Automata Theory and Game Semantics of Higher-Order Computation

Andrzej S. Murawski
UNIVERSITY OF WARWICK

reporting on joint work with
Conrad Cotton-Barratt, David Hopkins and Luke Ong
(ICALP'11, FOSSACS'15 + new)

Automata models for higher-order computation

Paradigm

- higher-order programming with state
- imperative programming with higher-order procedures

Languages

- CBN: Idealized Algol
- CBV: Reduced ML

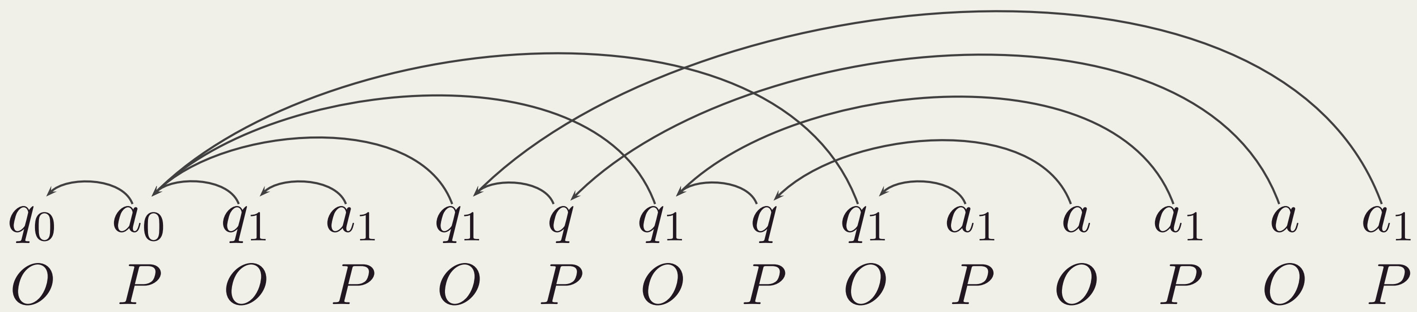
Fully-abstract semantic analysis

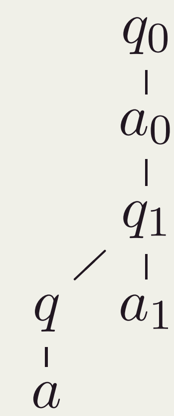
$$M \mapsto [\![M]\!]$$

1. What is the automata-theoretic nature of $[\![M]\!]$?
2. When can we decide $M_1 \cong M_2$?

Game Semantics

$[\![\vdash (\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}]\!]$

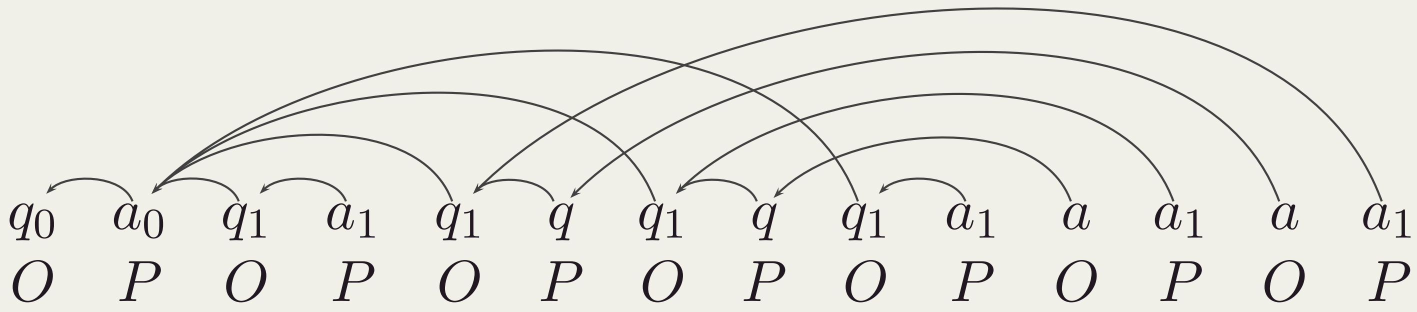




- alternation
- justification
- well-bracketing
- visibility

Full Abstraction

A play is **complete** if all questions have been answered.



