TECHNISCHE
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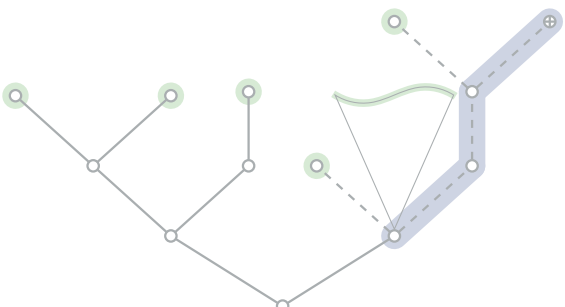
Interpolants from SAT solving certificates

Adrián Rebola-Pardo

TU Wien

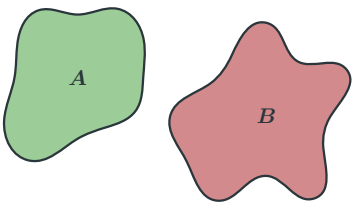


Automata, Logic and Games
Singapore

August 29th, 2016

Propositional interpolants

Let A, B be propositional formulae such that $A \wedge B$ is unsatisfiable.



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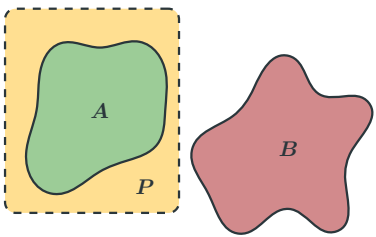
Let A, B be propositional formulae such that $A \wedge B$ is unsatisfiable.

Interpolants an (A, B)-interpolant is a propositional formula P such that:

- $A \models P$.
- $P \wedge B$ is unsatisfiable.
- P contains only variables occurring in both A and B.



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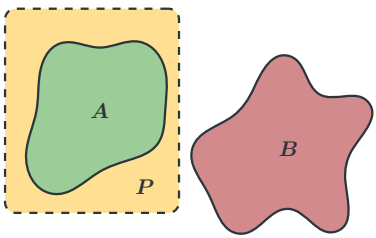


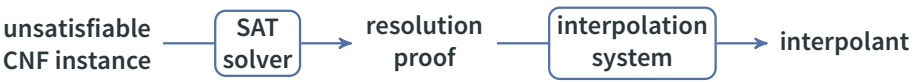
Interpolants are essential tools in formal methods and software verification:

- (Un)bounded model checking [McMillan '03]
- Boolean synthesis [Jiang et al. '09]
- Fault localization [Ermis et al. '12]
- Hardware verification [Keng Veneris '09]

Interpolation in practice

Interpolant generation

The good old times...



[Zhang, Malik '03]

[Huang '95]

Interpolant generation

The good old times are gone

[Huang '95]

Interpolant generation

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Properties of DRAT proofs

- ✔ **Shorter and easier to generate or check than resolution proofs.**
- ✔ **Allow to express satisfiability-preserving techniques.**

Interpolant generation

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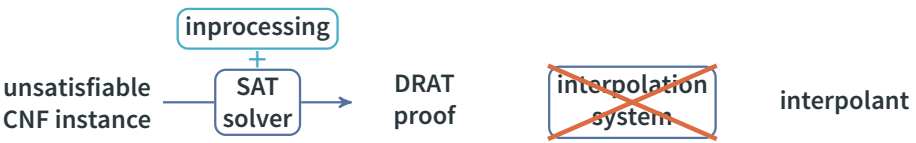


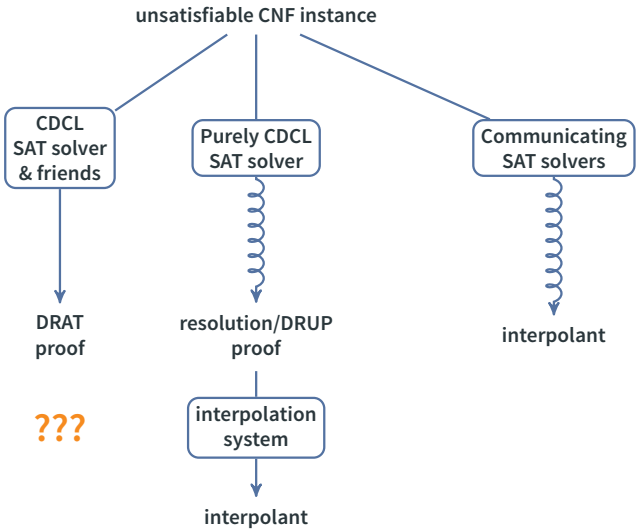
[Heule, Hunt, Wetzler '14]

Properties of DRAT proofs

- ✔ Shorter and easier to generate or check than resolution proofs.
- ✔ Allow to express satisfiability-preserving techniques.
- ✘ We do not know how to generate interpolants from DRAT proofs.

Interpolant generation from proofs

Three approaches



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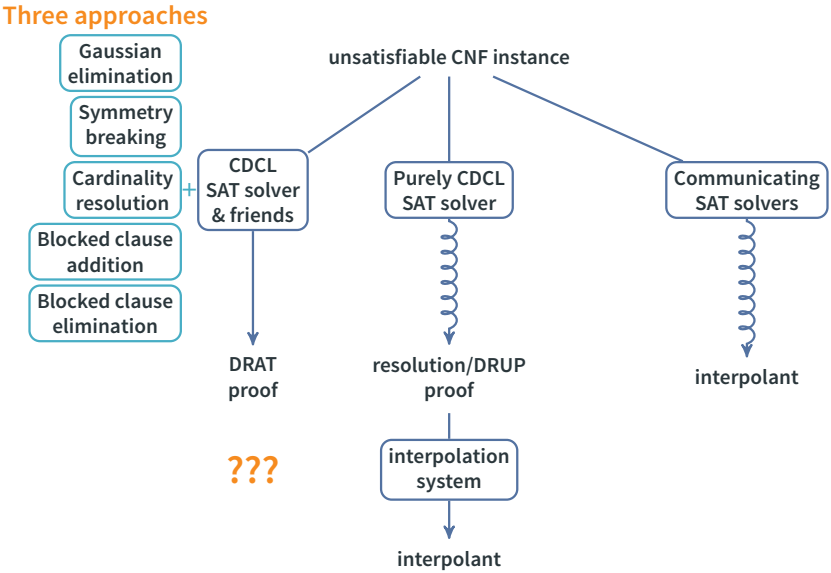

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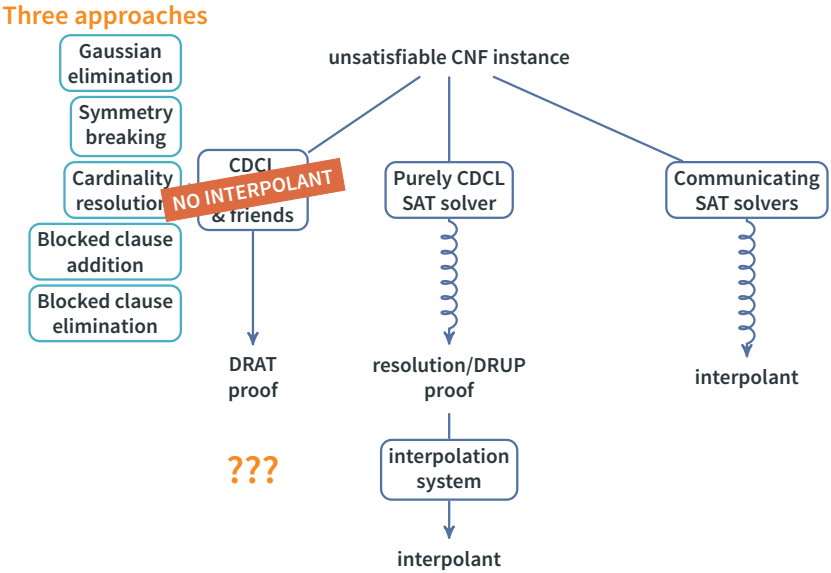

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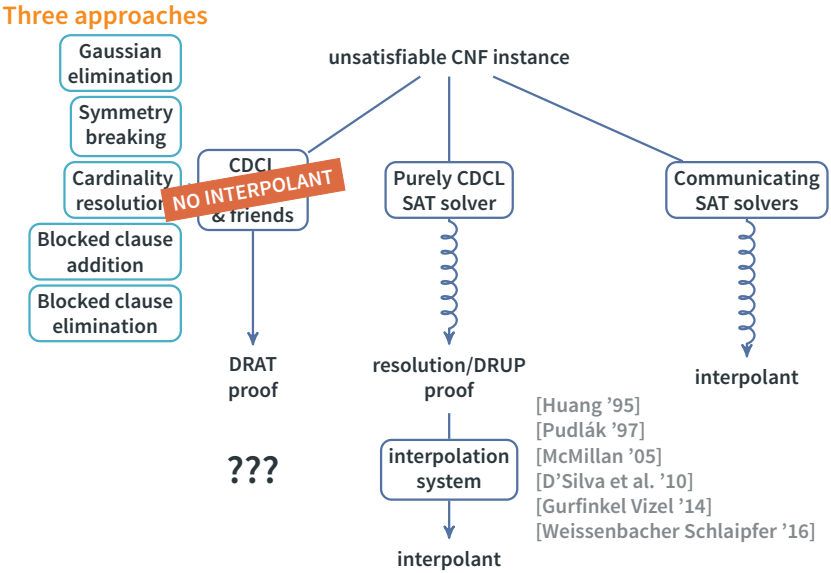

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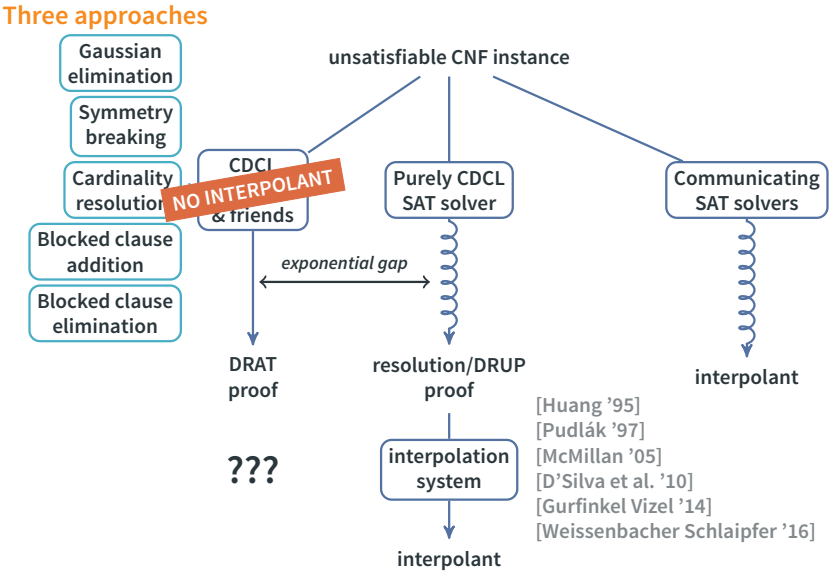

