Verification of parameterized shared-memory asynchronous systems

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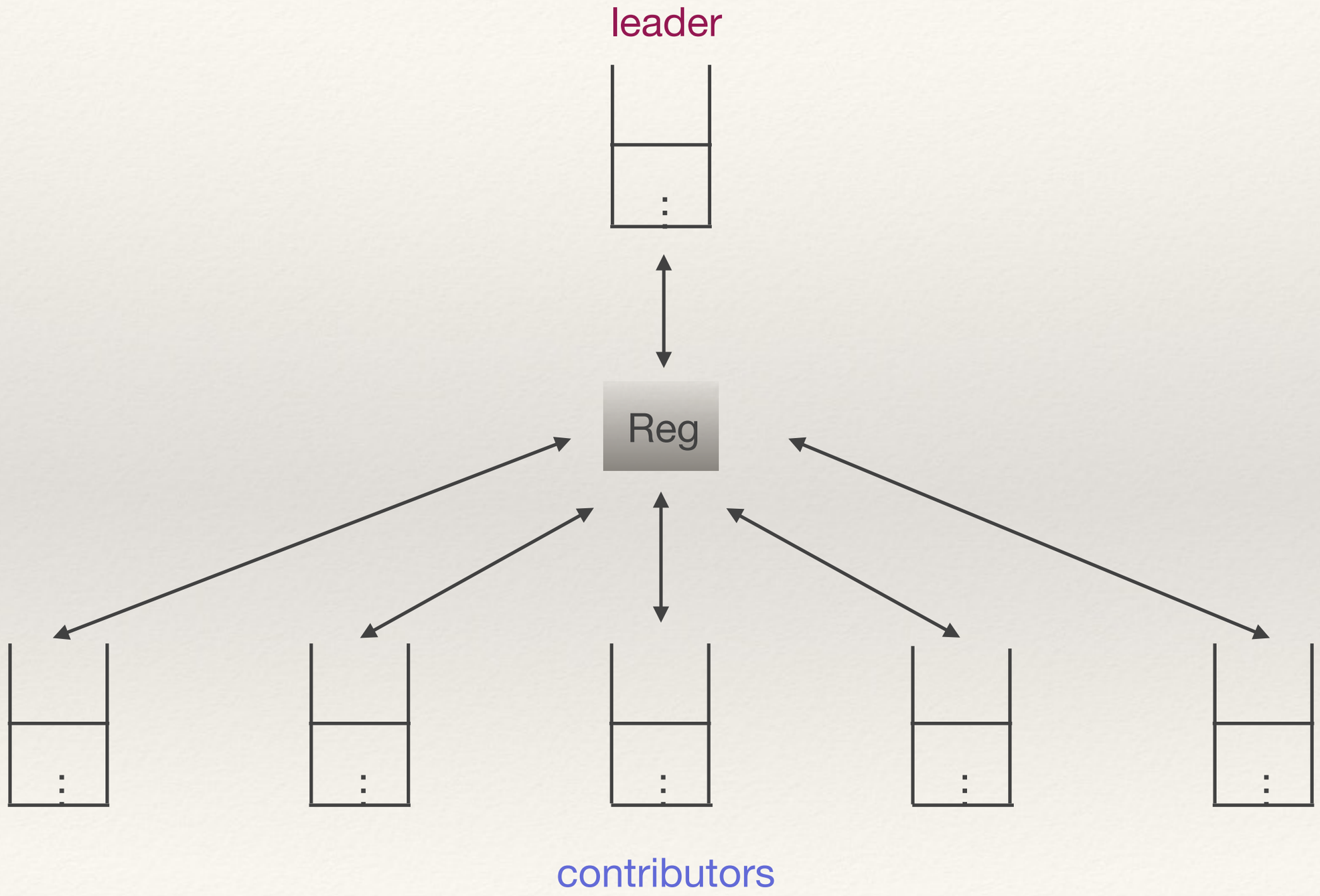
Joint work with: M. Fortin, S. LaTorre, I. Walukiewicz

Model: asynchronous, shared memory

Hague, 2011

Esparza, Ganty, Majumdar, 2013
Durand-Gasselin, Esparza, Ganty, Majumdar, 2015



Model: asynchronous, shared memory

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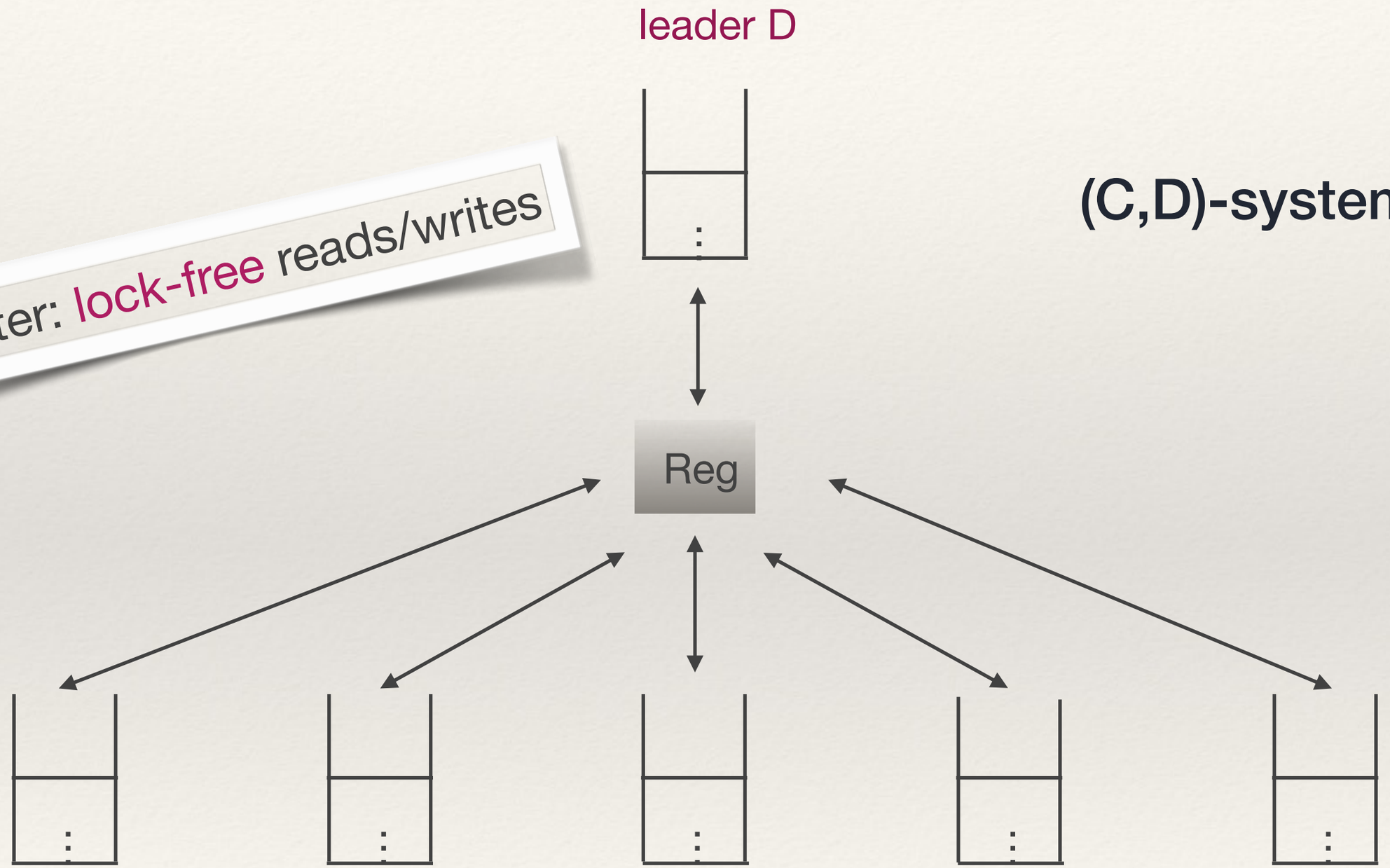
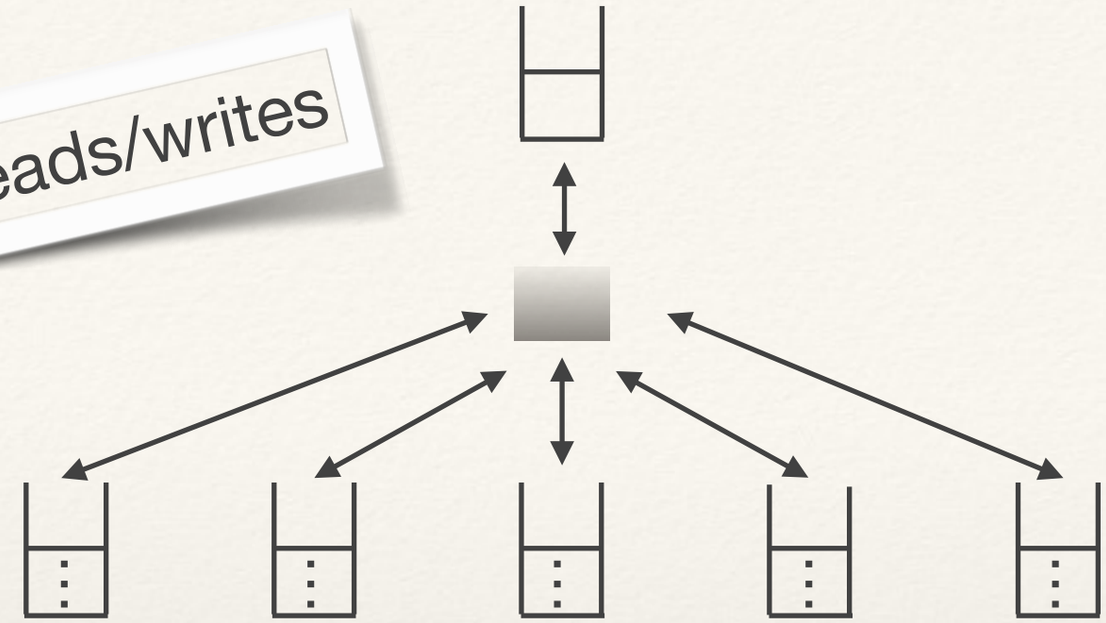
Register: lock-free reads/writes



(C,D)-systems

Register: lock-free reads/writes



(C,D)-systems

* Leader and contributors are pushdown processes.
* Contributors do not have identities.
* With locks: a unique contributor can be distinguished.
* With only one contributor: leader+contributor can simulate a Turing machine.
* Without locks and with an arbitrary number of contributors: the model becomes surprisingly manageable.

Parametrization as powerful abstraction (Kahlon, 2008)

Multiset semantics of (C,D)-systems

$$C = \langle S, \delta \subseteq S \times \Sigma_C \times S, s_{init} \rangle \qquad D = \langle T, \Delta \subseteq T \times \Sigma_D \times T, t_{init} \rangle \ .$$

contributor

leader

G: a finite set of register values

A configuration is (M, t, g), where $M \in \mathbb{N}^S$, $t \in T$, $g \in G$.

