etc.

$$\mathbf{E}\left[X\, H_F(X)\right] = \mathbf{E}\left[T_1\, H_F'(X + T_2)\right]$$

What is a Dirichlet structure ?



Stein representation formula

Kantorovitch-Rubinstein distance between P and Q

$$P_\infty F(\omega) - P_0 F(\omega) = \int_0^\infty \frac{d}{dt} P_t F(\omega) \mathrm{d}t$$

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Points solved

- Identify **P** as a solution of the functional problem
- Solve "Stein's equation" : $H_F = \int_0^\infty LP_tF\mathrm{d}t$

Gaussian measure on R^n

Standard Gaussian measure

- $\mathfrak{F} = \mathbf{R}^n$, $\mu = \mathcal{N}(0, \mathrm{Id})$
- $LF(x) = x.\nabla F(x) - \Delta F(x)$
- Semi-group (Mehler formula)

$$P_t F(x) = \int_{\mathbf{R}^n} F(e^{-t}u + \sqrt{1 - e^{-2t}}v)\, d\mu(v)$$

- $X = (X_1, \cdots, X_n)$ where X_k=Ornstein-Uhlenbeck process on $\mathbf{R}$

$$\mathrm{d}X_k(t) = -X_k(t)\mathrm{d}t + \sqrt{2}\,\mathrm{d}B_k(t)$$

- $D = \nabla$

Absolutely continuous measures

$\nu \ll \mu$

- μ= standard Gaussian measure on $\mathbf{R}^n$

$$\mathrm{d}\nu(x) = e^{-V(x)}\mathrm{d}\mu(x)$$

- $L^\nu F(u) = L^\mu F(x) - \nabla V(x).\nabla F(x)$
- Semi-group

$$P_t^\nu F(x) = e^{-V(x)/2} P_t^\mu \Big(e^{V(x)/2} F \Big)(x)$$

- $X = (X_1, \cdots, X_n)$ where

$$\mathrm{d}X_k(t) = -\Big(X_k(t) + \nabla_k V(X(t)) \Big)\mathrm{d}t + \sqrt{2}\, \mathrm{d}B_k(t)$$

Poisson distribution on $\mathfrak{N}$

Poisson

- $\mathfrak{F} = \mathbf{N}$, μ=Poisson $[\lambda]$
- $LF(n) = \lambda(F(n+1) - F(n)) + n(F(n-1) - F(n))$
- $X(t)$ = nb of occupied servers in M/M/∞
- Dist. $X(t)$ = Poisson$[\theta(t, X(0))]$ where

$$\theta(t,n) = e^{-t} n + (1 - e^{-t})\lambda$$

- Semi-group

$$P_t F(n) = \sum_{k=0}^{\infty} F(k) e^{-\theta(t,n)} \frac{\theta(t,n)^k}{k!}$$

- $DF(n) = F(n+1) - F(n)$

Poisson point process

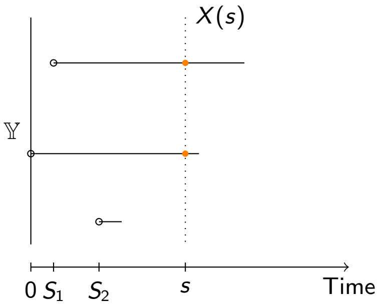
PPP over $\mathbb{Y}$

- $\mathfrak{F}$ = configuration space over $\mathbb{Y}$
- μ=dist. of PPP(Λ)
- Generator

$$LF(N) := \int_{\mathbb{Y}} \Big(F(N \cup \{y\}) - F(\omega) \Big) \, \mathrm{d}\Lambda(y) + \sum_{y \in N} F(N \backslash \{y\}) - F(\omega)$$

- X : Glauber process
- Dist. $X(t)$=PPP($(1-e^{-t})\lambda$) + e^{-t}-thinning of the I.C.