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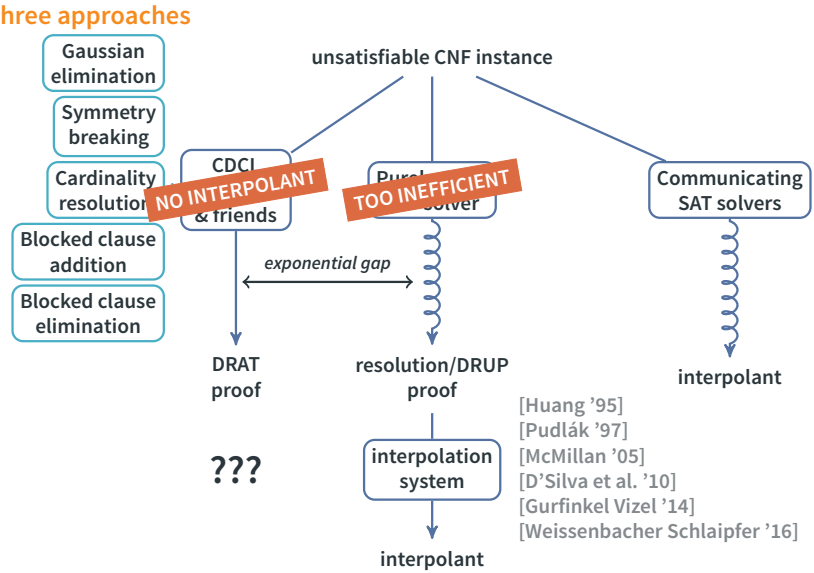

Interpolant generation from proofs

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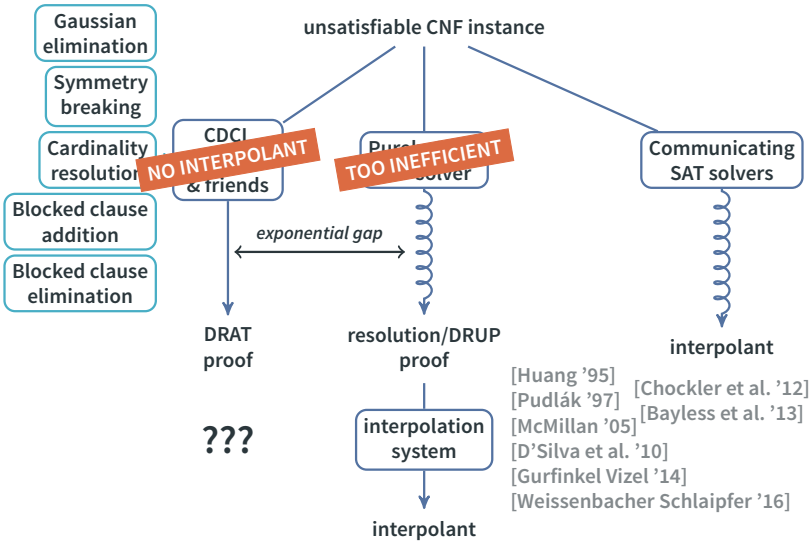

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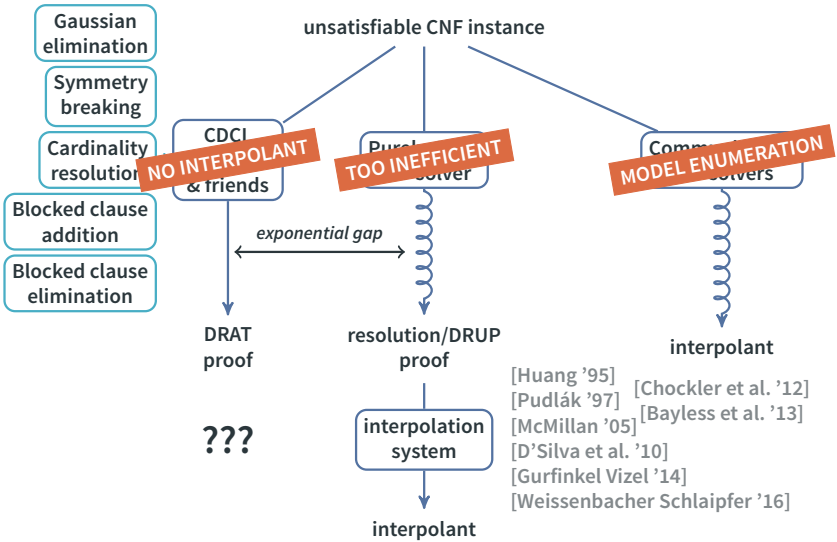

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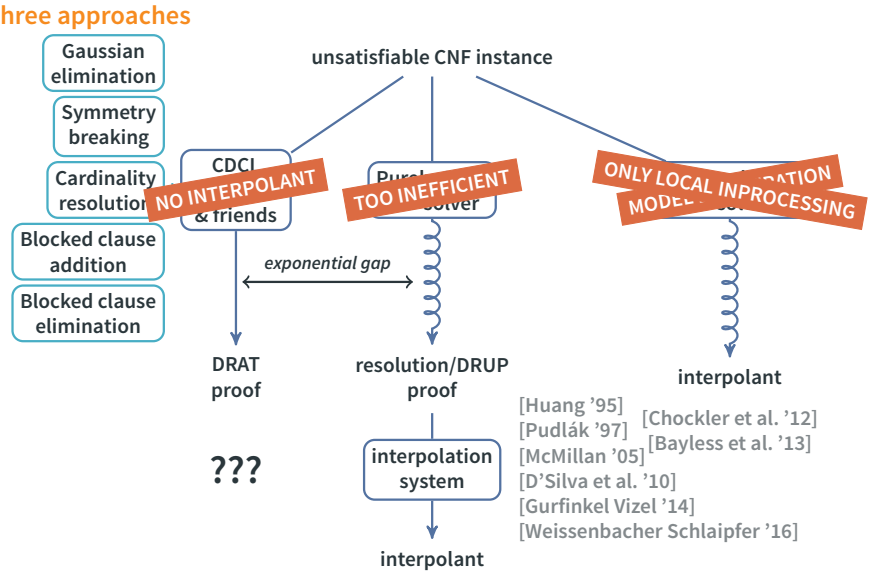

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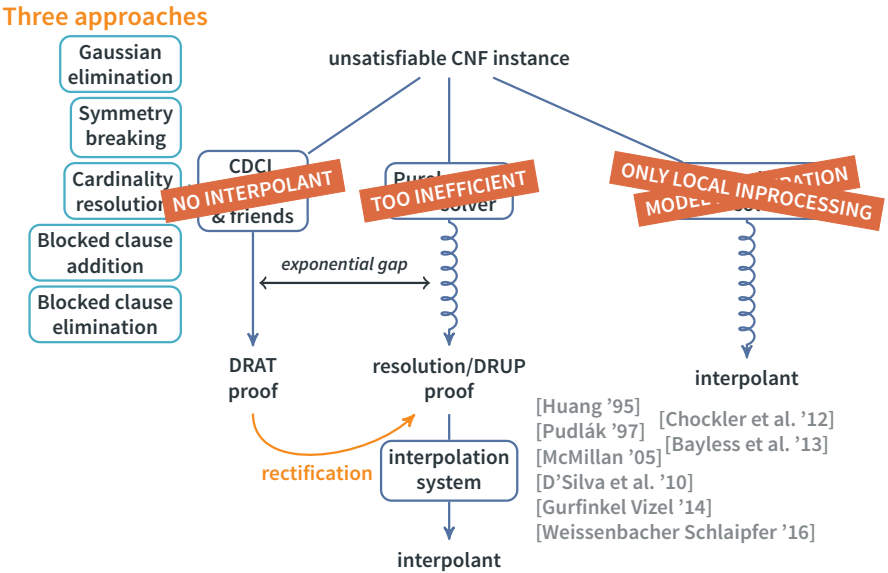
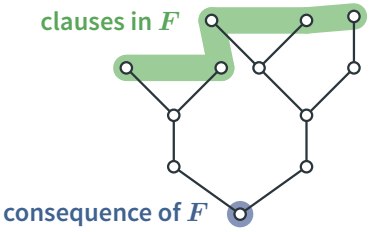


Proof systems for SAT solvers

DRUP proofs

Reverse Unit Propagation (RUP)



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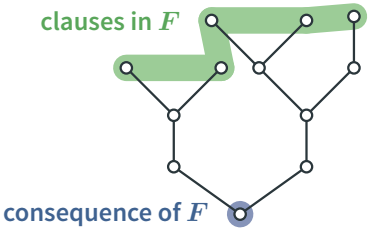

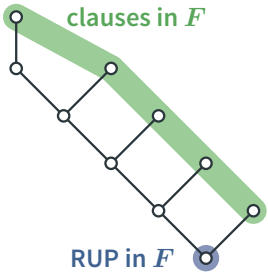

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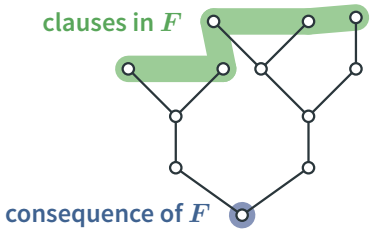

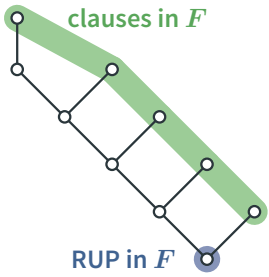

DRUP proof system **RUP introduction + arbitrary clause deletion**

DRUP proofs

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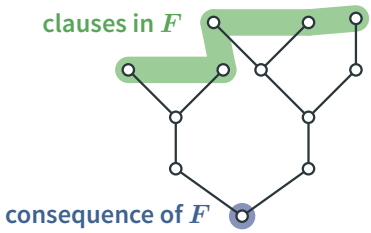

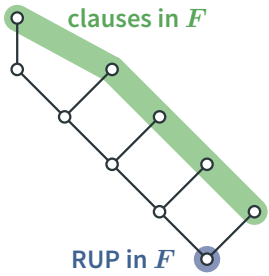

DRUP proof system RUP introduction + arbitrary clause deletion

- Essentially as powerful as resolution [Beame et al. '04]
- Interpolants can be easily generated [Gurfinkel Vizel '14]

Resolution asymmetric tautologies

A clause C is a resolution asymmetric tautology (RAT) in a CNF formula F upon a literal l if every resolvent $C \otimes D$ upon l, where $D \in F$, is a RUP in F.