$(M,t,g) \xrightarrow{w(h)} (M,t',h)$ if $t \xrightarrow{w(h)} t'$ in Δ ,

$(M,t,g) \xrightarrow{r(h)} (M,t',h)$ if $t \xrightarrow{r(h)} t'$ in Δ and $h = g$,

$(M,t,g) \xrightarrow{\bar{w}(h)} (M',t,h)$ if $M \xrightarrow{\bar{w}(h)} M'$ in δ ,

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where $M \xrightarrow{a} M'$ in δ if $s \xrightarrow{a} s'$ in δ and $M' = M - [s] + [s']$, for some $s, s' \in S$.

$$(M,t,g) \xrightarrow{w(h)} (M,t',h) \qquad \text{if } t \xrightarrow{w(h)} t' \text{ in } \Delta\,,$$

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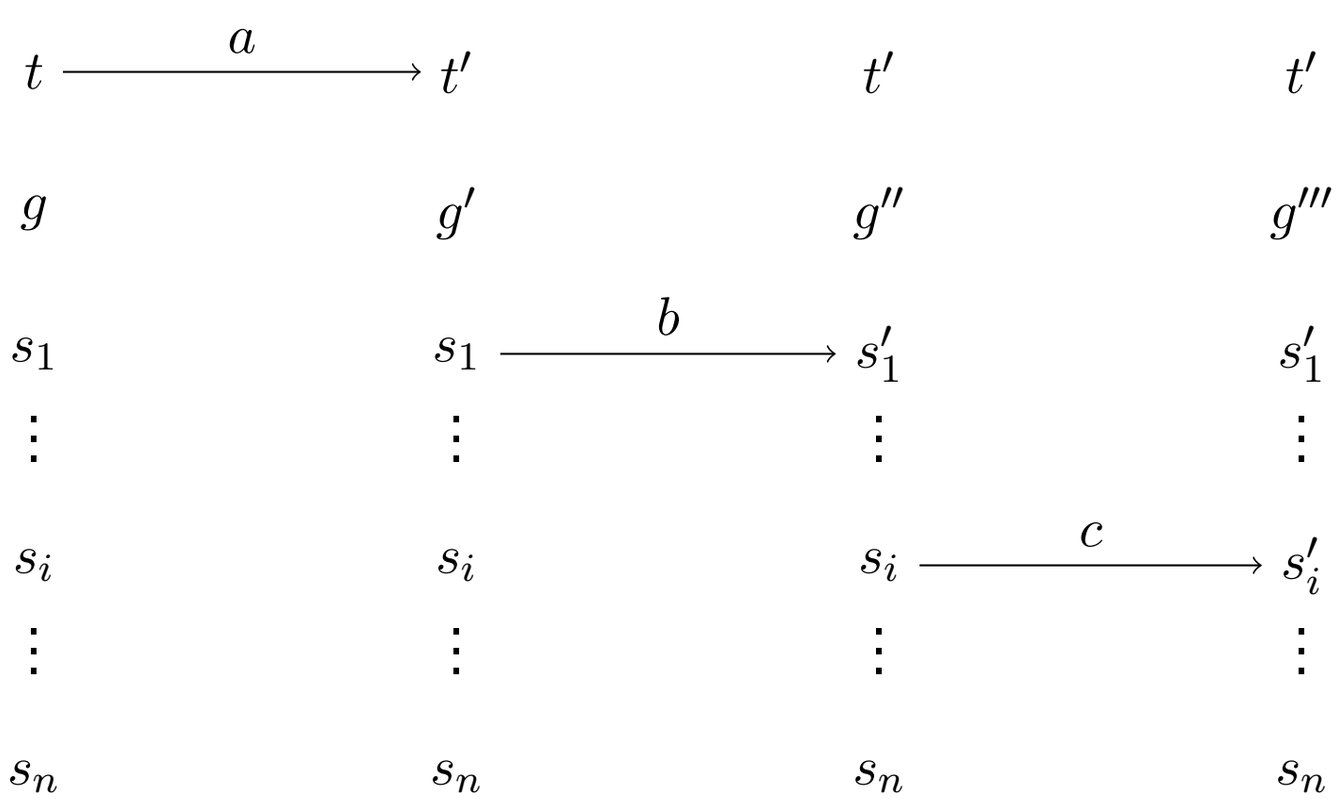


C,D may be infinite-state

$$C = \langle S, \delta \subseteq S \times \Sigma_C \times S, s_{init} \rangle \qquad D = \langle T, \Delta \subseteq T \times \Sigma_D \times T, t_{init} \rangle \ .$$

contributor

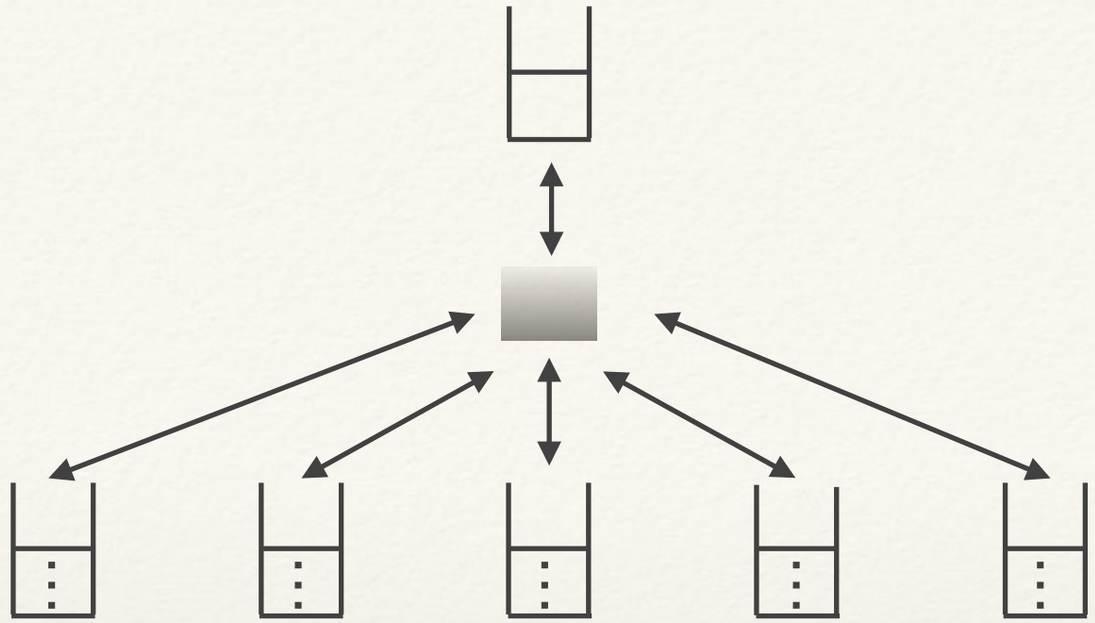
leader

Transition systems C and D need not to be finite.

In our case they are given by pushdown systems:

$$\mathcal{A}_C = \langle P, \Sigma_C, \Gamma_C, \delta, p_{init}, A^C_{init} \rangle \qquad \mathcal{A}_D = \langle Q, \Sigma_D, \Gamma_D, \Delta, q_{init}, A^D_{init} \rangle \ .$$

So $S = \{q\alpha : q \in P, \alpha \in \Gamma_C^*\}$



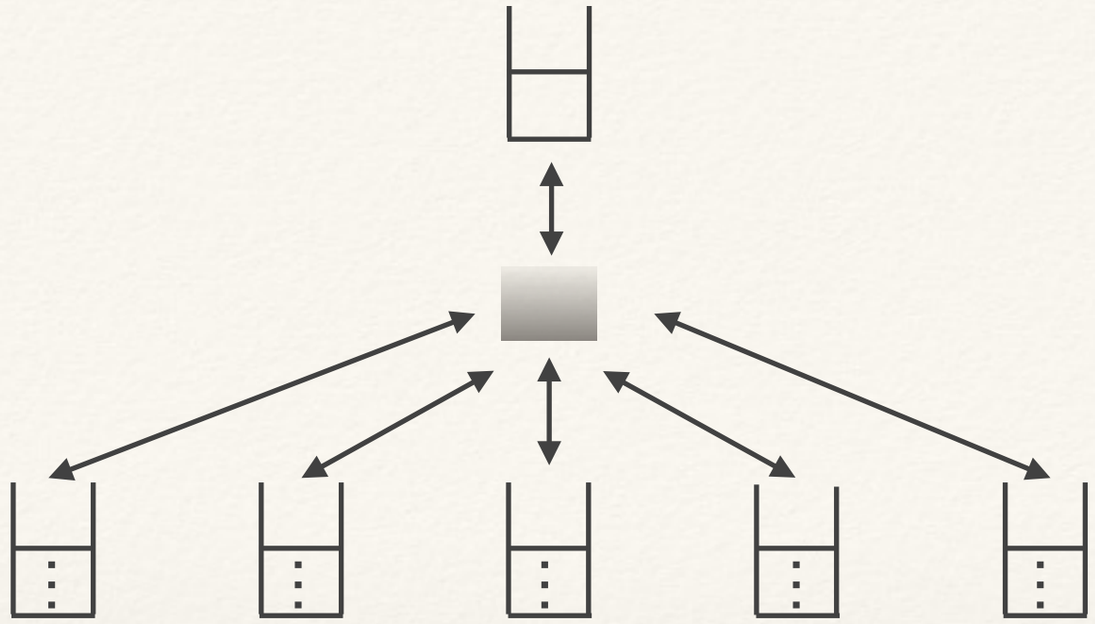
Example of system:

* Every contributor proposes a value
* Leader chooses one of these values
* The rest of the protocol uses the chosen value

Example of properties:

(for every n, for every run)

* Leader eventually decides on a value
* If the leader decides on a value, contributors use only this value
* On runs where only one value is used i.o. the protocol is correct



Example of a system:

* Contributors proposes values.
* Leader chooses one of these values.
* The rest of the protocol uses the chosen value.

Example properties:

(for every n, for every run)

* Leader eventually decides on a value
* If the leader decides on a value, contributors use only this value.
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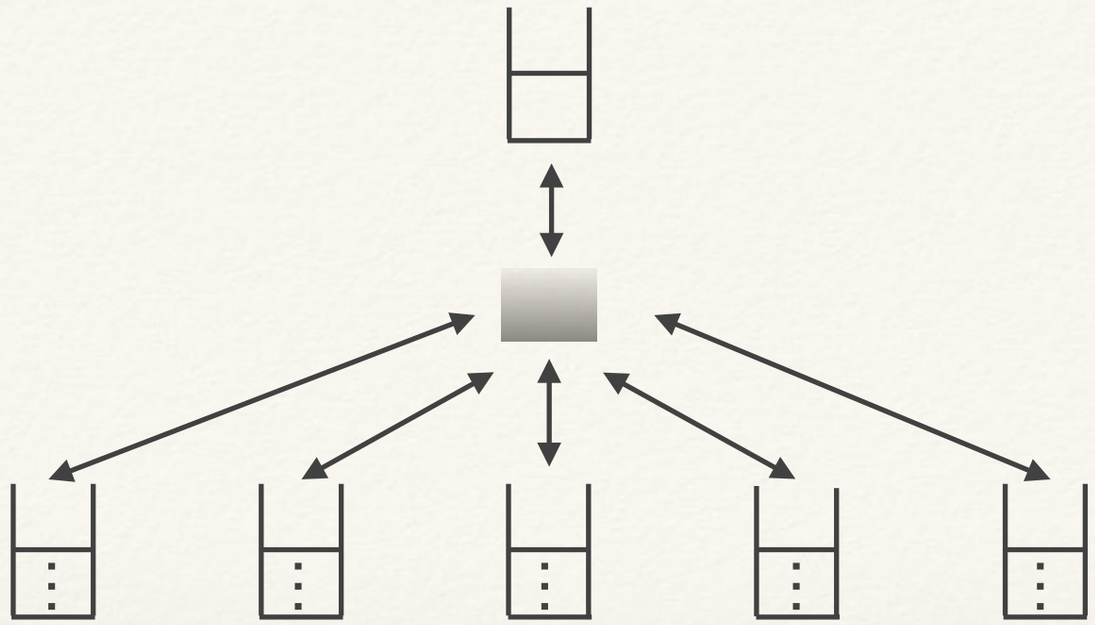
reachability

There is a run where the leader has decided on some value and afterwards a contributor uses a different value.



Example of a system:

* Contributors proposes values.
* Leader chooses one of these values.
* The rest of the protocol uses the chosen value.