Realization of a Glauber process



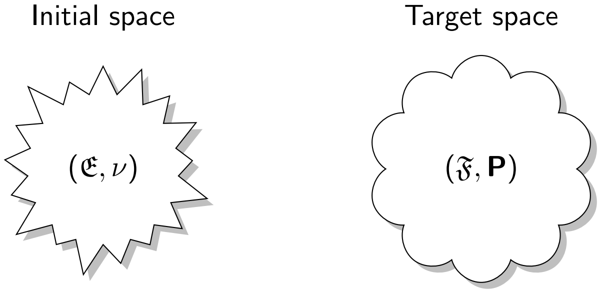
- $S_1, S_2, \cdots$: Poisson process of intensity $\Lambda(\mathbb{Y})\ ds$
- Lifetimes : Exponential rv of param. 1
- Remark : Nb of particles $\sim$ M/M/∞

What's next ?

Stein representation formula

$$\int F\mathrm{d}\mathbf{Q} - \int F\mathrm{d}\mathbf{P} = \int_0^\infty \left(\int LP_t F \, \mathrm{d}\mathbf{Q} \right) \mathrm{d}t$$

Generic scheme



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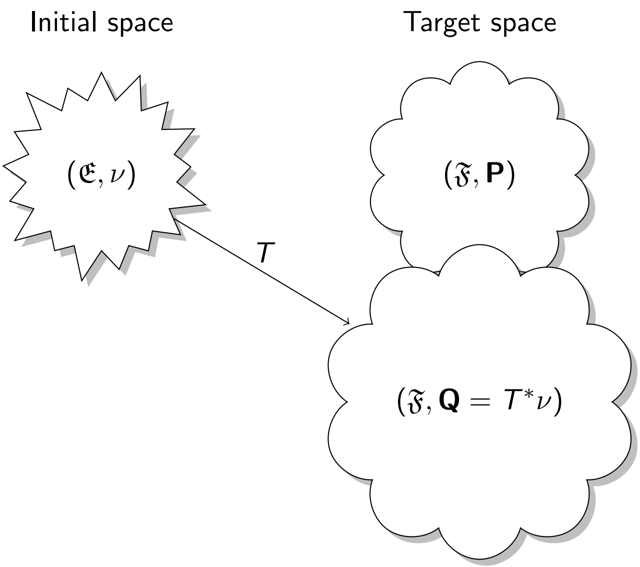

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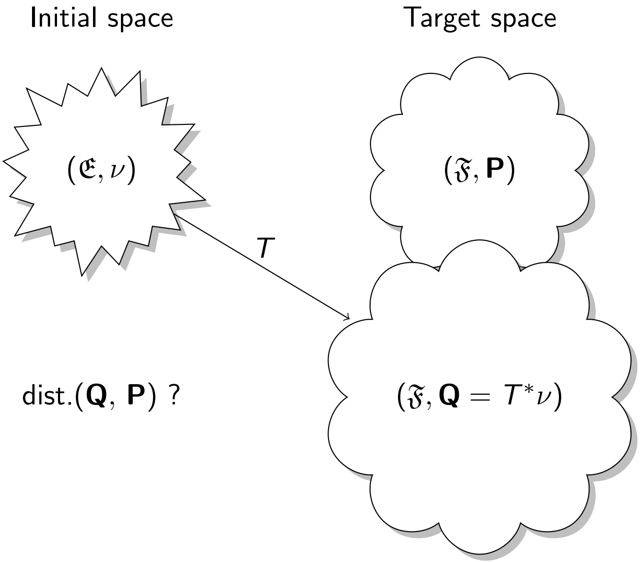

dist.$(\mathbf{Q}, \mathbf{P})$?