Complete plays capture contextual equivalence:

$$M_1 \cong M_2$$

if and only if

$$\text{comp}\,[\![M_1]\!] = \text{comp}\,[\![M_2]\!]$$

Idealized Algol (CBN)

fragment	status	language class
$\mathsf{IA}_0, \mathsf{IA}_1, \mathsf{IA}_2$	☺	regular
IA_3	☺	visibly pushdown
IA_4	☹	

Ghica, McCusker, M., Ong, Walukiewicz

Reduced ML (CBV)

Some surprises

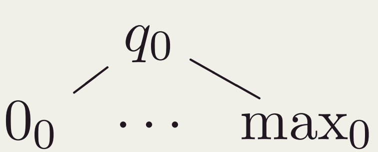
- no direct dependence on order
- need for automata over infinite alphabets
- links to unsolved decision problems

Regularity

- $\vdash M$: unit

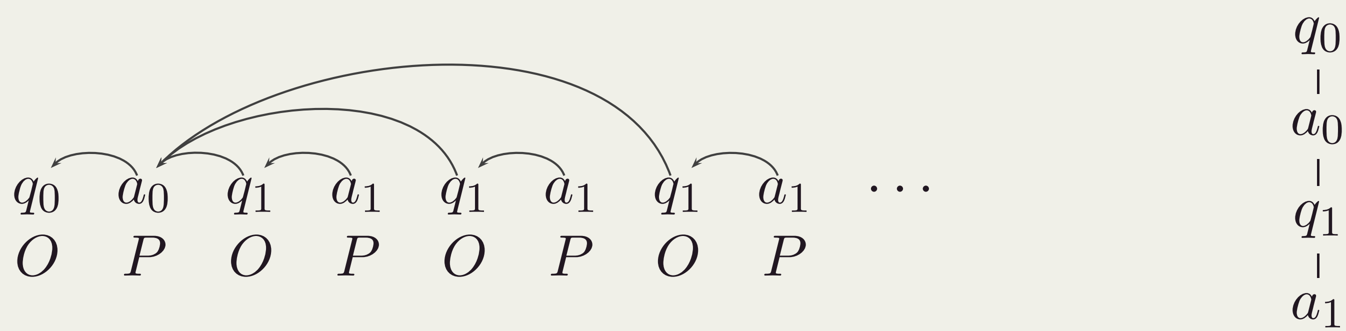
Single complete play: $\begin{matrix} q_0 & a_0 \\ O & P \end{matrix}$

- $\vdash M$: int

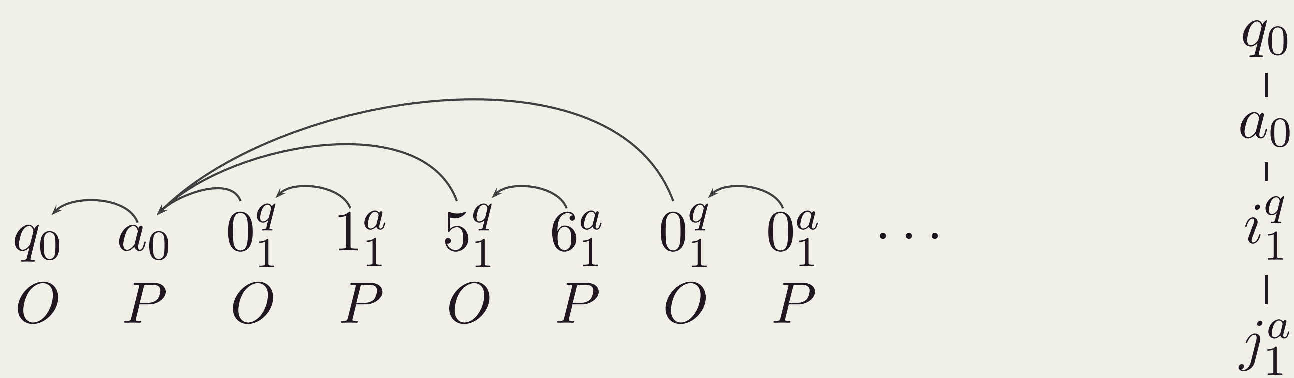


For each $i \in \{0, \cdots, \mathrm{max}\}$ there is a single complete play: $\begin{matrix} q_0 & i_0 \\ O & P \end{matrix}$