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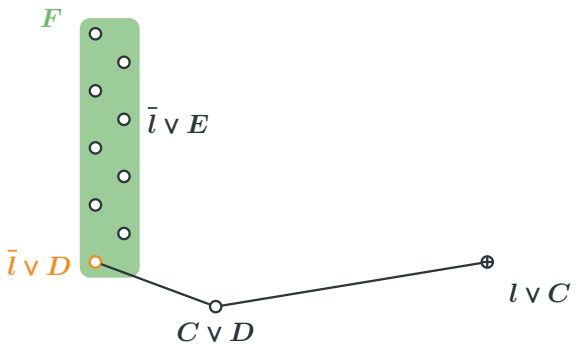

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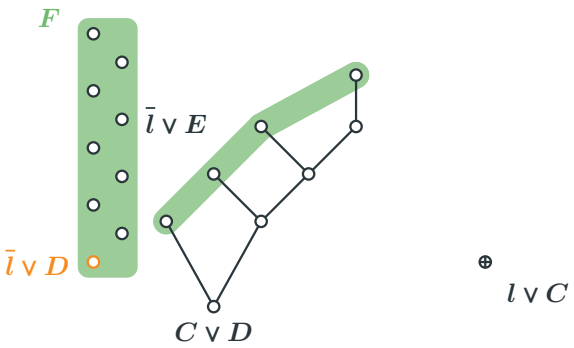

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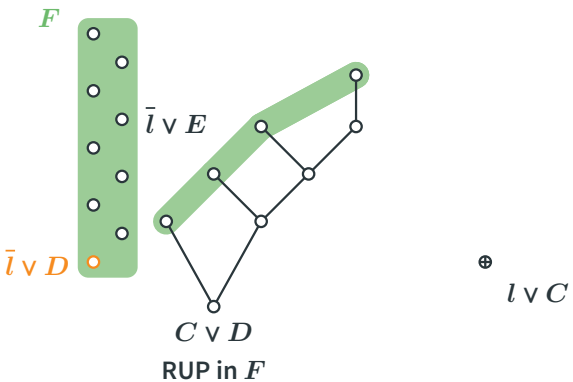

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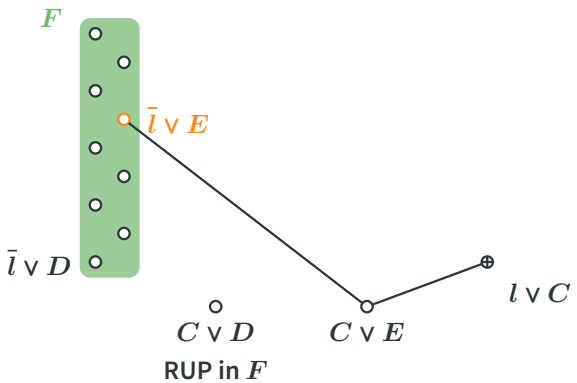

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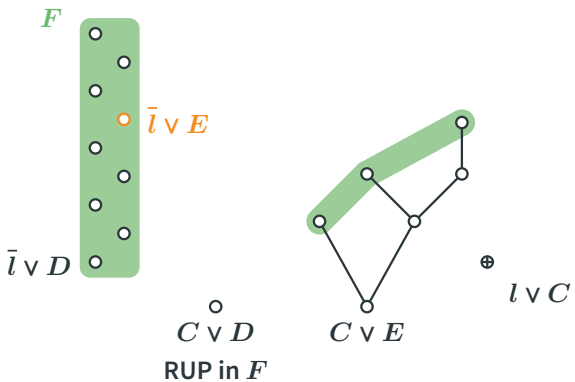

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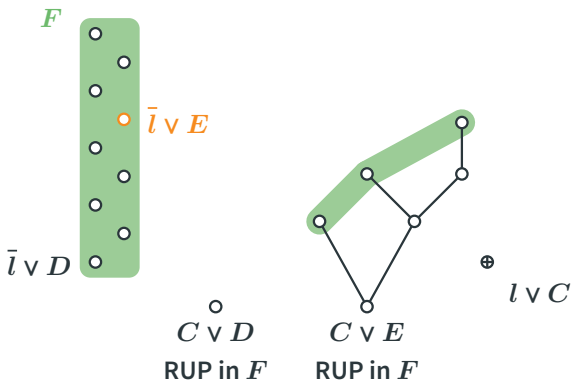

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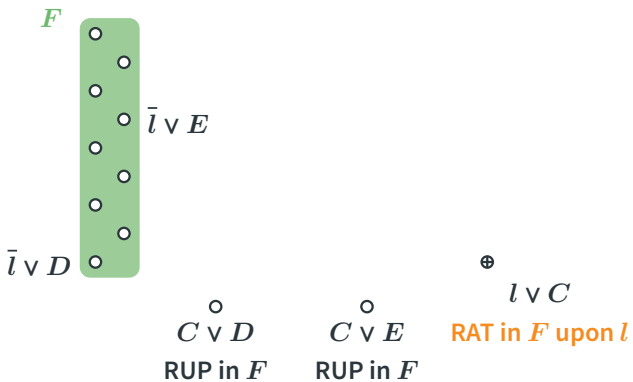

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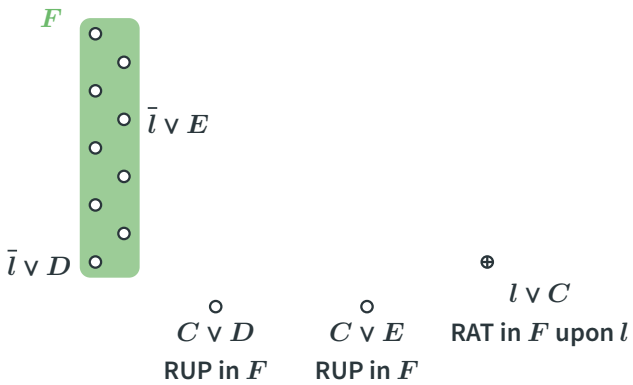

Theorem If C is a RAT in F, then F is satisfiable if and only if $F \cup \{C\}$ is.

RAT introduction can be used as an inference rule of a proof system.

The DRAT proof system

DRAT proof system RUP introduction + RAT introduction + arbitrary clause deletion

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Extended resolution resolution + definitions $p \leftrightarrow q \wedge r$ where p is fresh

Extended resolution can be simulated by DRAT

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Extended resolution can be simulated by DRAT

$$p \leftrightarrow q \wedge r \quad \equiv \quad (\neg p \vee q) \ \wedge \ (\neg p \vee r) \ \wedge \ (p \vee \neg q \vee \neg r)$$

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DRAT proof $(\neg p \vee q)^{\text{RAT}}$, $(\neg p \vee r)^{\text{RAT}}$, $(p \vee \neg q \vee \neg r)^{\text{RAT}}$

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no resolvents

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RAT upon $\neg p$

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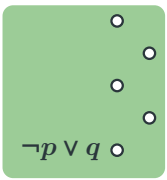

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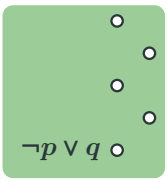

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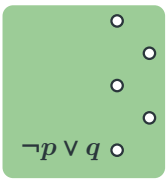

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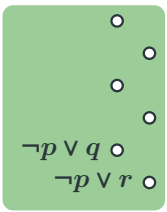

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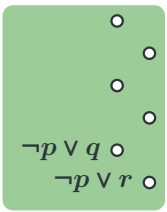

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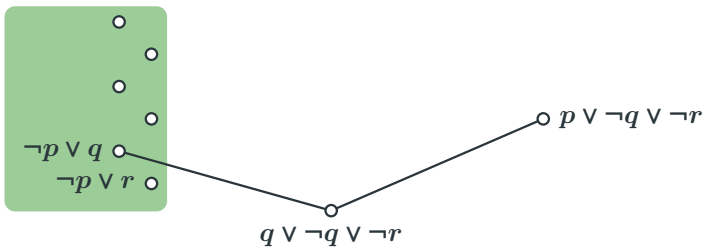

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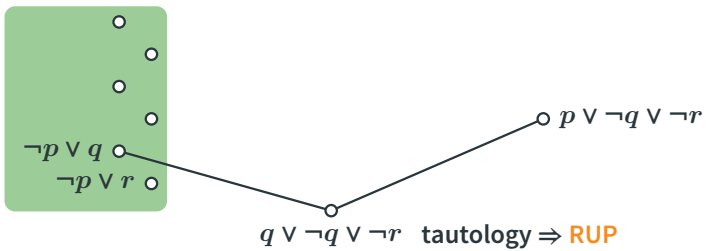

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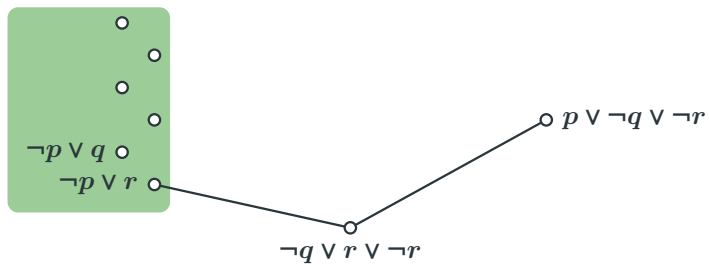

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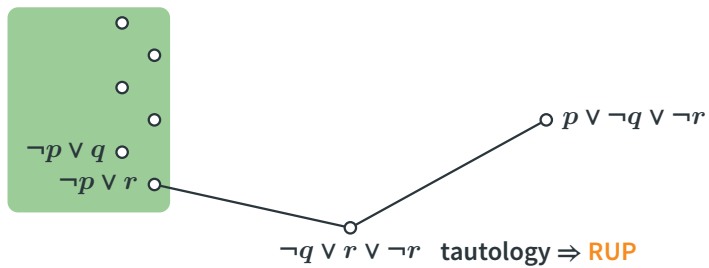

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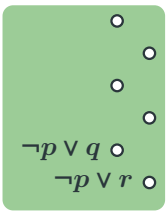

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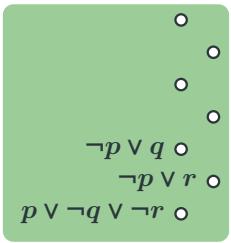

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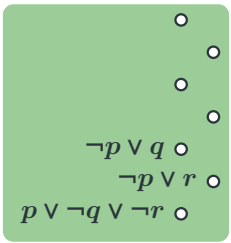

Properties of extended resolution

- No lower bound for length of extended resolution proofs is known.
- Used to express inprocessing techniques used in SAT solvers.
- Lacks the efficient interpolation property.
- No interpolation method is known.