Integration by parts

"Carré du champ" operator

$$\Gamma(F;G) = \frac{1}{2}\left(L(FG) - G\,LF - F\,LG\right)$$

For Gaussian measure, $\Gamma(F;G) = \nabla F.\nabla G$

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IBP

$$-\mathbf{E}_{\mu}\left[G\,LF\right] = \mathbf{E}_{\mu}\left[\Gamma(F;G)\right]$$

Remind that

$$LF(x) = x.\nabla F(x) - \Delta F(x)$$

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Bakry, Gentil, Ledoux : Analysis and geometry of Markov diffusion operators

Breaking the L

$$\int_{\mathbb{Y}} F \mathrm{d}\mathbf{Q} - \int_{\mathbb{Y}} F \mathrm{d}\mathbf{P} = \int_0^{\infty} \mathbf{E}_{\mathbf{Q}} \left[L P_t F \right] \mathrm{d}t$$

where

$$LF(N) := \int_{\mathbb{Y}} \Big(F(N \cup \{y\}) - F(N) \Big) \mathrm{d}\Lambda(y) + \sum_{y \in N} F(N \backslash \{y\}) - F(N)$$

Coupling or IBP'ing ?

- Coupling : size biased, exchangeable pair, etc. depends on ν and T

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Coupling or IBP'ing ?

- Coupling : size biased, exchangeable pair, etc. *depends on* ν *and* T
- Integration by parts *depends only on* ν
- In the sequel, ν=distr. of a general point process

Papangelou intensity

Theorem (Georgii-Nguyen-Zessin (GNZ) formula)

$$\mathbf{E}_\nu\left[\sum_{y\in N} u(y, N\backslash\{y\})\right] = \mathbf{E}_\nu\left[\int_{\mathbb{Y}} u(y, N)\, c(y, N)\mathrm{d}\Lambda(y)\right]$$

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Informally

$$c(x,\ N) \sim \mathbf{P}(N\cup\{x\}\,|\,N)$$

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Warning

$$c^{\Lambda'}(y, N) = c^{\Lambda}(y, N)\ \frac{\mathrm{d}\Lambda}{\mathrm{d}\Lambda'}(y)$$

Examples

Poisson process : $\nu = \pi_\Lambda$

$$c(x, N) = 1$$

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Pairwise interactions Gibbs process

If

$$d\nu(N) = \exp(-\sum_{x,y \in N} \phi(x, y))\, d\pi_\Lambda(N)$$

Then

$$c(x, N) = \exp(-\sum_{y \in N} \phi(x, y))$$

Integration by parts

Definition (Difference operator)

For $F : \mathfrak{N}_{\mathbb{Y}} \to \mathbf{R}$,

$$\begin{array}{rccl} DF: & \mathbb{Y} \times \mathfrak{N}_{\mathbb{Y}} & \longrightarrow & \mathbf{R} \\ & (y, N) & \longmapsto & D_y F(N) = F(N \cup \{y\}) - F(N \backslash \{y\}). \end{array}$$

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Theorem (LD-Flint)

For any bounded function F on $\mathfrak{N}_{\mathbb{Y}}$,

$$\mathbf{E}_\nu \left[F(N) \left(\int_{\mathbb{Y}} u(y, N \setminus y) \mathrm{d}N(y) - \int_{\mathbb{Y}} u(y, N)\, c(y, N) \mathrm{d}\Lambda(y) \right) \right] = \mathbf{E}_\nu \left[\int_{\mathbb{Y}} D_y F(N)\ u(y, N) c(y, N) \mathrm{d}\Lambda(y) \right]$$

Global picture

Stein-Dirichlet-Malliavin structure

- ν=dist. of point process of Papangelou intensity c
- D = difference operator, D^* its adjoint

$$D^* u = \int_{\mathbb{Y}} u(y, N \backslash y) \mathrm{d}N(y) - \int_{\mathbb{Y}} u(y, N)\, c(y, N) \mathrm{d}\Lambda(y)$$

- Generator

$$D^* DF(N) = \int_{\mathbb{Y}} D_y F(N) c(y, N) \, \mathrm{d}\Lambda(y) + \sum_{y \in N} F(N \backslash \{y\}) - F(N)$$