- $\vdash M$: unit $\to$ unit

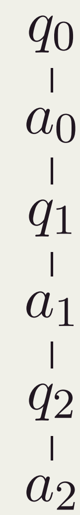


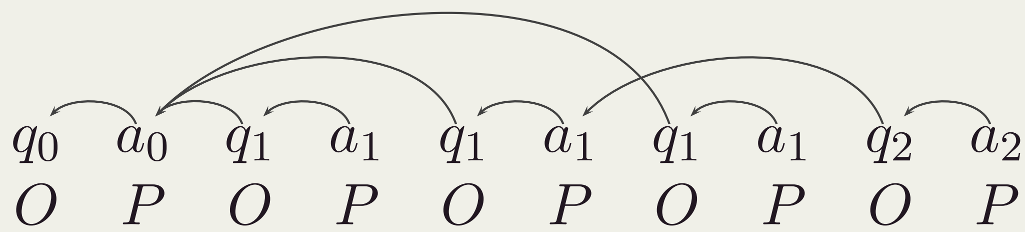
- $\vdash M$: int $\to$ int



NB: pointers are determined uniquely

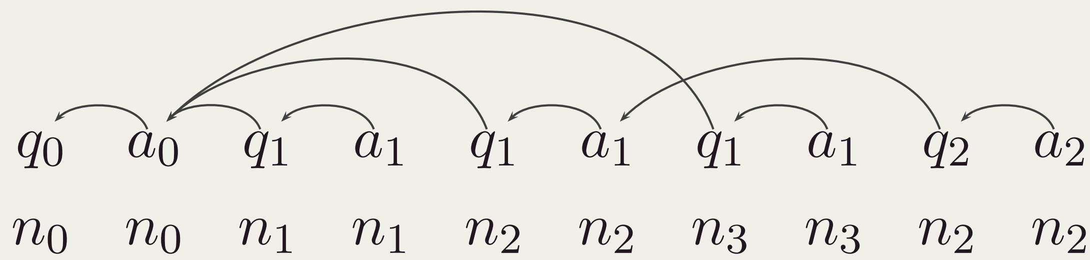
$\vdash M : \mathsf{unit} \to \mathsf{unit} \to \mathsf{unit}$





$\vdash M : \mathsf{unit} \to \mathsf{unit} \to \mathsf{unit}$

Name-based representation



$$(q_0, n_0)(a_0, n_0)(q_1, n_1)(a_1, n_1)(q_1, n_2)(a_1, n_2)(q_1, n_3)(a_1, n_3)(q_2, n_2)(a_2, n_2)$$

Observations

- communicating regular languages
- threads tagged with names (identifiers)

Class memory automata (Björklund & Schwentick)

$$(q_0, n_0)(a_0, n_0)(q_1, n_1)(a_1, n_1)(q_1, n_2)(a_1, n_2)(q_1, n_3)(a_1, n_3)(q_2, n_2)(a_2, n_2)$$

- global finite state
- access to last class state

We use a *weak* and deterministic variant of CMA: $(Q, \Sigma, q_0, \delta, F)$ with

$$\delta : Q \times \Sigma \times Q_\perp \longrightarrow Q$$

- Weakness corresponds to “thread completeness implies play completeness”.
- WDCMA are closed under all boolean operations.
- Emptiness decidable via VASS coverability (state-reachability).

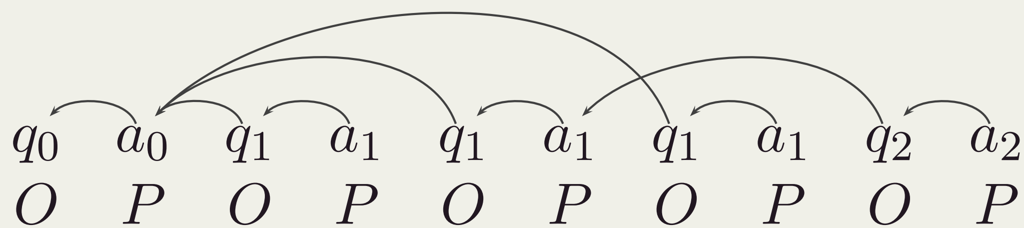
Aside

Equivalent to some other models over infinite alphabets:

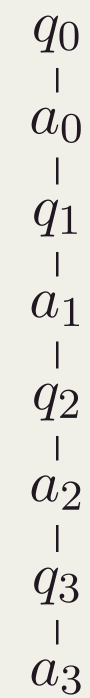
- Locally Prefix-Closed Data Automata (Decker, Habermehl, Leucker, Thoma)
- Class Counting Automata (Manuel, Ramanujam)
- Non-reset History Register Automata (Tzevelekos, Grigore)

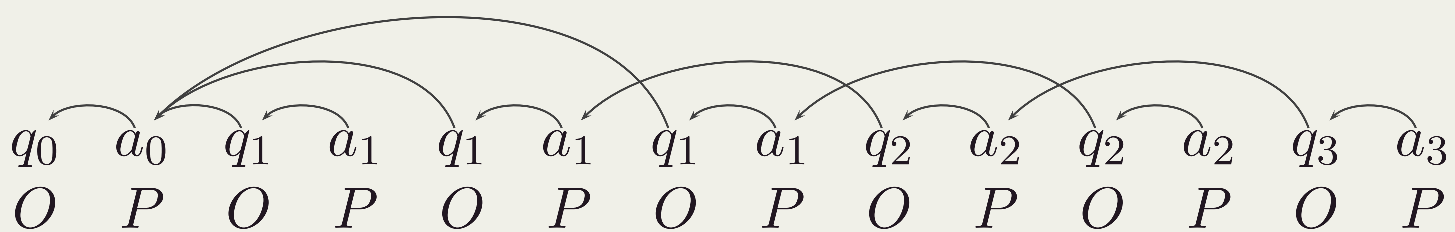
$\vdash M$: unit $\to$ unit $\to$ unit

q_0
|
a_0
|
q_1
|
a_1
|
q_2
|
a_2



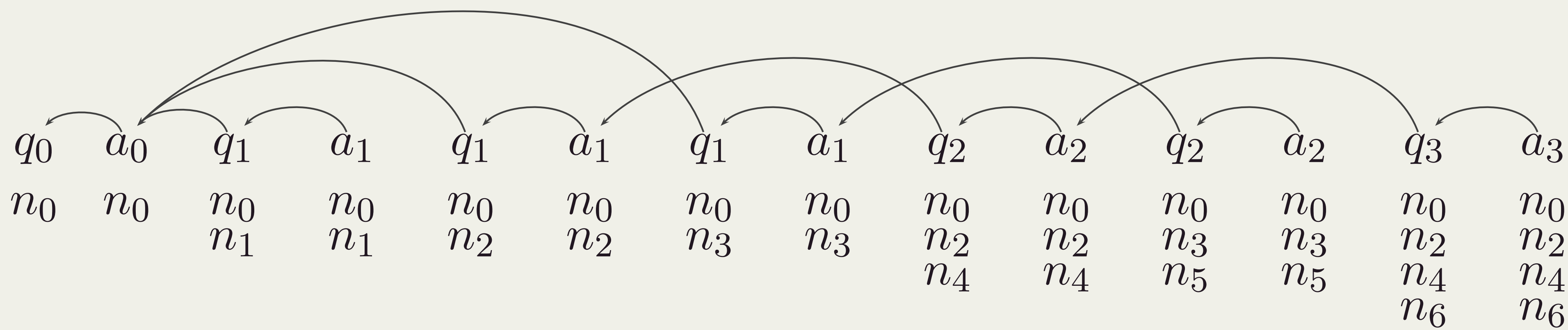
$\vdash M : \mathsf{unit} \to \mathsf{unit} \to \mathsf{unit} \to \mathsf{unit}$