Example properties:

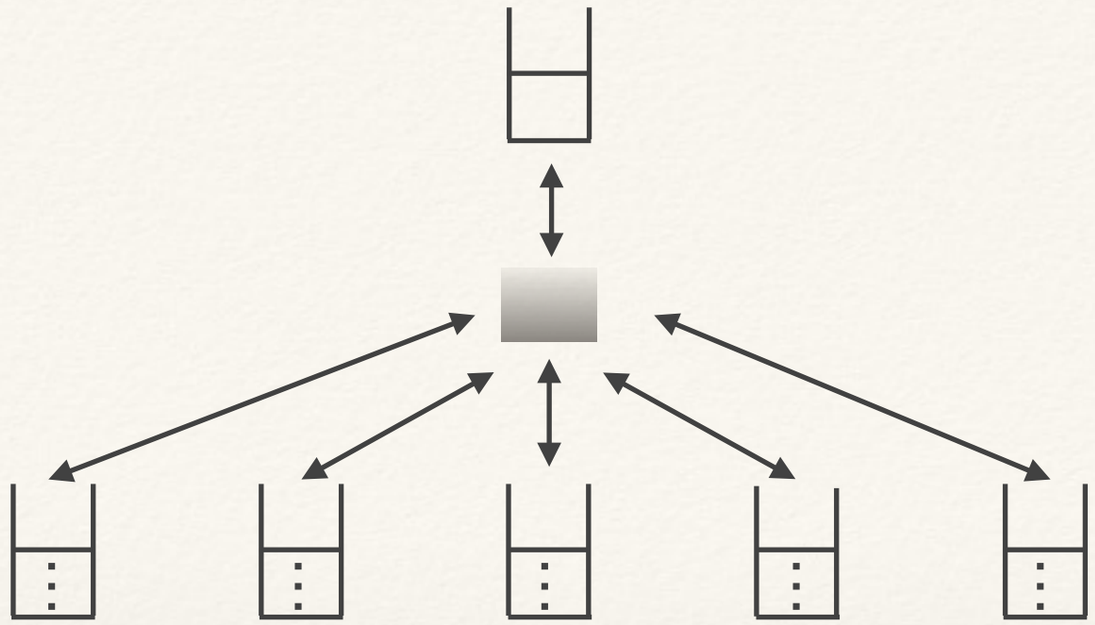
(for every n, for every run)

* Leader eventually decides on a value
* If the leader decides on a value, contributors use only this value.
* On runs where only one value is used i.o. the protocol is correct

safety

reachability

There is a maximal run where the leader does not decide on any value.



Example of a system:

* Contributors proposes values.
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Example properties:

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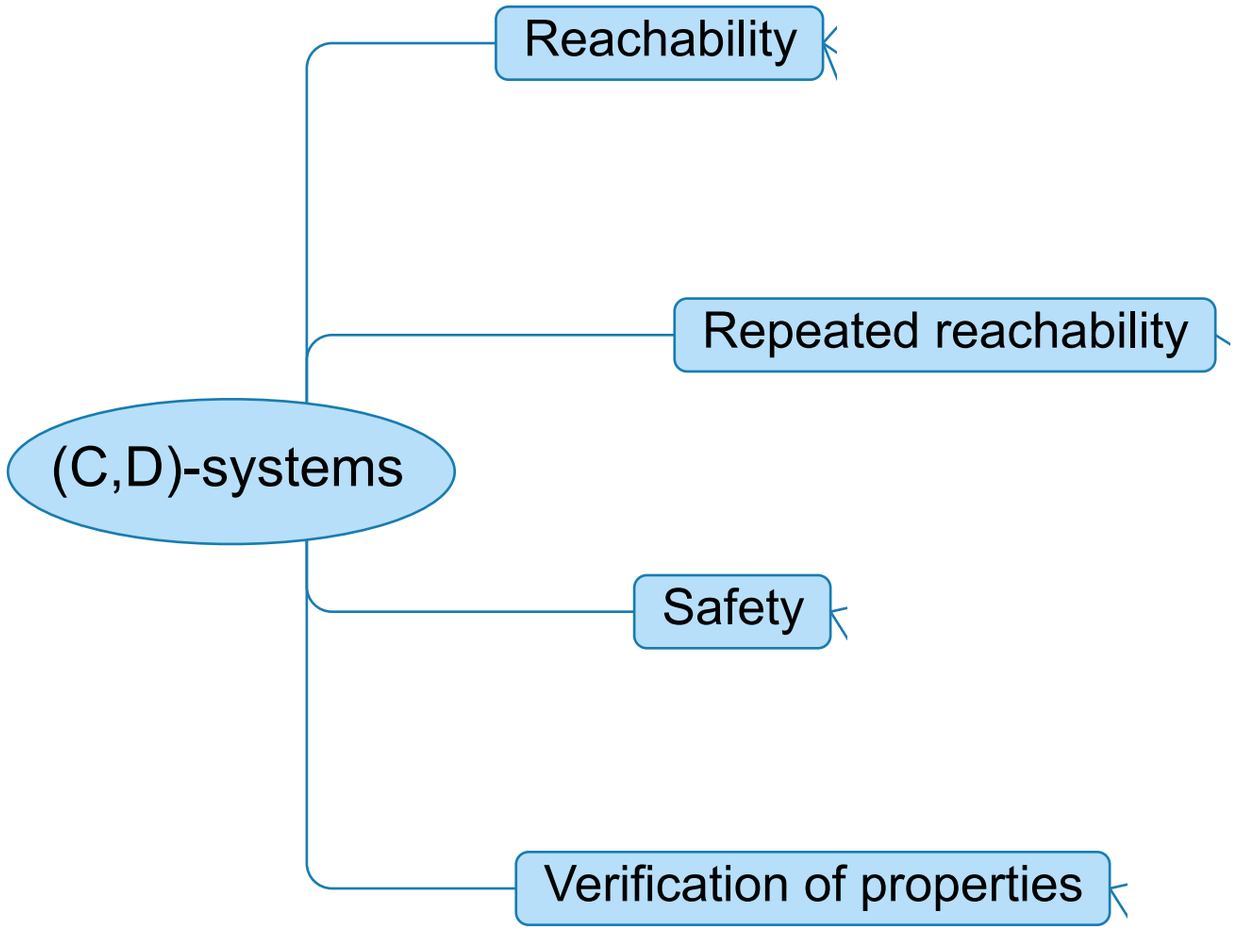
safe run

reachability

repeated reachability


Reachability
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(C,D)-systems
Safety
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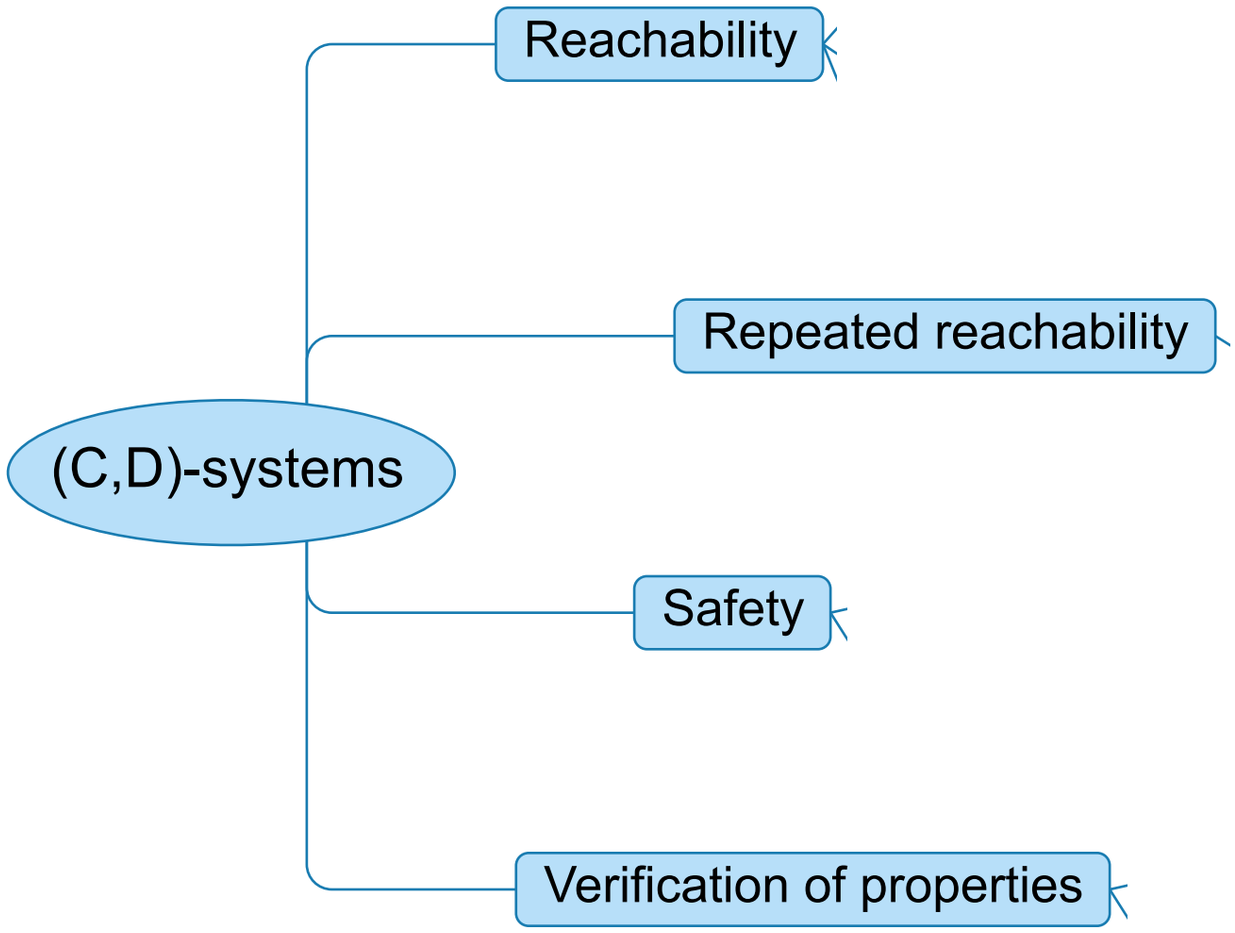
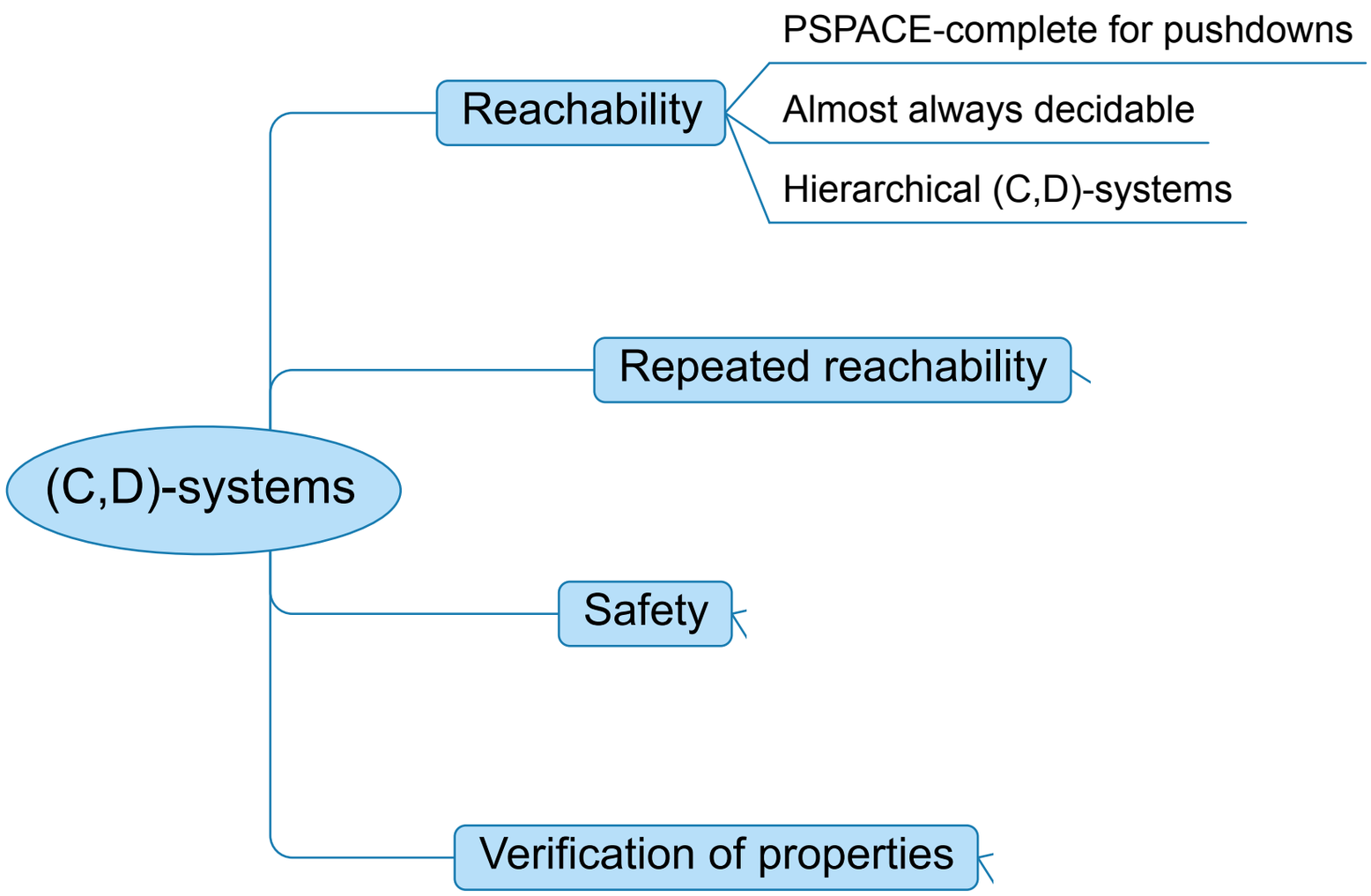