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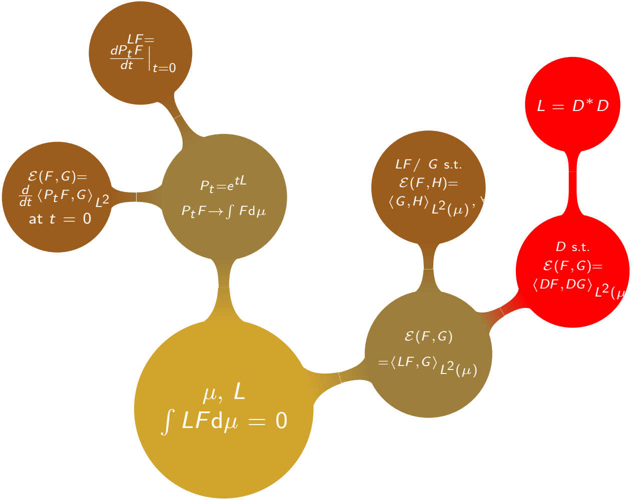
Dirichlet-Malliavin structure



Consequences

New problem solved

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almost equivalent to *integration by parts* or the existence of D.

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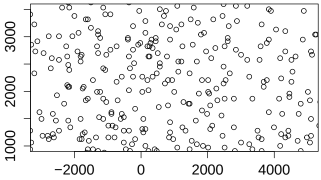
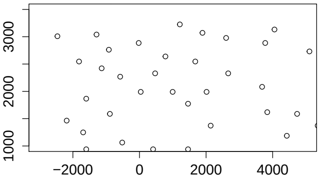
Hence

We need a Dirichlet-Malliavin structure on both *initial* and *target* spaces.

BTS deployement



Paris, all frequency bands



1 frequency band

Statistical analysis

Tests [Gomez et al.]

Locations of all BTS $\simeq$ Poisson point process

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Questions

- Which model for one frequency band ?

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Related works

- Deng, Zhou, Haenggi. The Ginibre Point Process as a Model for Wireless Networks with Repulsion
- Li, Baccelli, Dhillon, Andrews. Statistical modeling and probabilistic analysis of cellular networks with DPP

Ginibre process

Random matrices

$$N = \lim_{\text{size} \to \infty} \text{Eigenvalues of} \begin{pmatrix} \mathcal{N}_{\mathbf{C}}(0,1) & \dots & \dots & \mathcal{N}_{\mathbf{C}}(0,1) \\ \vdots & & & \vdots \\ \mathcal{N}_{\mathbf{C}}(0,1) & \dots & \dots & \mathcal{N}_{\mathbf{C}}(0,1) \end{pmatrix}$$

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Definition (Correlation functions)

$$\rho^{(n)}(x_1, \cdots, x_n) = \det\Big(K(x_i, x_j),\ 1 \le i, j \le n\Big)$$

where

$$K(x,y) = \frac{1}{\pi} \exp\Big(x\bar{y} - \frac{1}{2}(|x|^2 + |y|^2)\Big),\ x, y \in \mathbf{C}$$

Repulsion

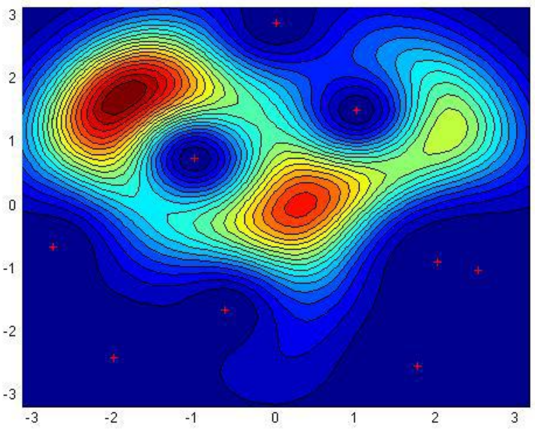


Figure: $c(x, \omega)$ where $|\omega| = 9$ (I. Flint©)