Partial soundness of DRAT

Partial soundness $F \vdash G \quad \not\Rightarrow \quad F \models G$

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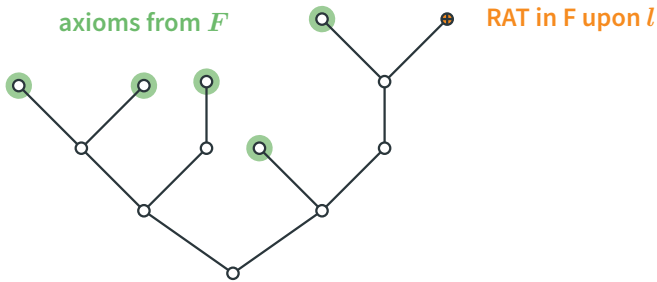
Interpolation and soundness

- Interpolation algorithms work because an induction invariant holds for partial interpolants.
- This invariant strongly requires soundness of the proof system.

Interpolation from DRAT proofs

RATs, consequences and bindings



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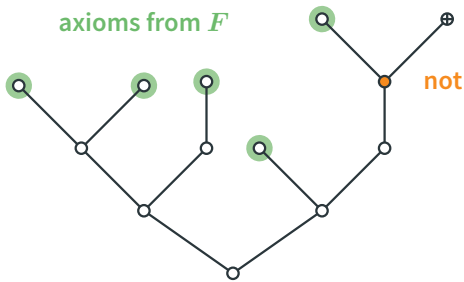

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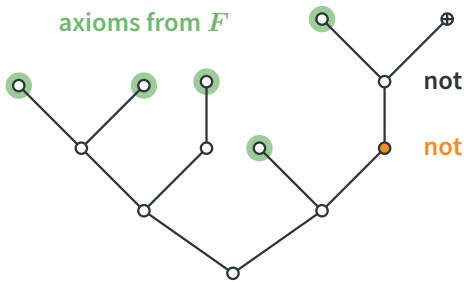

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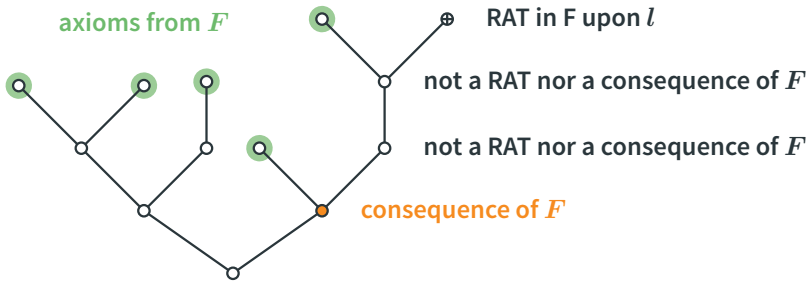

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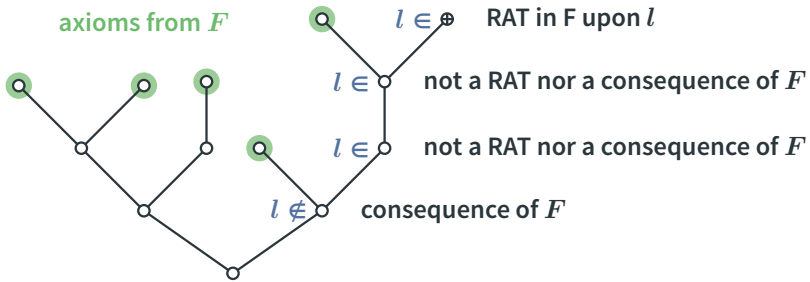

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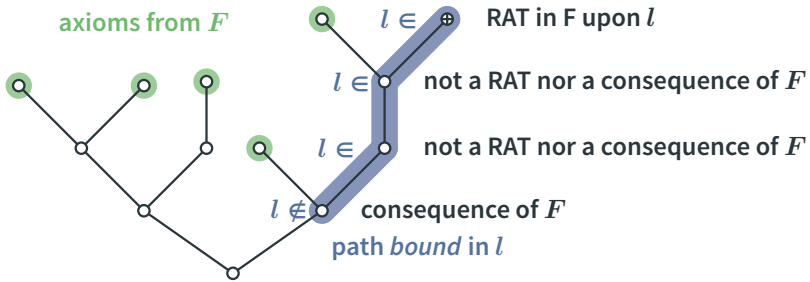

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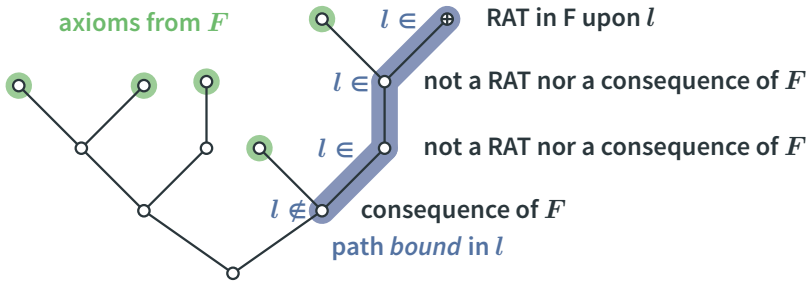

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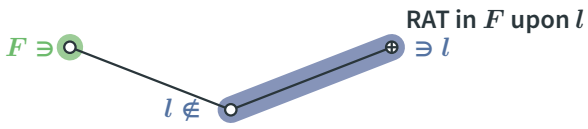

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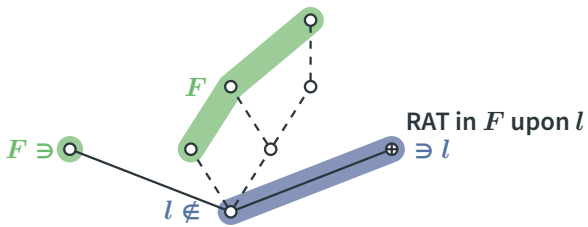

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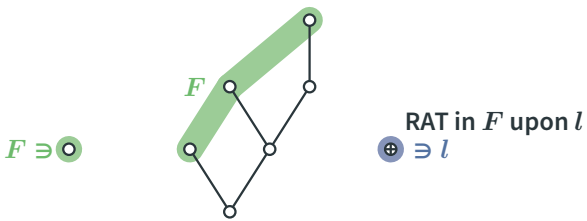

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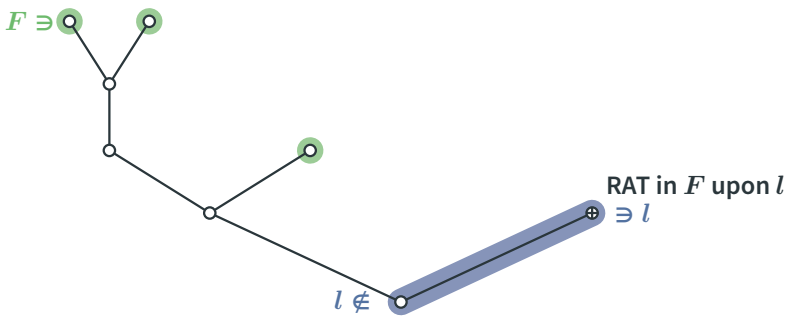

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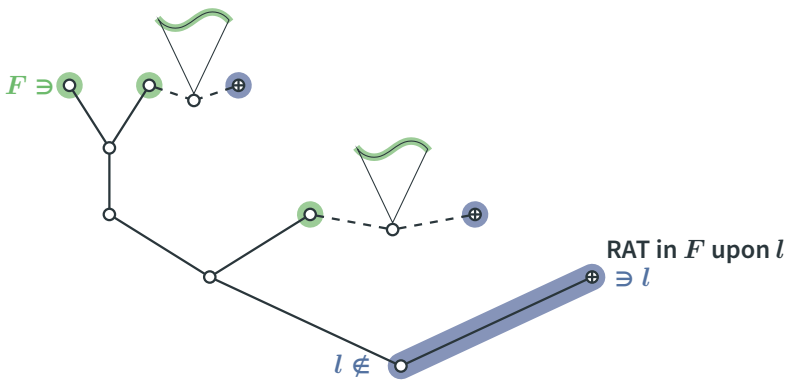

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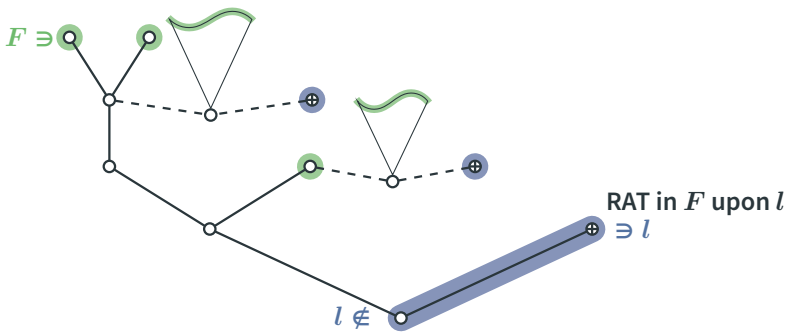

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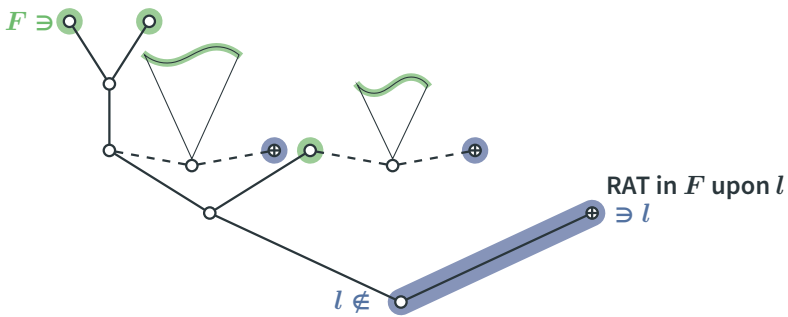