We are interested in the complexity of deciding these properties when C, D are pushdown systems.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PSPACE-complete for pushdowns
Almost always decidable
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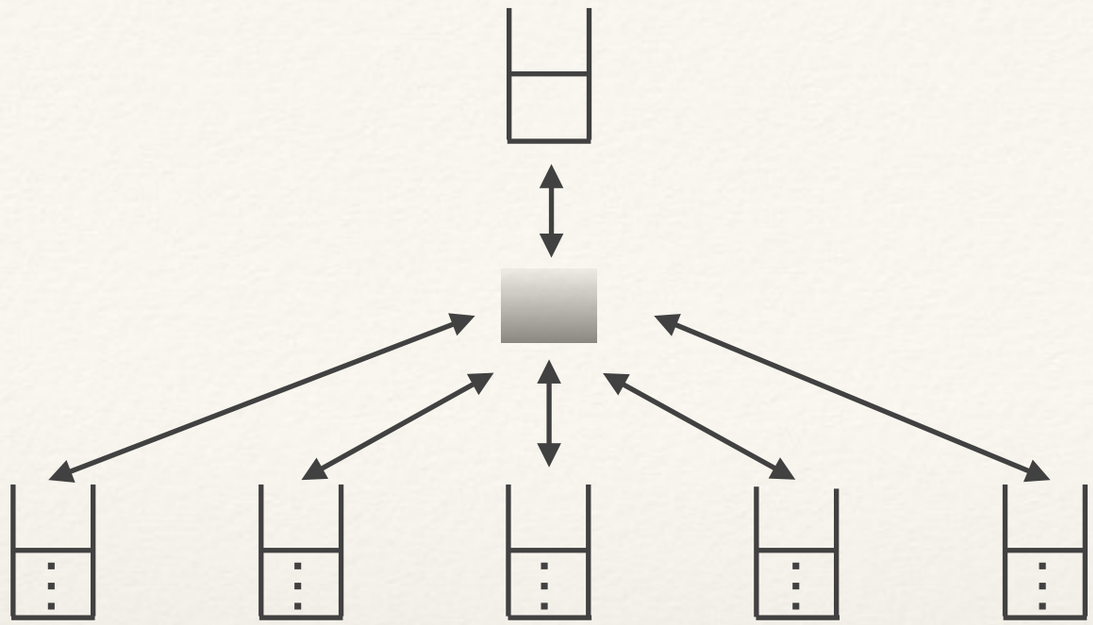
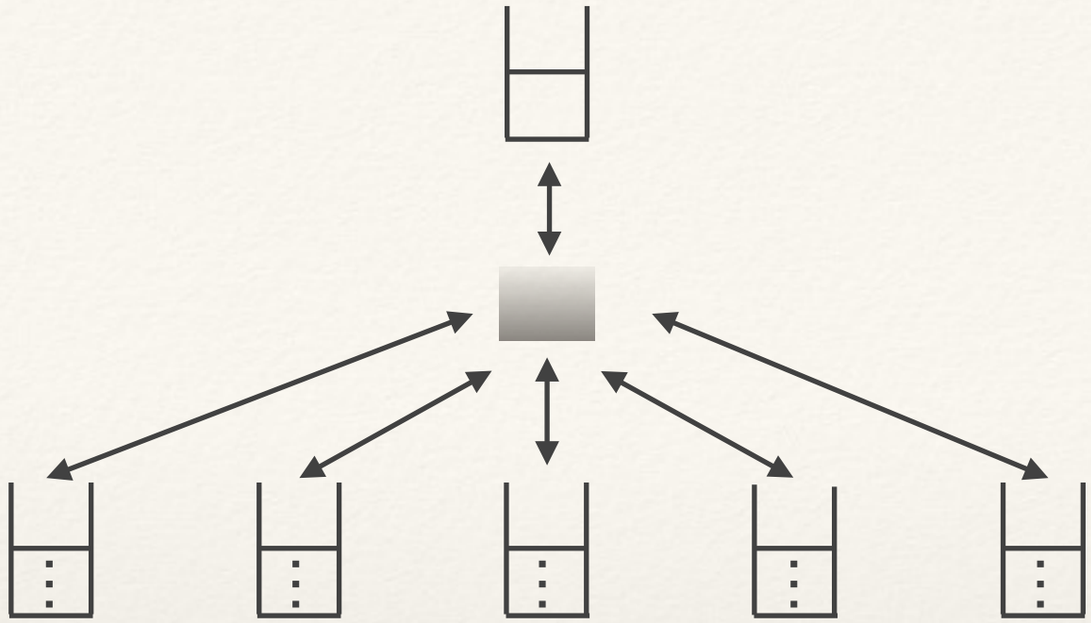


Reachability in (C,D)-systems

Given a leader D from some class of systems $\mathcal{D}$ and a contributor C from some class $\mathcal{C}$, is there some integer n such that D, together with n copies of C, writes a particular value into the register along some run?

Fact

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems and ***n* is known,** the problem is undecidable.



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Given a leader D from some class of systems $\mathcal{D}$ and a contributor C from some class $\mathcal{C}$, is there some integer n such that D, together with n copies of C, writes a particular value into the register along some run?

Thm [Hague, Esparza et al. 2013]

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems then the reachability problem is decidable (PSPACE-complete). If $\mathcal{C}$ is finite-state systems, the problem is NP-complete.

Accumulator semantics

$$(B,t,g) \xrightarrow{w(h)} (B,t',h) \qquad \text{if } t \xrightarrow{w(h)} t' \text{ in } \Delta\,,$$

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$B \subseteq S$

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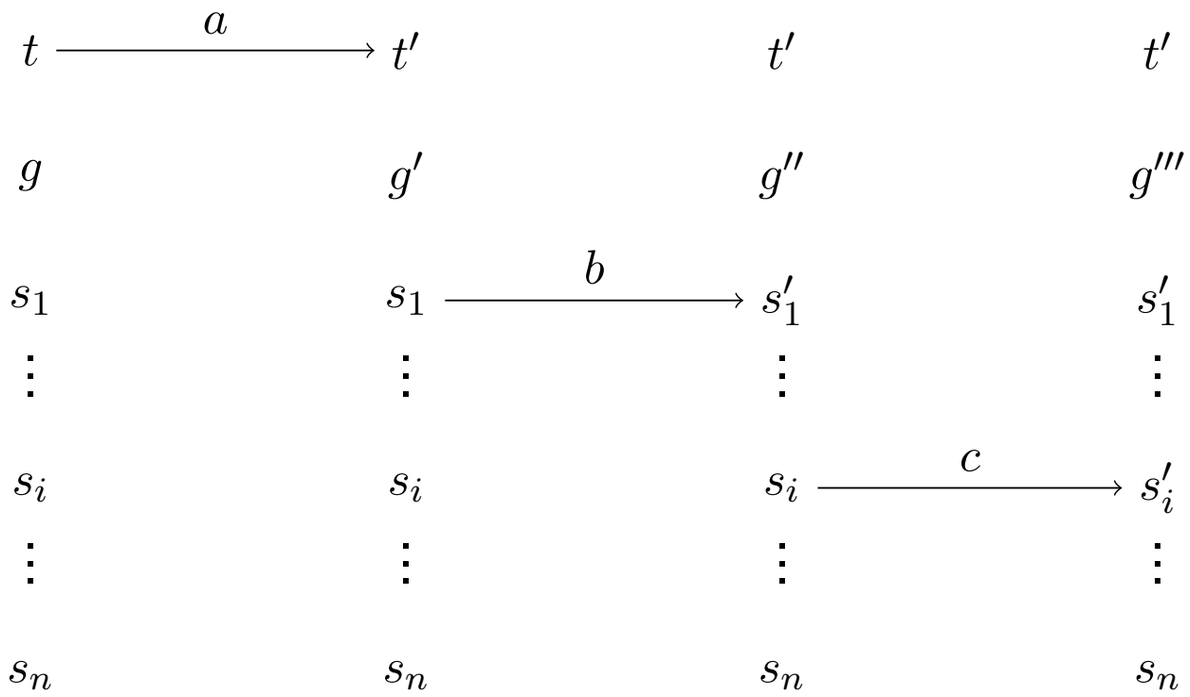
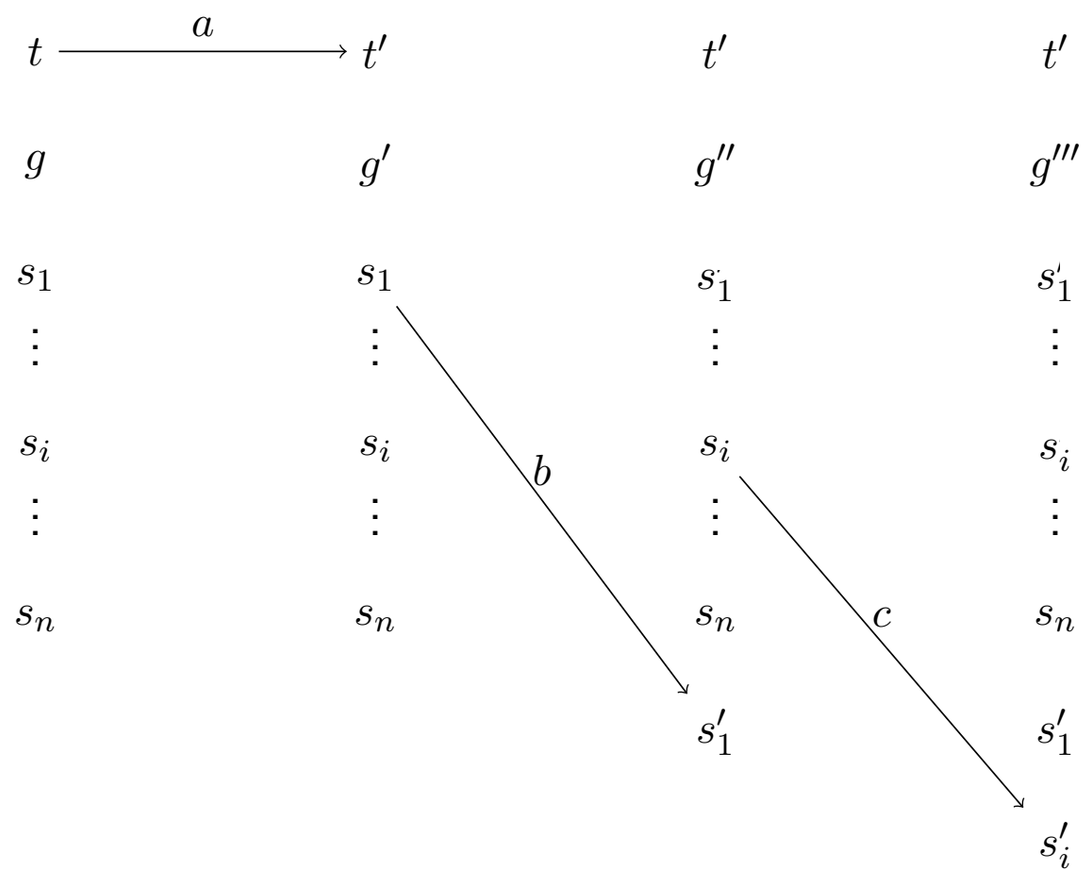
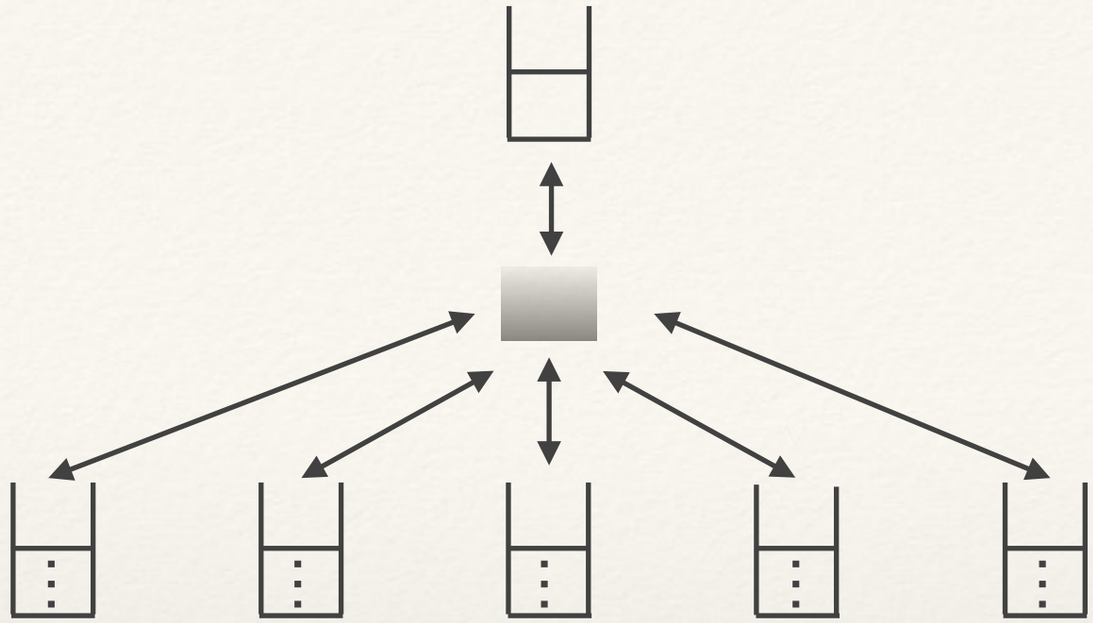
$M \xrightarrow{a} M'$ in δ if $s \xrightarrow{a} s'$ in δ and $M' = M - [s] + [s']$, for some $s, s' \in S$.