Randomization

(λ, β)-Ginibre

- Apply a β-thinning

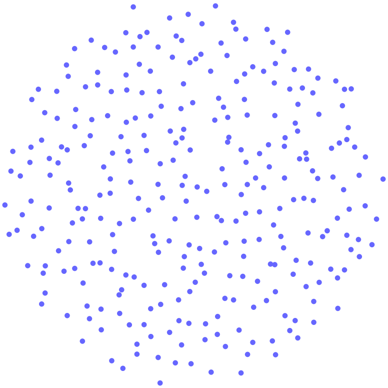
Randomization

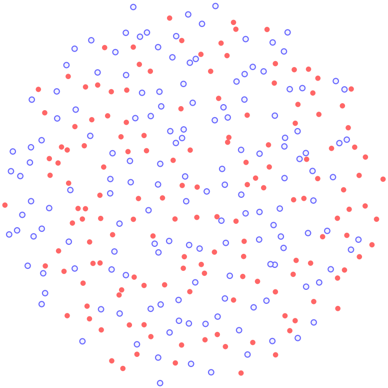
(λ, β)-Ginibre

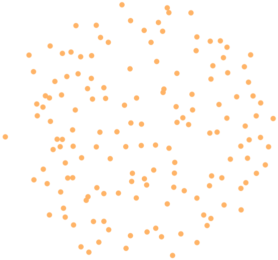
- Apply a β-thinning
- Apply a dilation of $\lambda\sqrt{\beta}$

Poisson as a $(\lambda, 0)$-Ginibre

$$(\lambda, \beta) - \text{Ginibre} \xrightarrow{\beta \to 0} \text{Poisson}(\pi^{-1}\lambda \, \mathrm{d}y)$$







Results [Gomez et al.]

Table: Numerical values of β and λ per technology and operator

		Orange	SFR	Bouygues	Free
GSM 900	β	0.81	0.76	0.65	NA
	λ	2.39	2.65	2.63	NA
GSM 1800	β	0.84	0.85	0.71	NA
	λ	3.00	2.39	3.59	NA
LTE 800	β	1.00	0.93	0.67	NA
	λ	0.67	1.65	1.87	NA
LTE 2600	β	0.93	0.67	0.63	0.89
	λ	2.80	2.76	2.46	1.05

Framework

Initial space

$(\mathfrak{N}_{\mathbb{Y}}^{K}, \otimes_{i=1}^{K}\nu_i)$

Target space

$(\mathfrak{N}_{\mathbb{Y}}, \mathrm{Pois}(\Lambda))$

Framework

Initial space

Target space

$(\mathfrak{N}_{\mathbb{Y}}^{K}, \otimes_{i=1}^{K} \nu_i)$

$(\mathfrak{N}_{\mathbb{Y}}, \mathrm{Pois}(\Lambda))$

T

$$T(N_1, \cdots, N_k) = \bigcup_{i=1}^{k} N_i$$

$(\mathfrak{N}_{\mathbb{Y}}, \mathfrak{v} = T^*(\otimes_{i=1}^{K} \nu_i))$

Our situation

Initial space

- ν_i= PP of intensity measure Λ_i and PI c_i^Λ

Target space

- $\mathbf{P}$= Poisson(Λ) with $\Lambda = \sum_{i=1}^K \Lambda_i$

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Initial space

- $\nu_i =$ PP of intensity measure Λ_i and PI c_i^Λ
- $T(N_1, \cdots, N_N) = \bigcup_{i=1}^K N_i$