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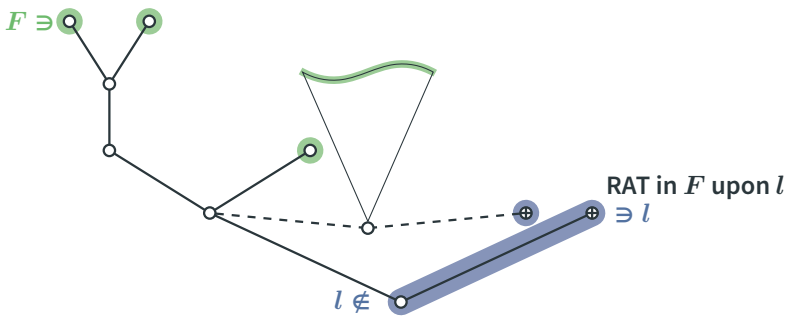

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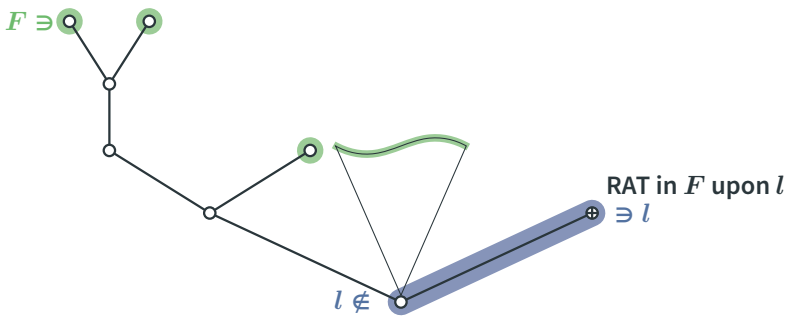

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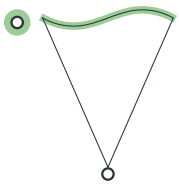

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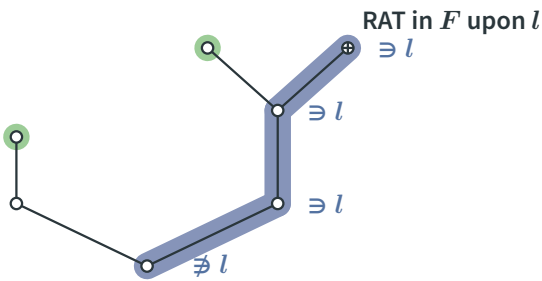

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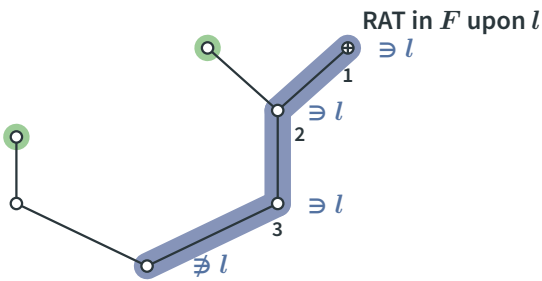

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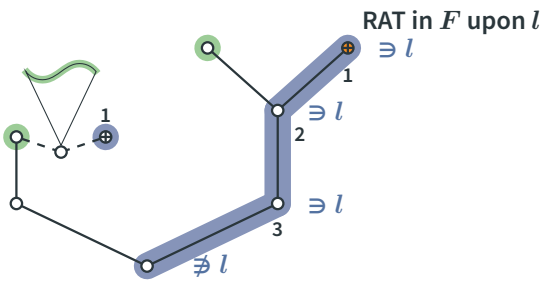

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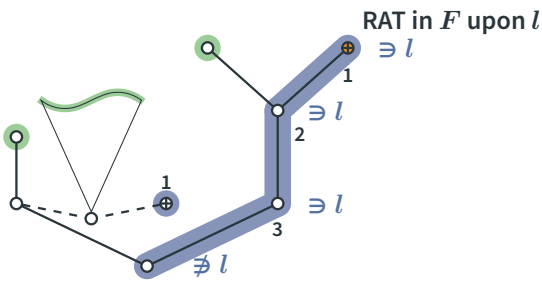

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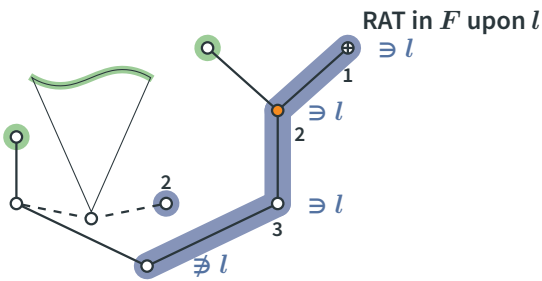

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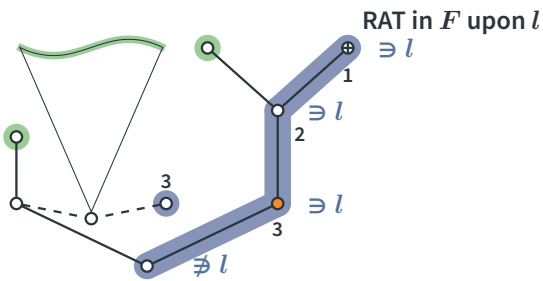

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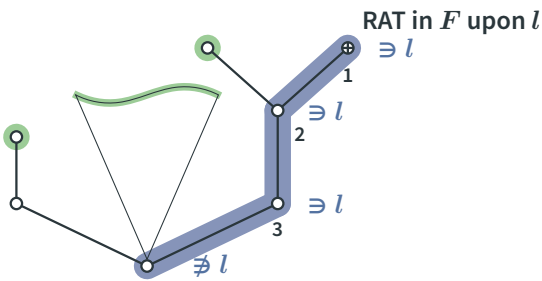

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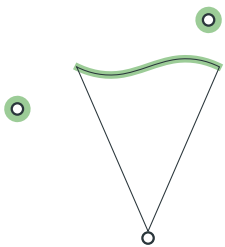
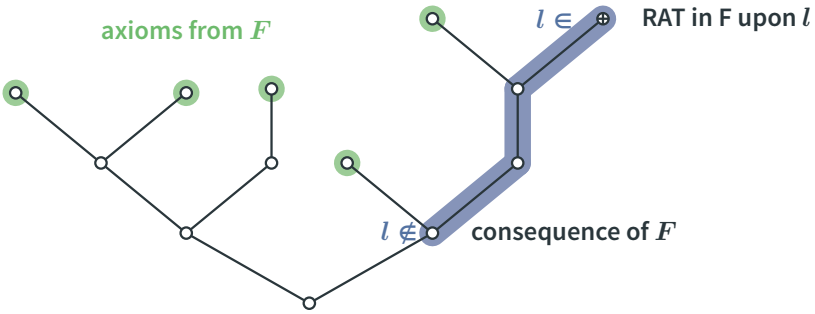


Interpolant generation from DRAT proofs

Interpolation by rectification into a resolution proof



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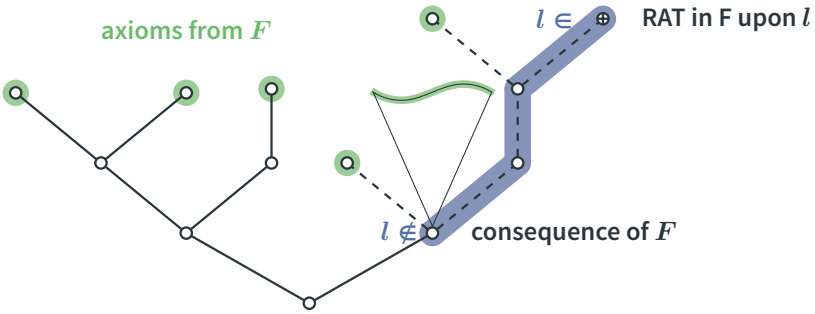

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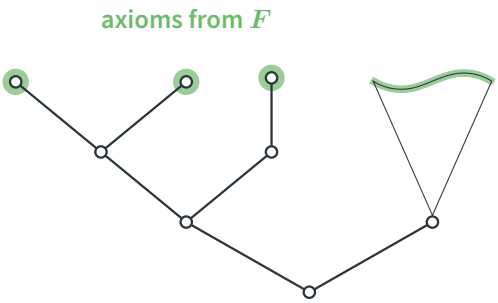

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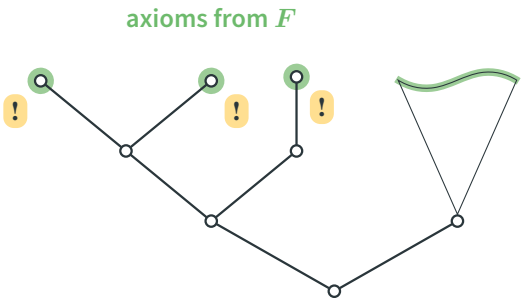

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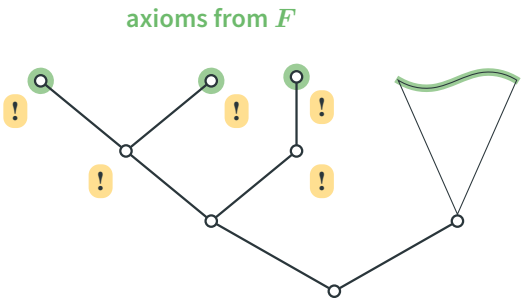

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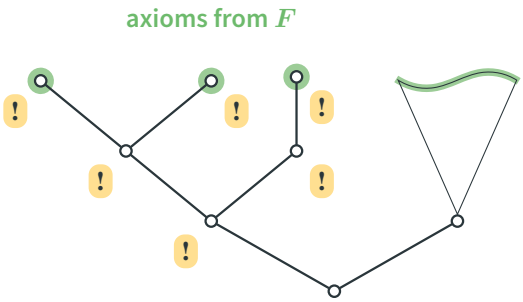

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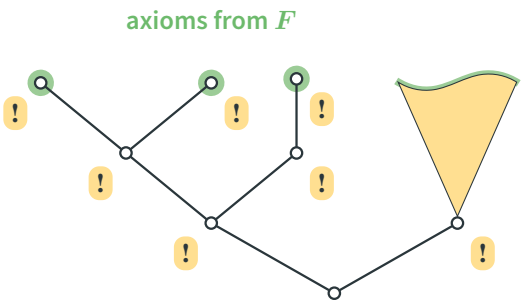

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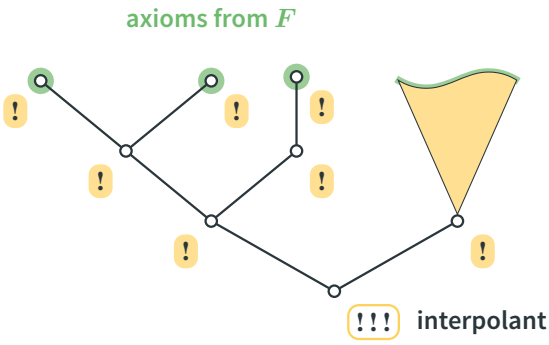

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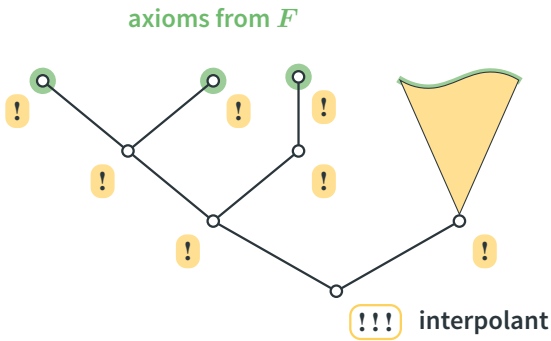

Issues

- **The interpolant may be exponential with respect to the DRAT proof.**
  But DRAT proofs can be exponentially shorter than DRUP proofs!
- **Currently we only eliminate RATs and bound paths one by one.**
  For a general enough case, the number of required sweeps is reduced.
- **Fully rectified DRAT proofs are huge and cannot be held in memory.**
  We try to store only necessary information and exploit RUPs to compress it.