Multiset



Accumulator





Thm [La Torre, M., Walukiewicz 2015]

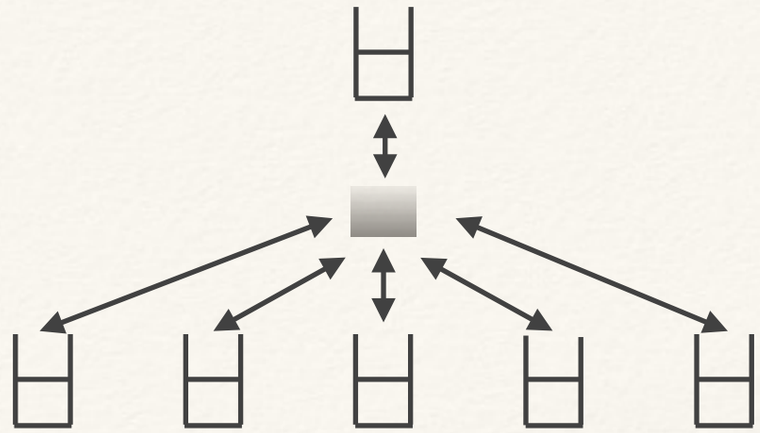
Let $\mathcal{C}$ and $\mathcal{D}$ be both effectively closed under synchronized product with finite-state systems.
If $\mathcal{C}$ has a decidable reachability problem and $\mathcal{D}$ has effective downward closure, the reachability for $(\mathcal{C},\mathcal{D})$-systems is decidable.

Thm

Let $\mathcal{C}$ and $\mathcal{D}$ be both effectively closed under synchronized product with finite-state systems.
If $\mathcal{C}$ has a decidable reachability problem and $\mathcal{D}$ has effective downward closure, then reachability for $(\mathcal{C},\mathcal{D})$-systems is decidable.

$\mathcal{C}$ is effectively closed under synchronized product with finite-state systems:
given M from $\mathcal{C}$ and a finite automaton A, the synchronized product of M and A belongs to $\mathcal{C}$ and can be effectively constructed.

$\mathcal{D}$ has effective downward closure:
given M from $\mathcal{D}$, the finite automaton accepting all (scattered) subwords of traces of M can be constructed effectively.



Effective downward closure:

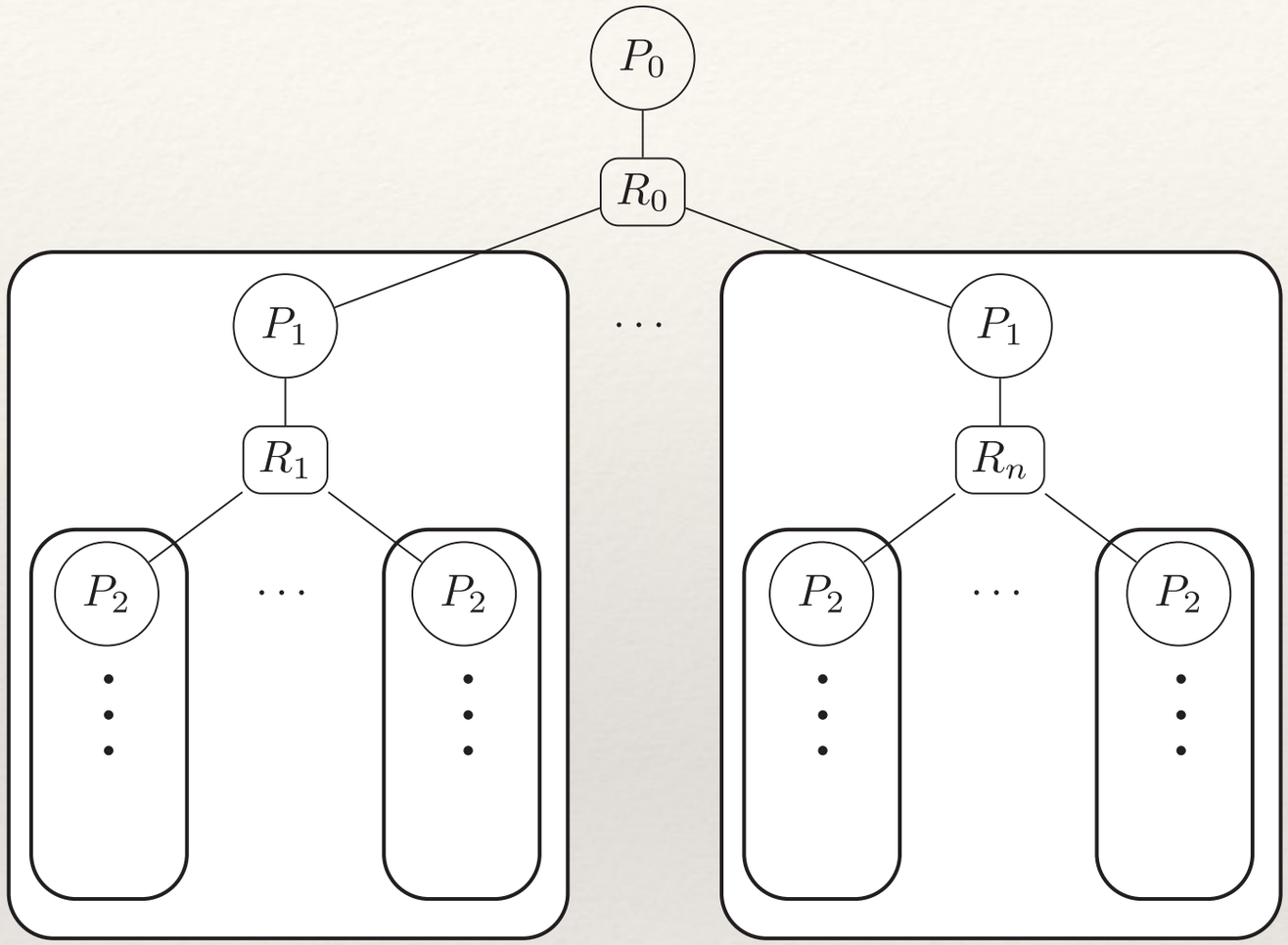
- pushdown automata [Courcelle 1991]
- Petri nets [Habermehl et al. 2010]
- stacked counter automata [Zetzsche 2015]
- higher-order pushdown wo / with collapse
  [Hague, Kochems, Ong 2016] [Clemente, Parys, Salvati, Walukiewicz 2016]