Target space

- $\mathbf{P} =$ Poisson(Λ) with $\Lambda = \sum_{i=1}^K \Lambda_i$
- $\mathfrak{v} = T^* \otimes_{i=1}^K \nu_i$ has PI

$$c^\Lambda(y, \cup_{i=1}^K N_i) = \sum_{i=1}^K c_i^\Lambda(y, N_i)$$

Lipschitz functionals

Distance between configurations

$c(N, M) = \mathrm{dist}_{\mathrm{TV}}(N, M) =$ number of different points

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Definition

$F : \mathfrak{N}_{\mathbb{Y}} \to \mathbf{R}$ is TV–Lip_1 if

$$|F(N) - F(M)| \le \text{dist}_{\text{TV}}(N, M)$$

$$\text{Example}: \; N \longmapsto N(A)$$

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Definition (Kantorovich-Rubinstein distance)

$$\mathrm{d}_{\mathrm{KR}}(\mathbf{P}, \mathbf{Q}) := \sup_{F \in \mathrm{TV}\text{–}\mathrm{Lip}_1} \big(\mathbf{E}_{\mathbf{P}}[F] - \mathbf{E}_{\mathbf{Q}}[F]\big),$$

Second step

$$\mathbf{E}_{\mathfrak{v}}\left[\sum_{y\in N} P_tF(N\backslash\{y\}) - P_tF(N)\right]$$

$$= -\mathbf{E}_{\mathfrak{v}}\left[\sum_{y\in N} D_yP_tF(N)\ c(y,N)\mathrm{d}\Lambda(y)\right]$$

$$= -\sum_{i=1}^{K}\mathbf{E}_{\nu_i}\left[\int_{\mathbb{Y}} D_yP_tF(\cup_{i=1}^{K}N_i)\ c_i^{\Lambda}(y,N_i)\ \mathrm{d}\Lambda(y)\right]$$

Putting all together

Partial conclusion

$$\mathbf{E}_{\mathfrak{v}}\left[\int_0^\infty LP_tF(N)\mathrm{d}t\right]$$
$$= \mathbf{E}_{\otimes \nu_i}\left[\int_0^\infty \int_{\mathbb{Y}} D_yP_tF(\cup_{i=1}^K N_i)\left|1-\sum_{i=1}^K c_i^\Lambda(y,N_i)\right| \mathrm{d}\Lambda(y)\ \mathrm{d}t\right]$$

Putting all together

Partial conclusion

$$\mathbf{E}_{\mathfrak{v}}\left[\int_0^\infty LP_tF(N)\mathrm{d}t\right]$$
$$= \mathbf{E}_{\otimes\nu_i}\left[\int_0^\infty \int_{\mathbb{Y}} e^{-t}P_tD_yF(\cup_{i=1}^K N_i)\left|1-\sum_{i=1}^K c_i^\Lambda(y,N_i)\right| \mathrm{d}\Lambda(y)\ \mathrm{d}t\right]$$

Putting all together

Partial conclusion

$$\mathbf{E}_{\mathfrak{v}}\left[\int_0^{\infty} LP_tF(N)\mathrm{d}t\right]$$

$$= \mathbf{E}_{\otimes\nu_i}\left[\int_0^{\infty}\int_{\mathbb{Y}} e^{-t}P_tD_yF(\cup_{i=1}^{K}N_i)\left|1-\sum_{i=1}^{K}c_i^{\Lambda}(y,N_i)\right|\mathrm{d}\Lambda(y)\,\mathrm{d}t\right]$$

Theorem

$$\mathrm{d}_{KR}(\mathfrak{v},\pi_\Lambda) \le \mathbf{E}\left[\int_{\mathbb{Y}}\left|1-\sum_{i=1}^{K}c_i^{\Lambda}(y,N_i)\right| d\Lambda(y)\right]$$