Conclusion

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- State-of-the-art SAT solvers do not (and most likely, *will not*) produce resolution proofs, because of inprocessing techniques.

Conclusions

- State-of-the-art SAT solvers do not (and most likely, *will not*) produce resolution proofs, because of inprocessing techniques.
- The *de facto* standard DRAT certificates can be rectified into resolution proofs, and then interpolants can be extracted.

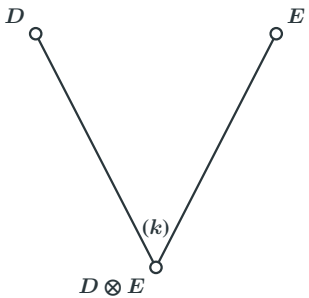
Conclusions

- State-of-the-art SAT solvers do not (and most likely, *will not*) produce resolution proofs, because of inprocessing techniques.
- The *de facto* standard DRAT certificates can be rectified into resolution proofs, and then interpolants can be extracted.
- Our efforts now are directed towards an efficient implementation of the algorithm by storing minimal information and using restrictive but general enough versions of DRAT.

Backup slides

A closer look into rectification

C RAT upon F in l
D, E clauses mutually resolvable upon k.



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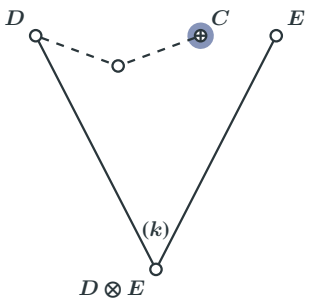
D is resolvable with C upon l



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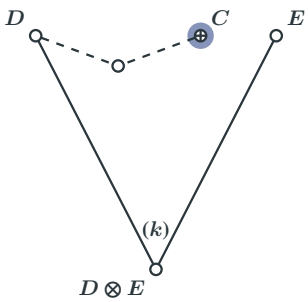
E is not resolvable with C upon l



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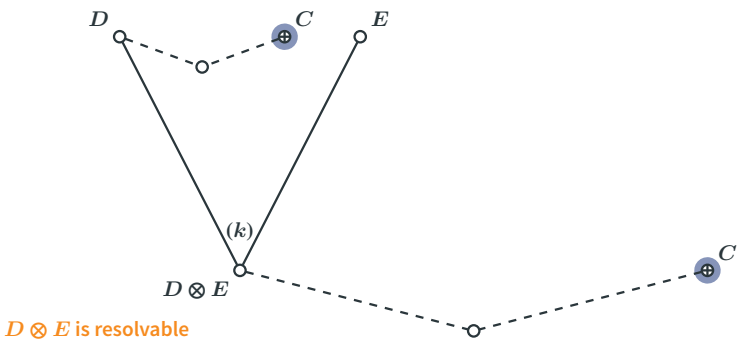

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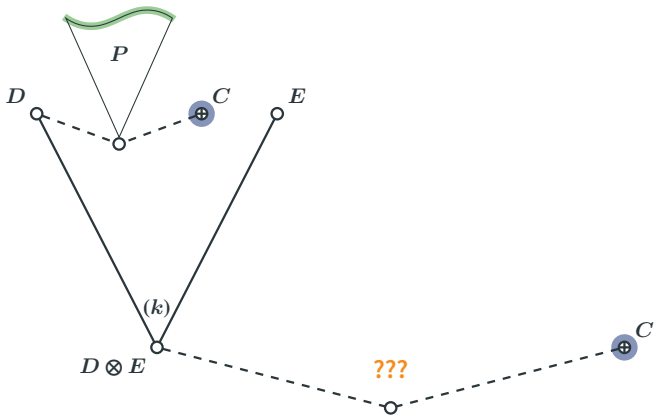

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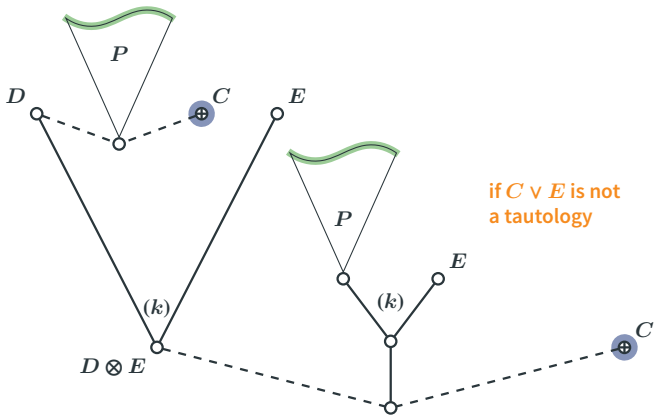

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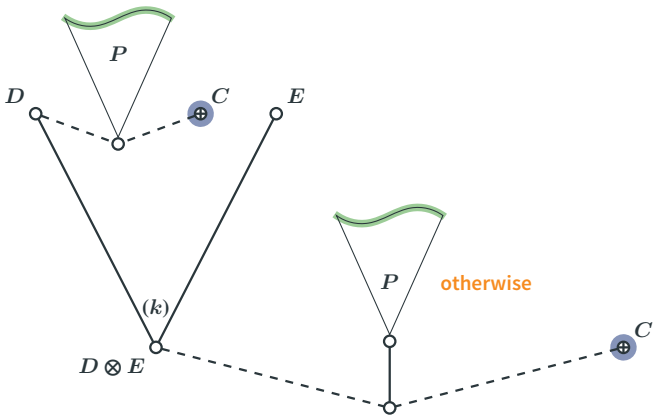
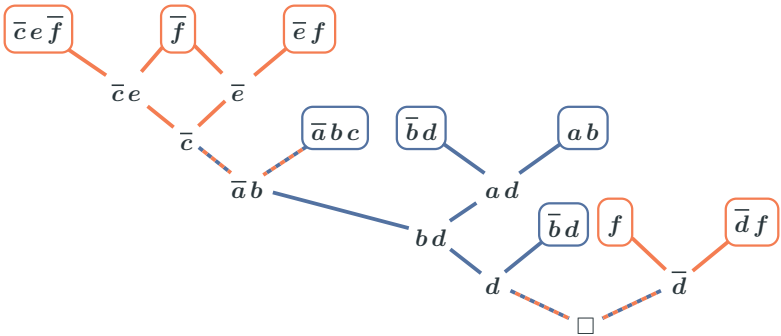


Interpolants from resolution proofs

Example

$A = (\overline{a}\, b\, c) \wedge (\overline{b}\, d) \wedge (a\, b)$ $\quad$ $B = (\overline{c}\, e\, f) \wedge (\overline{e}\, f) \wedge (\overline{d}\, f) \wedge (\overline{f})$



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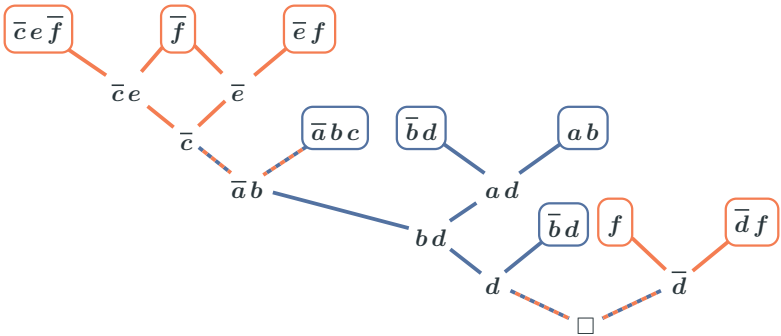
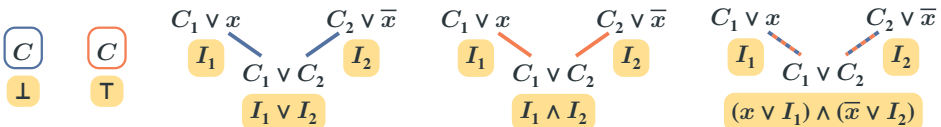


Interpolation rules [Huang '95]



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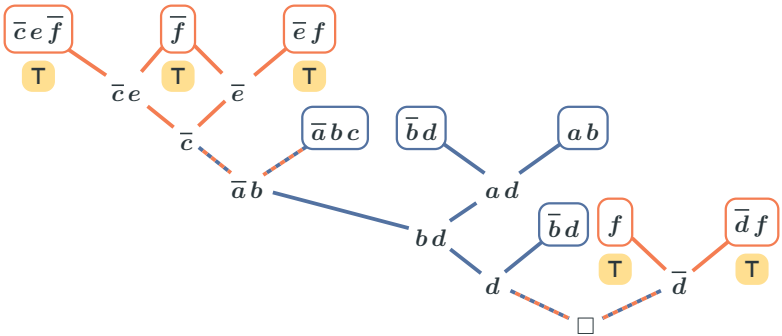

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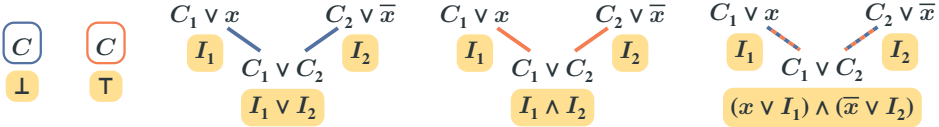