Theorem applies to

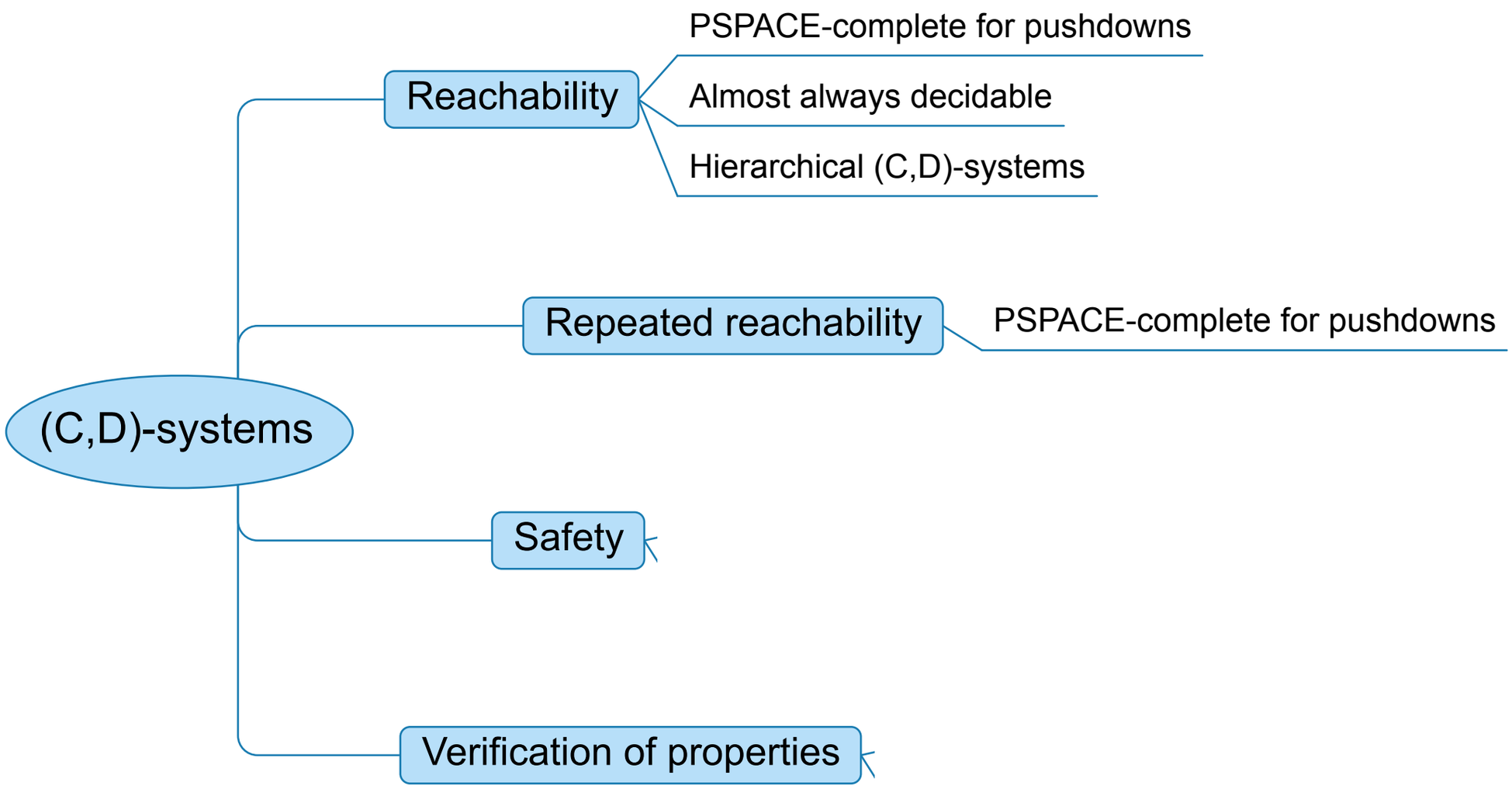
leader: pushdown automata, Petri nets, decidable subclasses of multi-stack, stacked counter automata.

contributors: any of the above, lossy channel systems,
hierarchical composition of (C,D)-systems.

Hierarchical composition of (C,D)-systems



leader P_0 and each subtree $(\mathcal{C},\mathcal{D})$-system is contributor


(C,D)-systems
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Hierarchical (C,D)-systems
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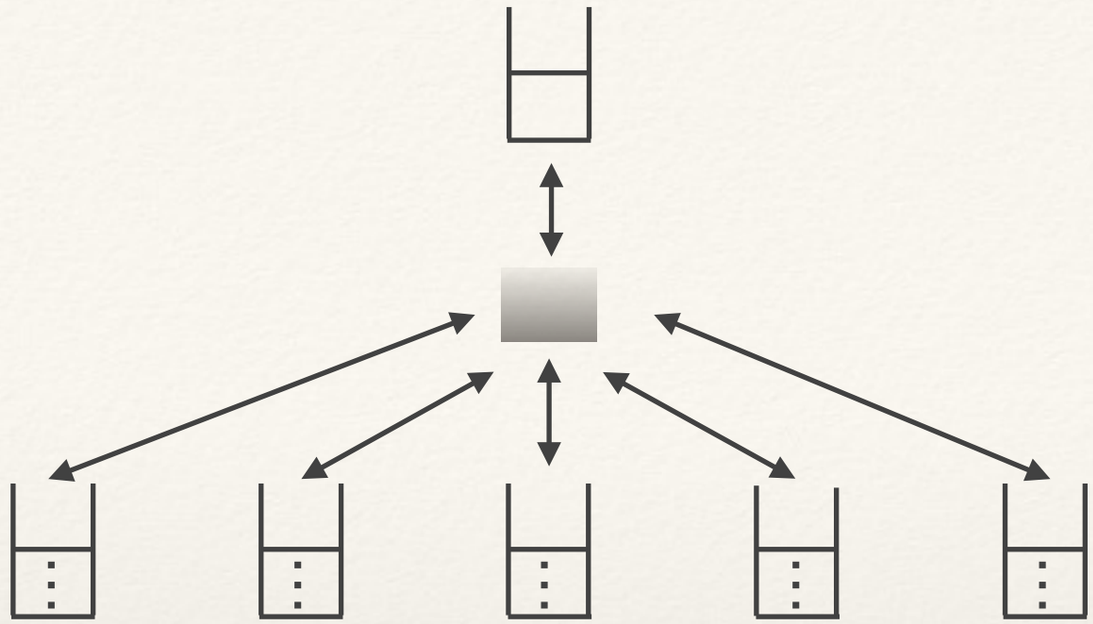
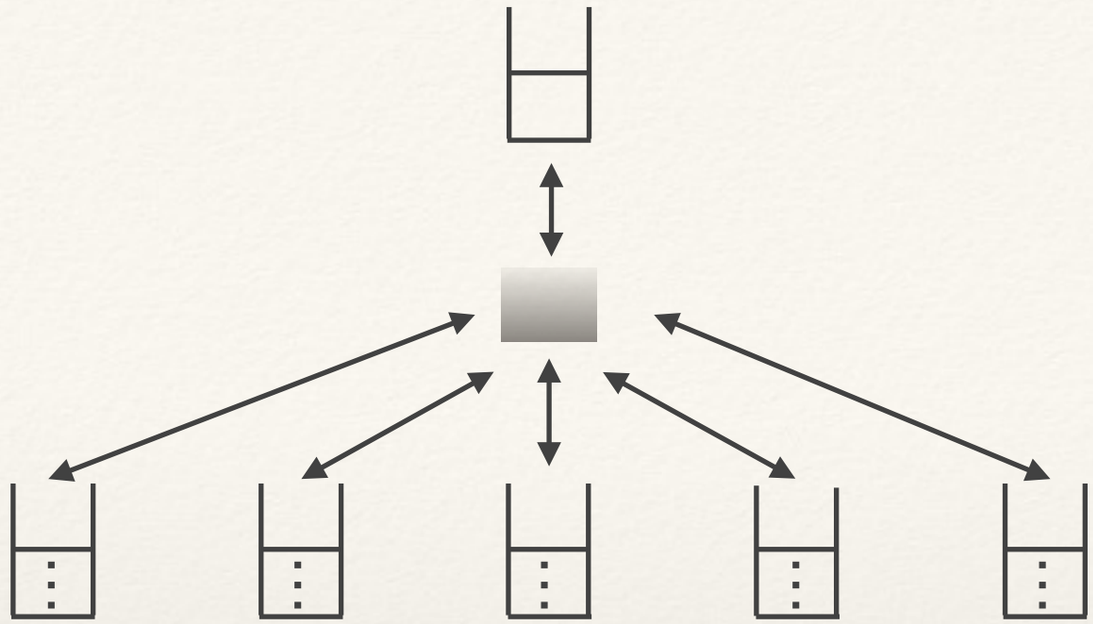


Repeated reachability in (C,D)-systems

Given a leader D from some class of systems $\mathcal{D}$ and a contributor C from some class $\mathcal{C}$, is there some integer n such that D, together with n copies of C, writes a particular value into the register infinitely often along some run?

Thm [Durand-Gasselin, Esparza, Ganty, Majumdar 2015]

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems: the repeated reachability problem is PSPACE-hard and in NEXPTIME.



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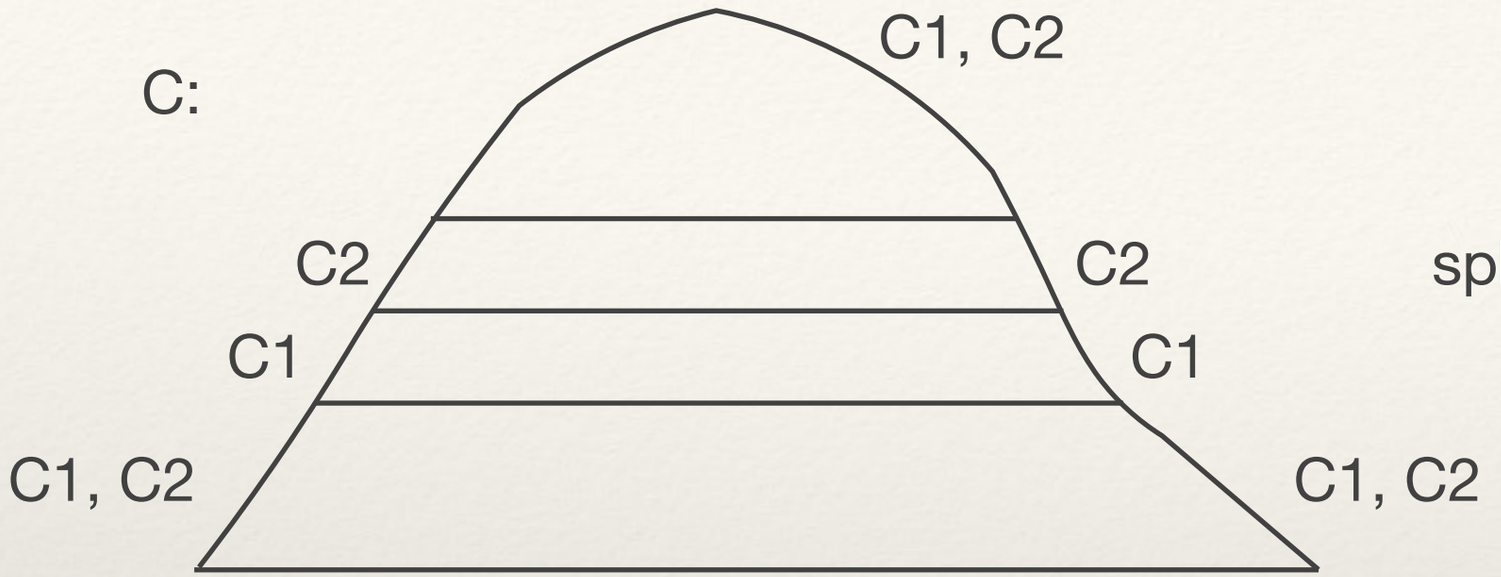
Thm [Fortin, M., Walukiewicz 2016]

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems: the repeated reachability problem is PSPACE-complete.

NEXPTIME (Esparza et al.): reduce the problem to C = NFA of exponential size by bounding the stack



split C into C1 and C2

The $(\mathcal{C},\mathcal{D})$-system has a live run iff there is some Büchi trace uv^ω s.t.