Repulsive point processes

Definition (Repulsivity)

$$M \subset N \Longrightarrow c(x, N) \leq c(x, M)$$

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Weak repulsivity

$$c(x, N) \le c(x, \emptyset)$$

Convergence of weak repulsive processes

Theorem

If $\nu_{K,i},\ \ i=1,\cdots,K$ *are w-repulsive*

$$\mathrm{d}_{KR}\left(T^*(\otimes_{i=1}^{K}\nu_{K,i}), Poisson(\Lambda)\right) \le \int_{\mathbb{Y}} \left| \sum_{i=1}^{K} \rho^{(1)}_{K,i}(y) - 1 \right| \mathrm{d}\Lambda(y) + 2\sum_{i=1}^{K}(1-\nu_{K,i}(\{\emptyset\}))^2$$

Proof

Lemma

If N w-repulsive

$$c(y,\emptyset) \geq \rho^{(1)}(y) \geq c(y,\emptyset)\,\mathbf{P}(N=\emptyset)$$

Proof

Lemma

If N w-repulsive

$$c(y,\emptyset) \geq \rho^{(1)}(y) \geq c(y,\emptyset)\,\mathbf{P}(N=\emptyset)$$

Proof.

- w-repulsive $\Longrightarrow c(y,N) \leq c(y,\emptyset)$
- GNZ $\Longrightarrow \mathbf{E}\left[c(y,N)\right] = \rho^{(1)}(y)$
- On the other hand

$$\begin{aligned}\mathbf{E}\left[c(y,N)\right] &= \mathbf{E}\left[c(y,\emptyset)\mathbf{1}_{N=\emptyset}\right] + \mathbf{E}\left[c(y,N)\mathbf{1}_{|N|\geq 1}\right]\\ &\geq c(y,\emptyset)\mathbf{P}(N=\emptyset)\end{aligned}$$

□

Superposition

Theorem (LD-A. Vasseur)

$$\mathrm{d}_{KR}\left(\bigoplus_{i=1}^{K}(\lambda_{K,i},\beta_{K,i})\text{-}Ginibre,\ Poisson(\pi^{-1}\sum_{i=1}^{K}\lambda_{K,i}\mathrm{d}y)\right) \le \frac{c}{K}\sup_i \beta_{K,i}$$

Superposition

Theorem (LD-A. Vasseur)

$$\mathrm{d}_{KR}\left(\bigoplus_{i=1}^{K}(\lambda_{K,i},\beta_{K,i})\text{-}\mathit{Ginibre},\ \mathit{Poisson}(\pi^{-1}\sum_{i=1}^{K}\lambda_{K,i}\mathrm{d}y)\right) \le \frac{c}{K}\ \sup_i \beta_{K,i}$$

$$\mathrm{d}_{KR}\Big((\lambda,\beta)-\mathit{Ginibre}, \mathit{Poisson}(\pi^{-1}\lambda)\Big) \le c\beta$$

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure

Initial space

Conclusion

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Initial space

- Gradient and its adjoint for the initial measure

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure
- $|DP_tF| \le \psi(t)P_t|DF|$ with $\psi \in L^1(\mathbf{R}^+)$

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Conclusion

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  Characterization of the target measure
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Initial space

- Gradient and its adjoint for the initial measure
  Exchangeable pairs, bias coupling, etc

Conclusion

Target space

- Dirichlet Malliavin structure for the target measure
  Characterization of the target measure
- $|DP_tF| \le \psi(t)P_t|DF|$ with $\psi \in L^1(\mathbf{R}^+)$
  Properties of the solution of the Stein equation, gradient bounds

Initial space

- Gradient and its adjoint for the initial measure
  Exchangeable pairs, bias coupling, etc

Conclusion

One could put them first of all as you said them: "Beautiful marchioness, your lovely eyes make me die of love." Or else: "Of love to die make me, beautiful marchioness, your beautiful eyes." Or else: "Your lovely eyes, of love make me, beautiful marchioness, die." Or else: "Die, your lovely eyes, beautiful marchioness, of love make me." Or else: "Me make your lovely eyes die, beautiful marchioness, of love."

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