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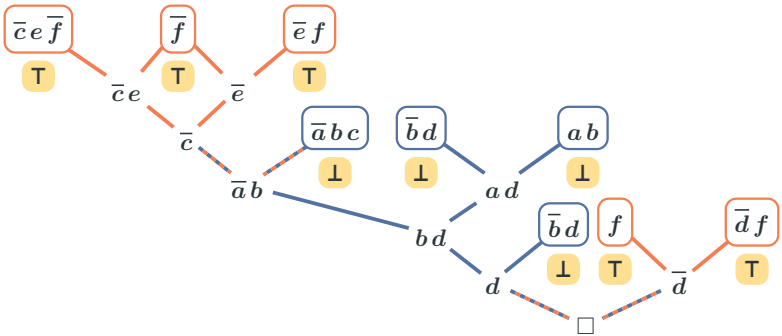

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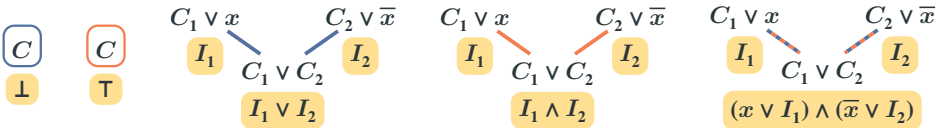

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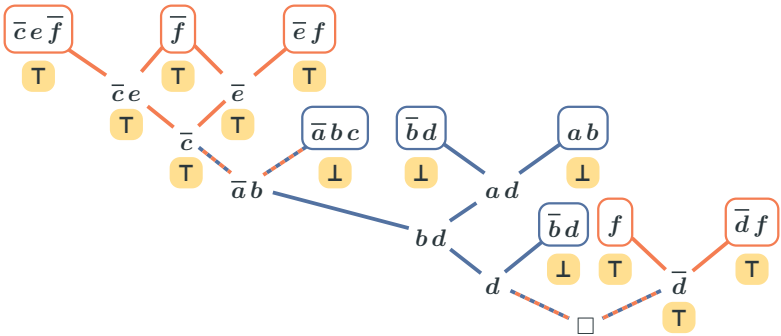

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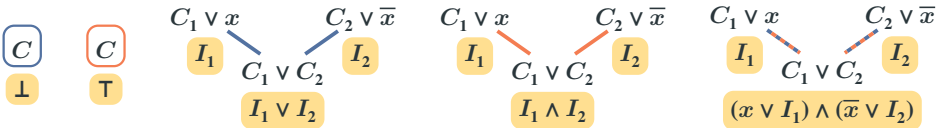

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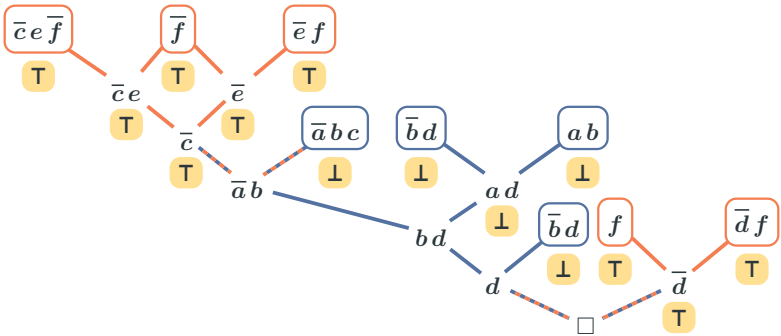

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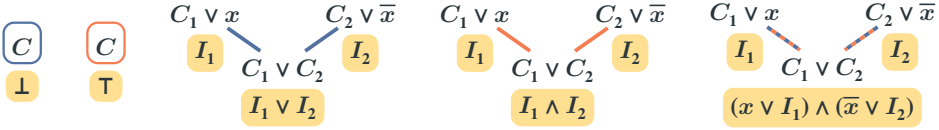

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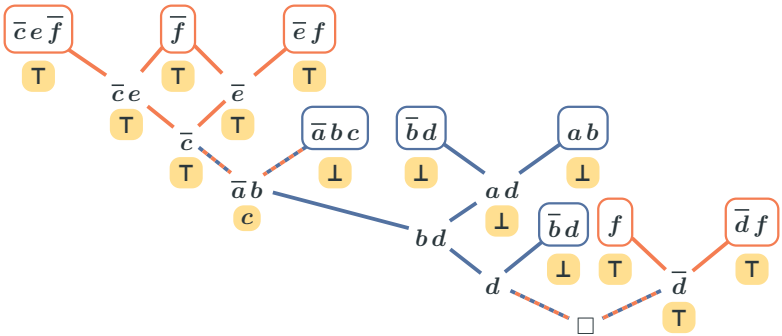

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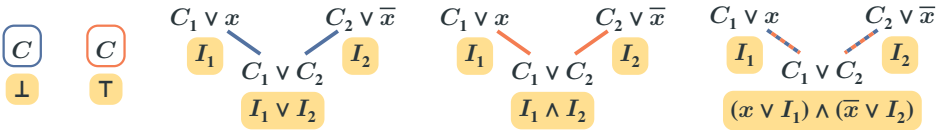

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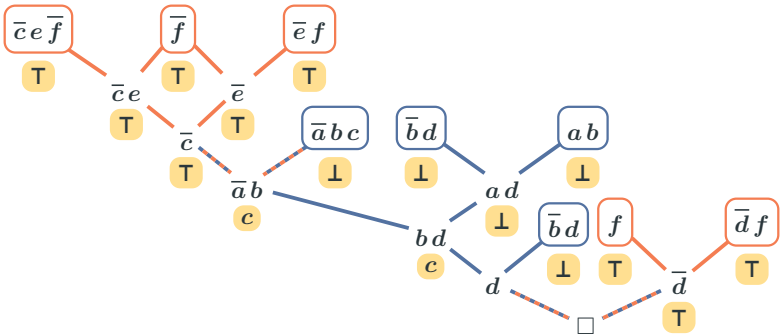

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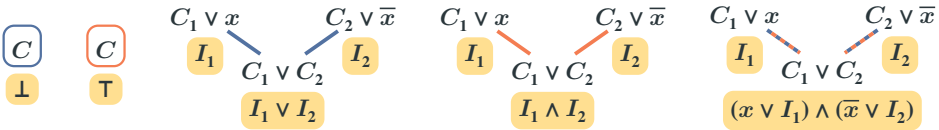

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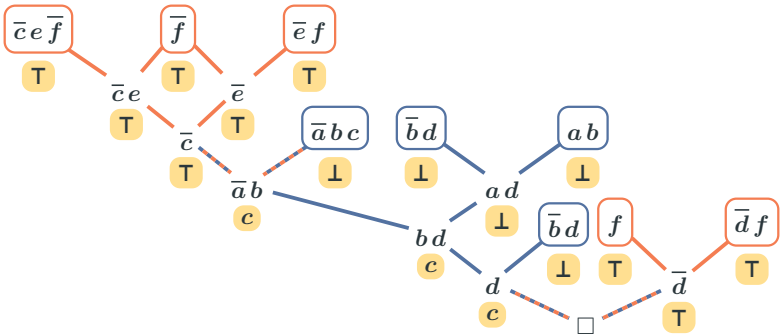

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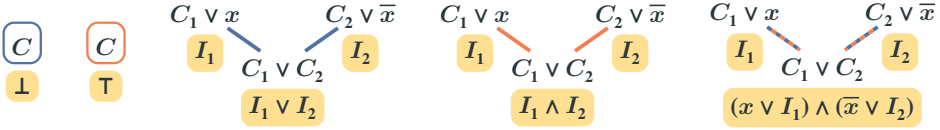

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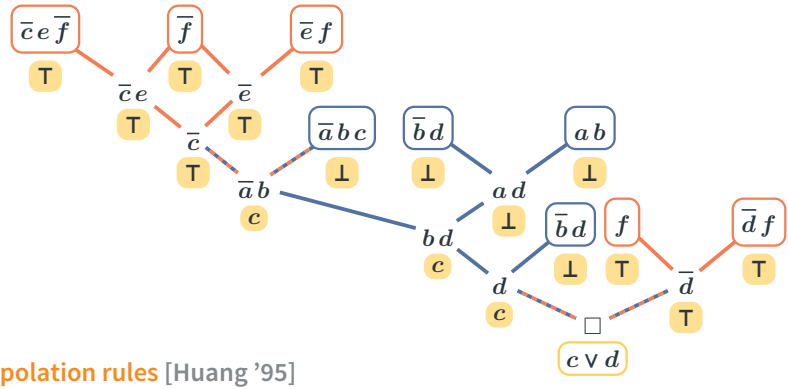

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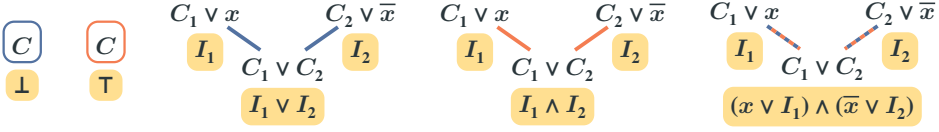

Proofless interpolation

Interpolation through communicating SAT solvers [Chockler Ivrii Matsliah '12]

A B

Proofless interpolation

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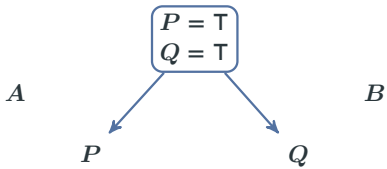

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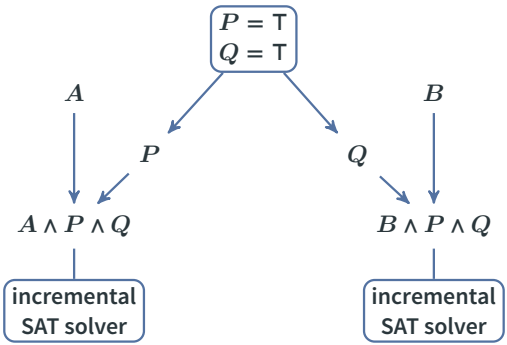

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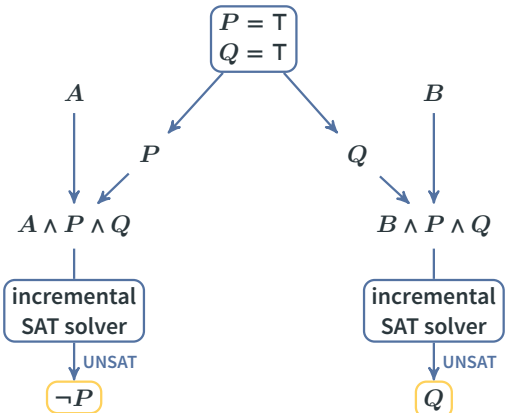