- leader pushdown has effective stack height 1 after each uv^k and same (state, top of stack)
- multiset of contributor states is the same after each uv^k

Separate leader D and contributor C according to the first writes of C:

Trace of D

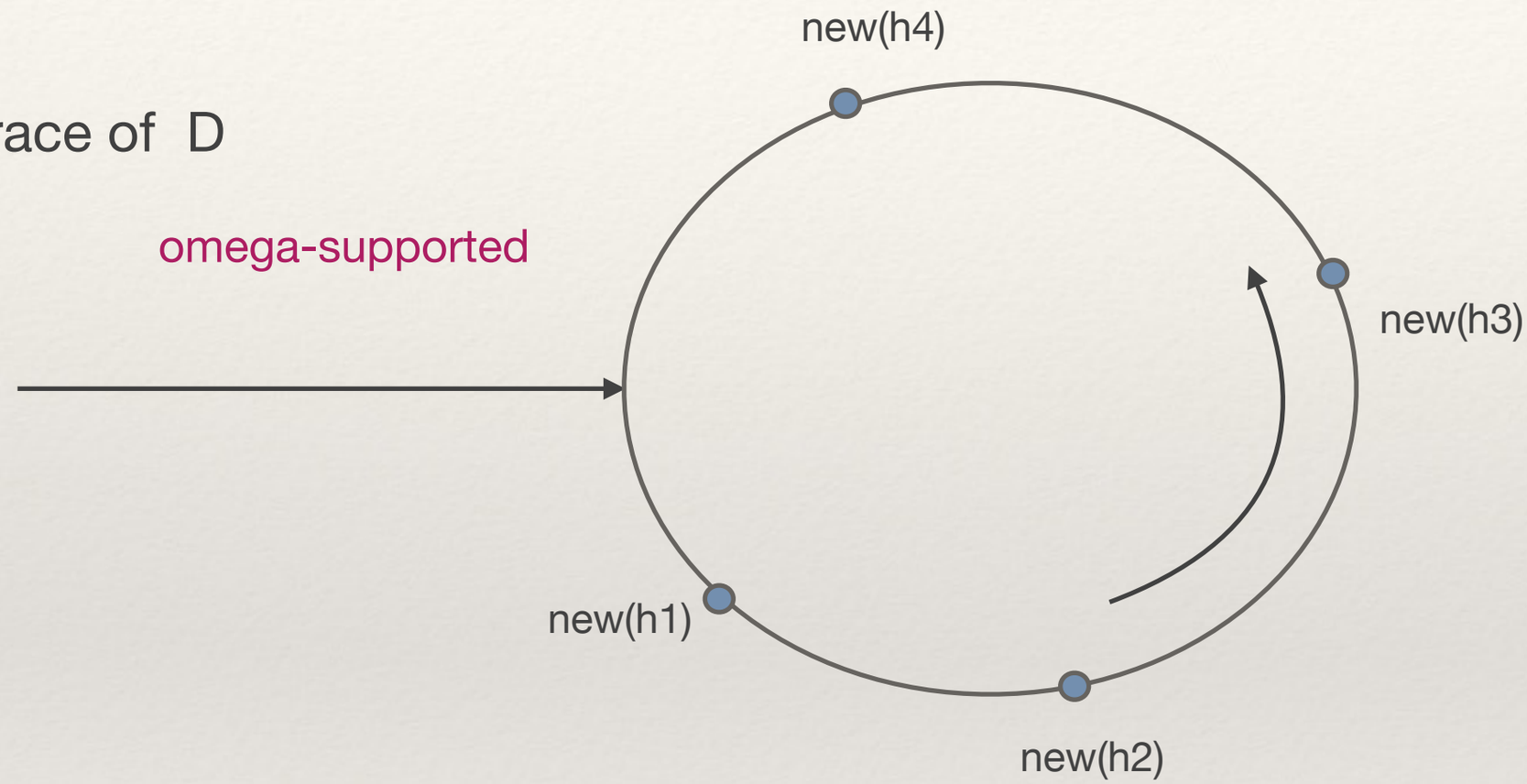
$$v_1 \,\mathrm{new}(h_1) v_2 \,\mathrm{new}(h_2) \cdots v_k \,\mathrm{new}(h_k) v_{k+1}$$

is omega-supported if for every $1 \leq j \leq k$ there exists a loop of C

$$u_1 \,\mathrm{new}(h_1) u_2 \cdots u_j \,\mathrm{new}(h_j) \overline{\mathbf{w}}(\mathbf{h_j}) \cdots v_k \,\mathrm{new}(h_k) u_{k+1}$$

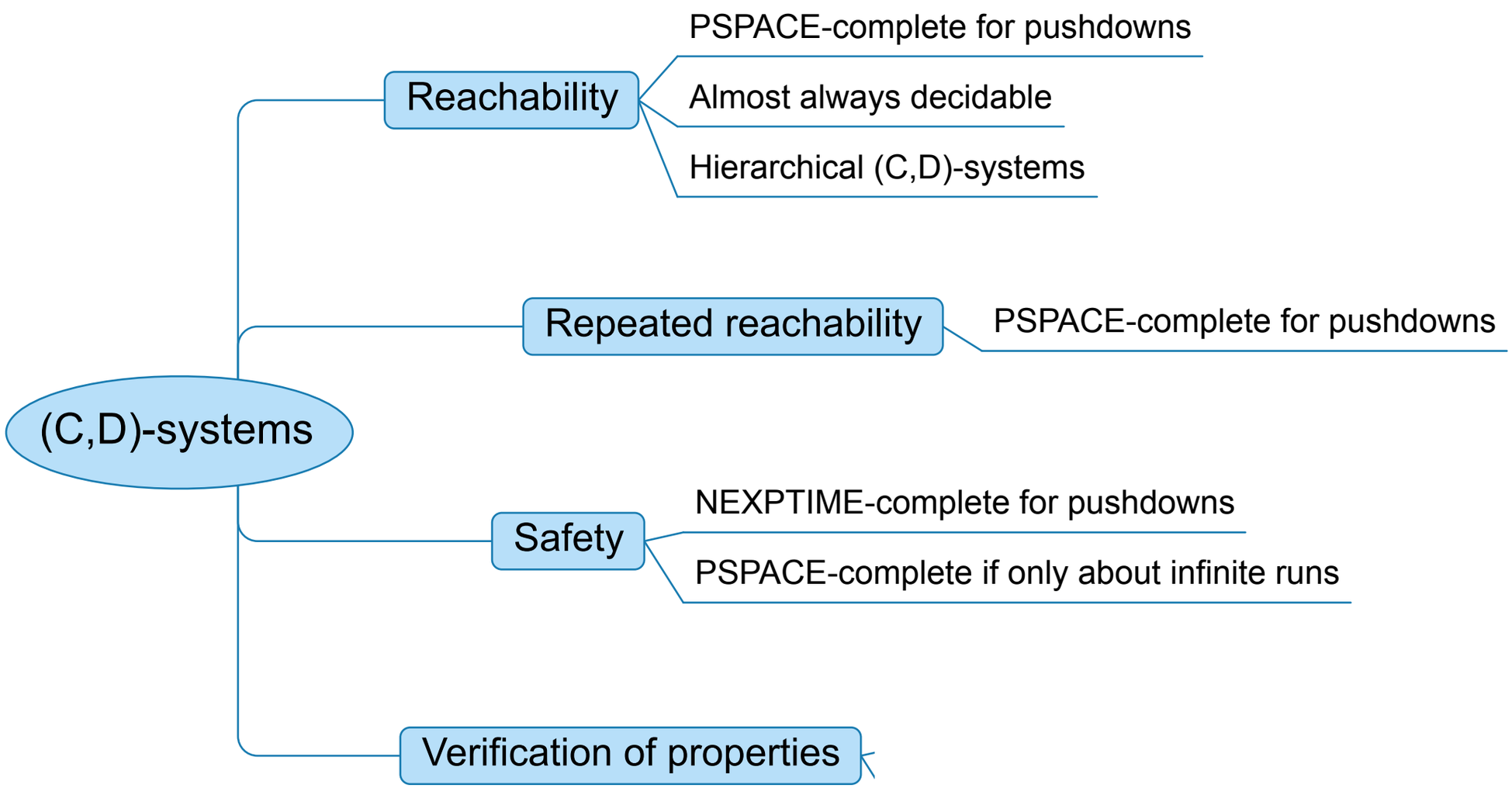
such that $v_i = \,\mathrm{projection\ of}\, u_i \,\mathrm{on\ alphabet\ of}\, D$ for every i

Trace of D



For each *j*: contributor run *rho(j)* supporting new(*hj*).

Use (2n+1) copies of *rho(j)* to support all n reads of *hj* in the loop.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PSPACE-complete if only about infinite runs
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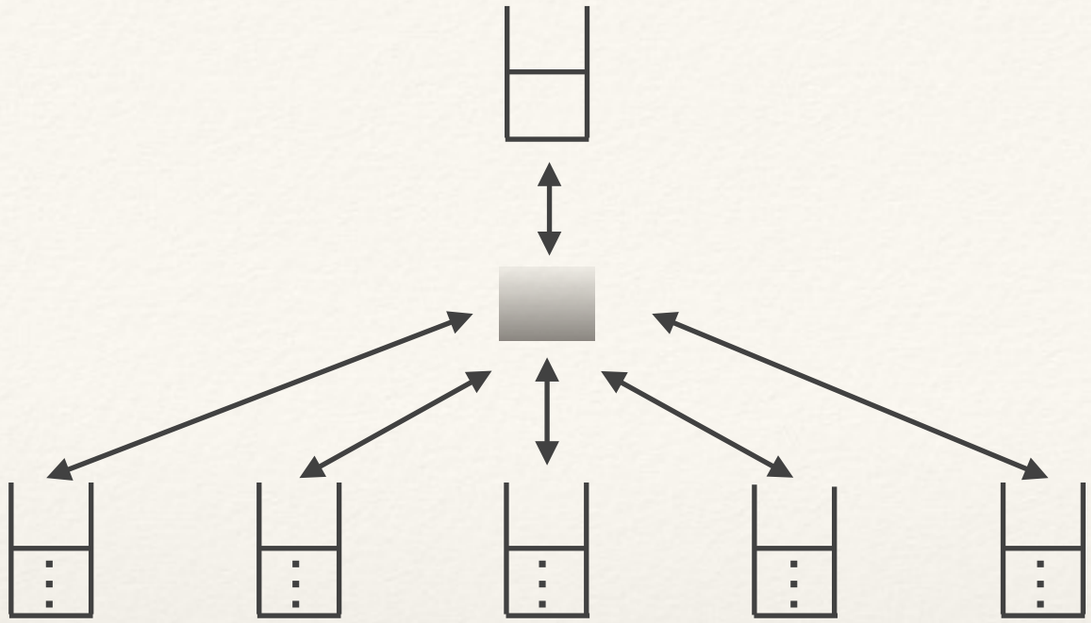

Safety in (C,D)-systems

Given a leader D from some class of systems $\mathcal{D}$ and a contributor C from some class $\mathcal{C}$, is there some integer n such that D, together with n copies of C, does not write a particular value into the register along some maximal run?

Thm

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems then the safety problem is NEXPTIME-complete.

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If $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems the safety problem is NEXPTIME-complete.

Thm

Let $\mathcal{C}$ and $\mathcal{D}$ be pushdown systems.
Knowing if there is some infinite safe run is PSPACE-complete.
Knowing if there is some maximal finite safe run is NEXPTIME-complete.

Thm

If$\mathcal{C}$ is finite-state systems and $\mathcal{D}$ is pushdown systems the problems are NP-complete.

Set semantics

$$(B,t,g) \xrightarrow{w(h)} (B,t',h) \quad \text{if } t \xrightarrow{w(h)} t' \text{ in } \Delta$$

$$(B,t,g) \xrightarrow{r(h)} (B,t',h) \quad \text{if } t \xrightarrow{r(h)} t' \text{ in } \Delta \text{ and } h = g$$

$$(B,t,g) \xrightarrow{\overline{w}(h)} (B',t,h) \quad \text{if } B \xrightarrow{\overline{w}(h)} B' \text{ in } \delta$$

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$$B \subseteq S$$

$B \xrightarrow{a} B'$ in δ if $s \xrightarrow{a} s'$ in δ, and B' is either $B \cup \{s'\}$ or $(B \cup \{s'\}) \setminus \{s\}$ for some $s \in B$.

The (C,D)-system has a finite, maximal run ending in configuration (M,t,g) in the multiset semantics iff it has a finite, maximal run ending in (B,t,g) in the set semantics, with B = support(M).

Prop

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems, the existence of a maximal finite safe run is NEXPTIME-hard.

Reduction from tiling problem:
Find a tiling with symbols from Σ of a 2^nx2^n square.
The tiling should respect neighborhood relations H,V⊆ΣxΣ.

Leader writes: $A_{1,1}, \overline{A_{1,1}}, A_{1,2}, \overline{A_{1,2}}, \ldots, A_{1,2^n}, \overline{A_{1,2^n}}, \ldots, A_{2^n,2^n}\ \overline{A_{2^n,2^n}}\ (\$\overline{\$})^{2^n} \diamond$.

and checks the horizontal dependencies.

Prop

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then the existence of a maximal finite safe run is NEXPTIME-hard.

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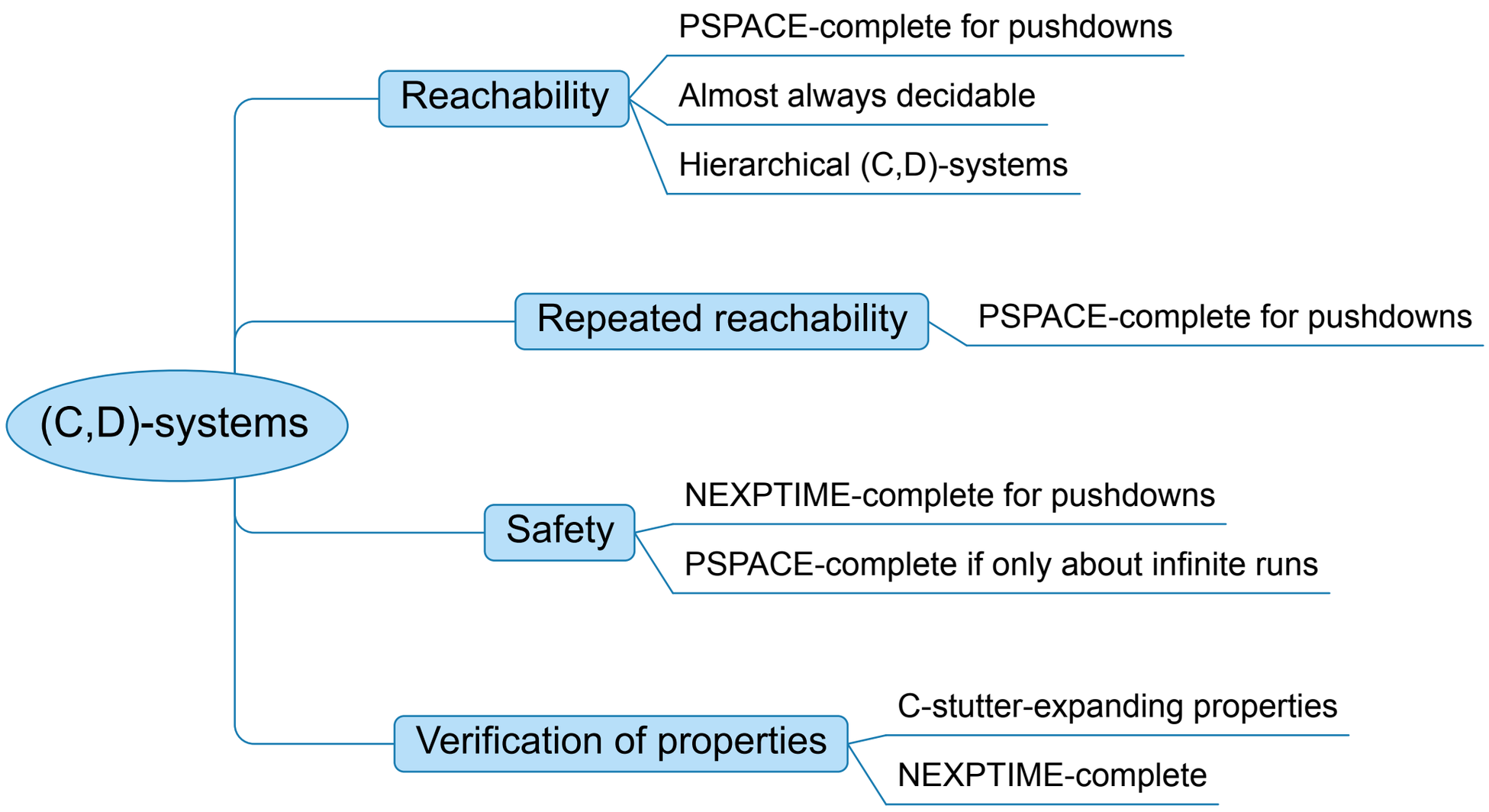
and checks the horizontal dependencies.

Contributors check vertical dependencies, using counting.

Leader ensures that

- every vertical dependency is checked by some contributor, and
- inconsistencies lead to write an error value in the register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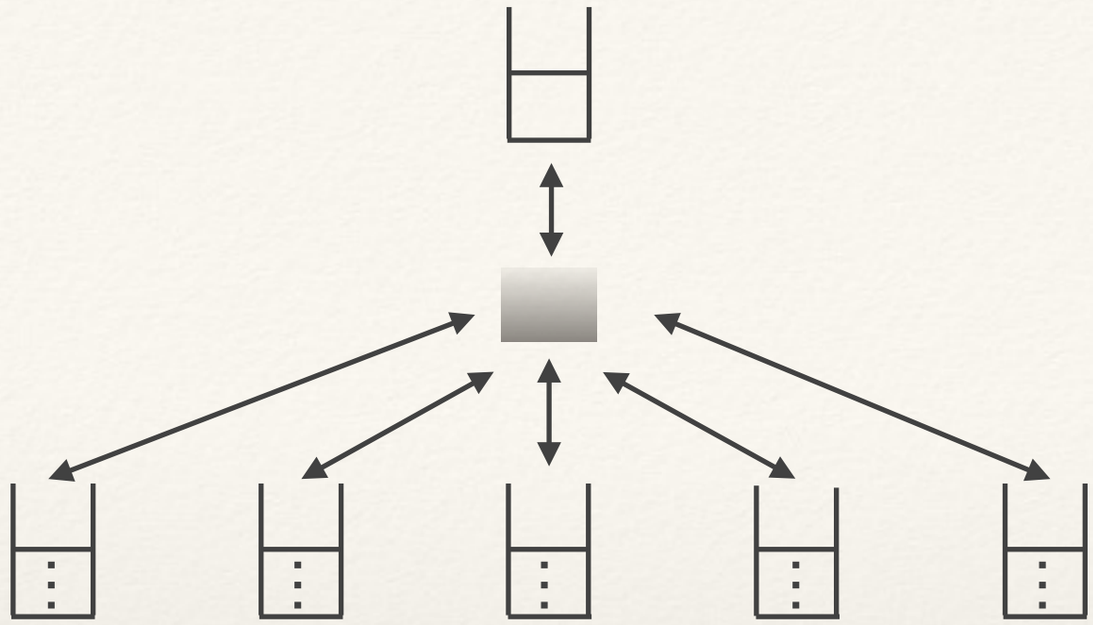
Hierarchical (C,D)-systems
Repeated reachability
PSPACE-complete for pushdowns
Safety
NEXPTIME-complete for pushdowns
PSPACE-complete if only about infinite runs
Verification of properties
C-stutter-expanding properties
NEXPTIME-complete



A trace is a sequence of register operations during a run.
Maximal trace comes from a maximal run (finite or infinite).

A property of traces is $P \subseteq (\Sigma_{\mathcal{D}} \cup \Sigma_{C})^{\infty}$
A property is C-stutter-expanding if it is closed under replicating contributor actions.

$$\text{If} \quad x\ \bar{w}(g)\ y \in P \quad \text{then} \quad x\ \bar{w}(g)\bar{w}(g)\ y \in P$$

Verification of properties of (C,D)-systems

Given a C-stuttering-expanding property P. Given a leader D from some class of systems $\mathcal{D}$ and a contributor C from some class C, is there some integer n such that D, together with n copies of C, has a maximal trace in P?

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All previously considered properties are special instances:

- reachability: P is the set of traces containing the special action.
- repeated reachability: P is the set of traces containing the special action infinitely often.
- safety: P is the set of traces without the special action.

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A property of traces is $P \subseteq (\Sigma_{\mathcal{D}} \cup \Sigma_{\mathcal{C}})^{\infty}$.

A property is C-stutter-expanding if it is closed under replicating actions of contributors.

$$\text{If} \quad x\ \bar{w}(g)\ y \in P \quad \text{then} \quad x\ \bar{w}(g)\bar{w}(g)\ y \in P$$

Rem: Verification of arbitrary regular properties is undecidable, as with a property we can require that there is only one copy of a contributor.

Thm

When $\mathcal{C}$ and $\mathcal{D}$ are pushdown systems then the verification of regular properties of (C,D)-systems is NEXPTIME-complete.

A property of traces is $P \subseteq (\Sigma_{\mathcal{D}} \cup \Sigma_C)^\infty$.

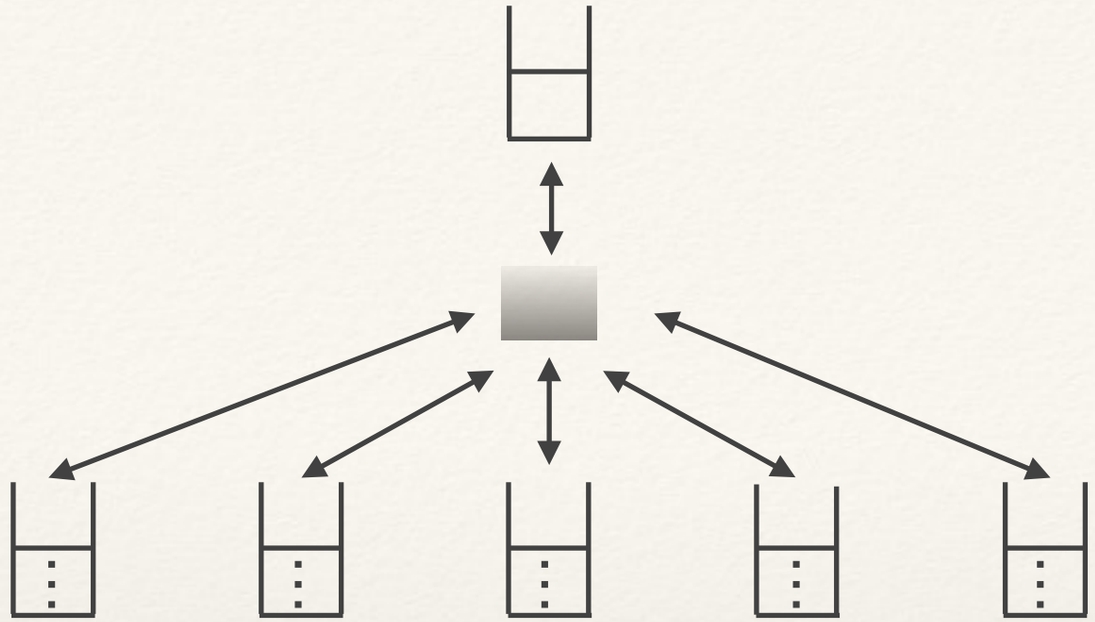
A property is C-stutter-expanding if it is closed under duplicating actions of contributors.

$$\text{If} \quad x\ \bar{w}(g)\ y \in P \quad \text{then} \quad x\ \bar{w}(g)\bar{w}(g)\ y \in P$$

The verification of C-stutter-expanding properties of (C,D)-systems reduces to the verification of properties over leader actions.

Leader can keep a local copy of the register and simulate contributor actions: action-requests from contributors are acknowledged by leader, information flows through the register.

Conclusion



Changing from one to arbitrary many contributors turns the problem from undecidable to manageable.

(C,D)-systems of pushdown process have very good algorithmic properties

- Verification of C-stutter-expanding properties is decidable in NEXPTIME
- For some relevant subclasses it is PSPACE.

NEXPTIME-hardness shows that they can exhibit a quite nontrivial behavior.