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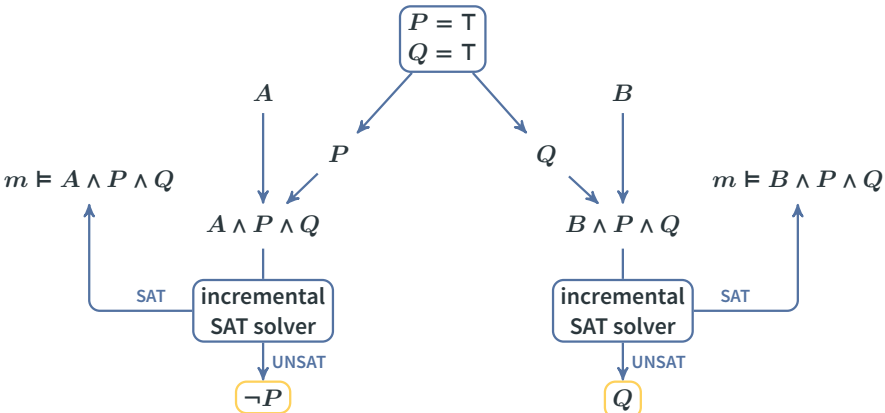

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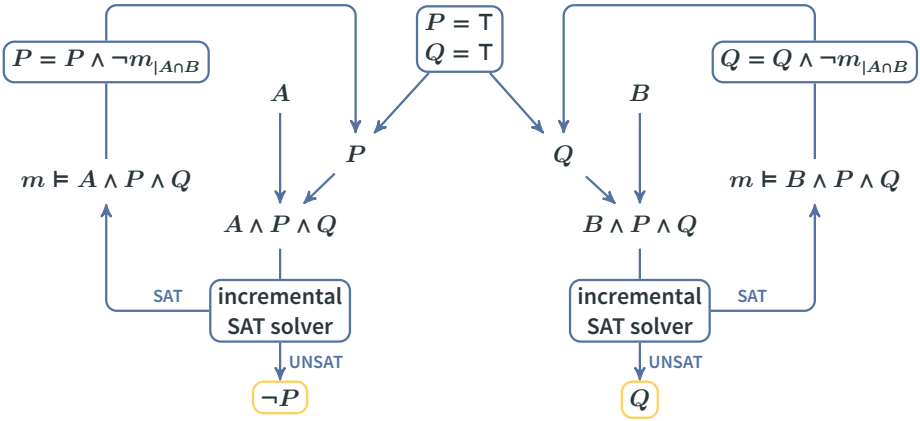

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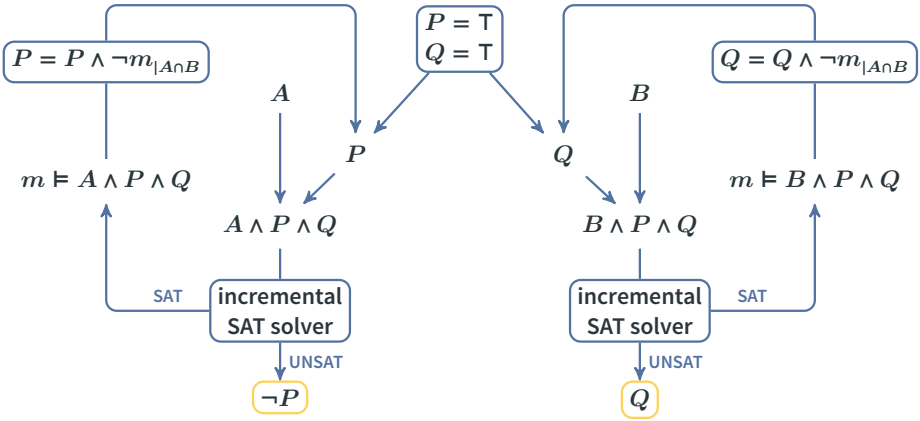
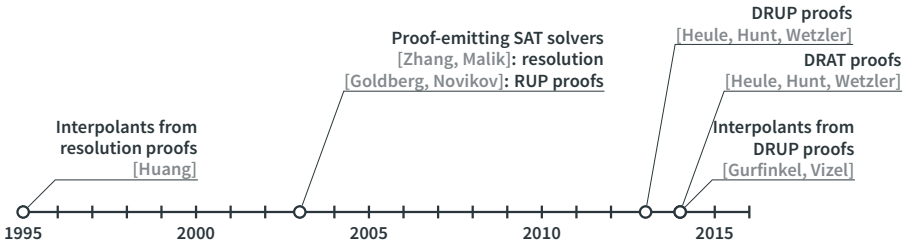


Disadvantages

- Simplification techniques can only be applied locally.
- Obtained interpolants are in DNF or CNF.
- Clause minimization is required to obtain reasonably-sized interpolants.

Proof systems for SAT solvers

A timeline of proof logging and interpolation for SAT solvers



Properties of proof systems

	Resolution	RUP	DRUP	DRAT
Manageable proof size				
Easily expresses CDCL				
Efficient verification				
Expressive enough for inprocessing				
Interpolant generation				

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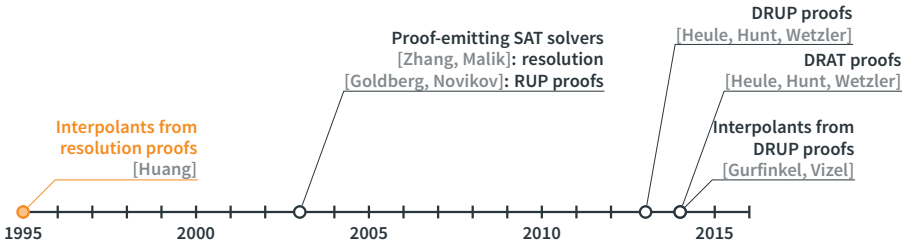

Properties of proof systems

	Resolution	RUP	DRUP	DRAT
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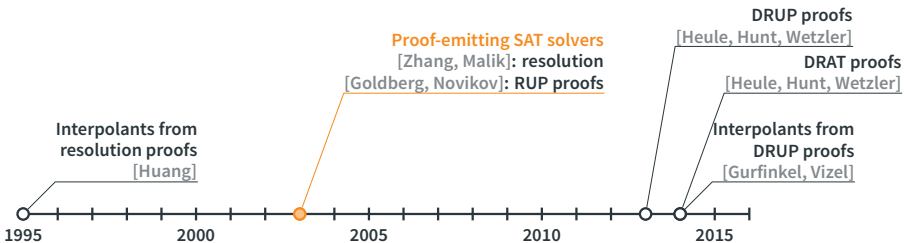

Properties of proof systems

	Resolution	RUP	DRUP	DRAT
Manageable proof size	✘	✔		
Easily expresses CDCL	✘	✔		
Efficient verification	✔	✘		
Expressive enough for inprocessing	✘	✘		
Interpolant generation	✔			

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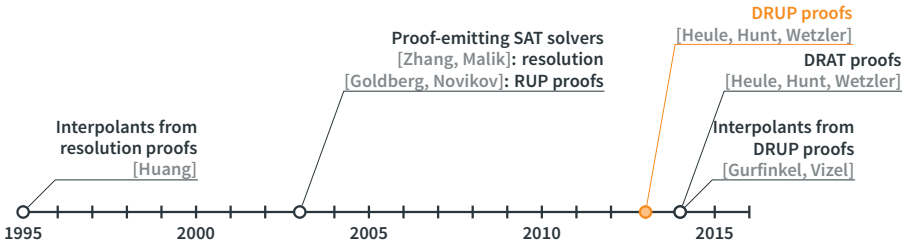

Properties of proof systems

	Resolution	RUP	DRUP	DRAT
Manageable proof size	✘	✔	✔	
Easily expresses CDCL	✘	✔	✔	
Efficient verification	✔	✘	✔	
Expressive enough for inprocessing	✘	✘	✘	
Interpolant generation	✔			

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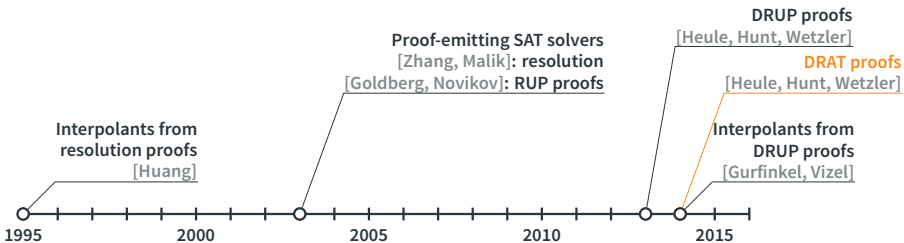

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	Resolution	RUP	DRUP	DRAT
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Easily expresses CDCL	✘	✔	✔	✔
Efficient verification	✔	✘	✔	✔
Expressive enough for inprocessing	✘	✘	✘	✔
Interpolant generation	✔			

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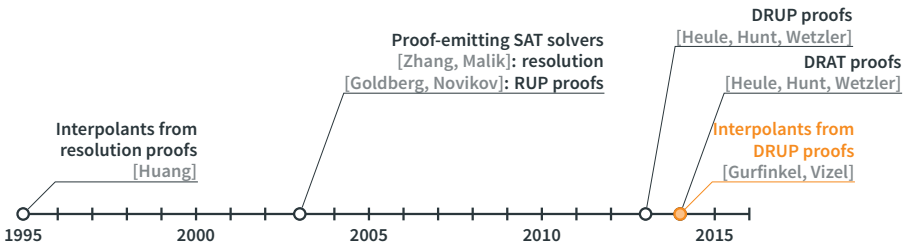

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Efficient verification	✔	✘	✔	✔
Expressive enough for inprocessing	✘	✘	✘	✔
Interpolant generation	✔	✔	✔	

Proof systems for SAT solvers

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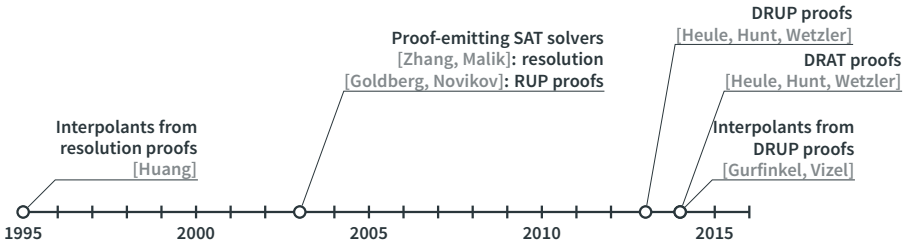

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Efficient verification	✔	✘	✔	✔
Expressive enough for inprocessing	✘	✘	✘	✔
Interpolant generation	✔	✔	✔	???