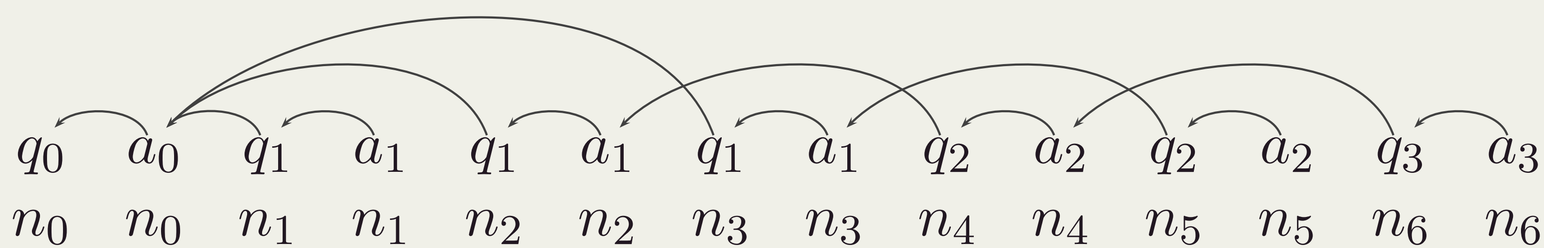


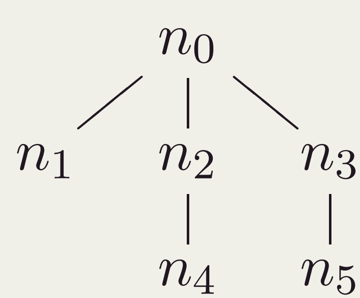


$\vdash M$: unit $\to$ unit $\to$ unit $\to$ unit

Name-based representation







Deterministic Weak Nested Class Memory Automata (FOSSACS’15)

Data level l

$$(Q, \Sigma, \delta, q_0, F)$$

$$\delta = \sum_{i=1}^{l} \delta_i$$

$$\delta_i : Q \times \Sigma \times (Q_\perp)^i \longrightarrow Q$$

- deterministic weak
- decidable emptiness (via WSTS coverability), inclusion and equivalence
- non-primitive recursive complexity

Equivalent Automata

Locally Prefix-Closed Nested Data Automata

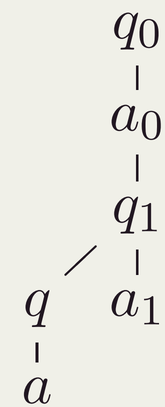
Decker, Habermehl, Leucker, Thoma; CONCUR'14

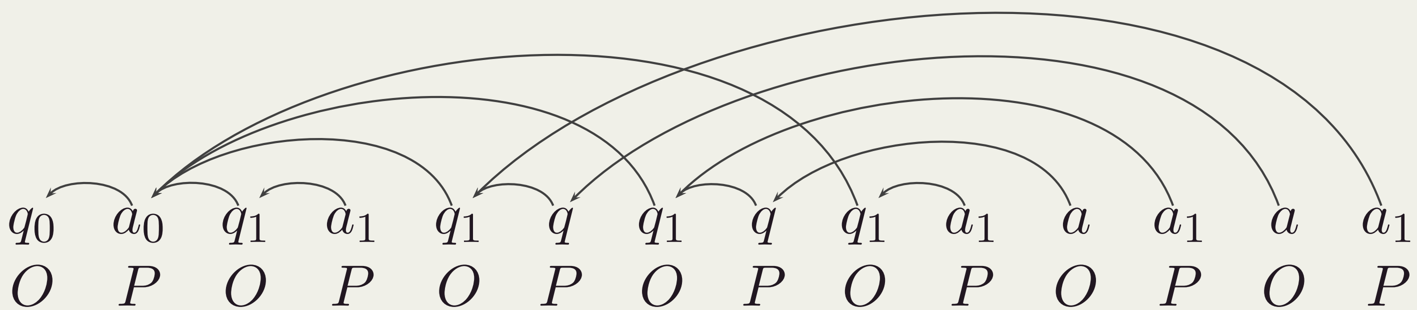
Summary (first-order types)

$$\vdash M : \mathsf{unit} \to \cdots \to \mathsf{unit}$$

- Semantics can be captured by *deterministic weak nested word automata*, whose emptiness problem is decidable.
- Contextual equivalence is decidable but non-primitive-recursive complexity.

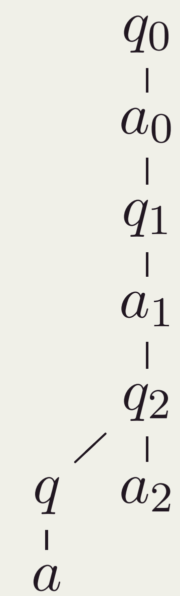
$\vdash M : (\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}$

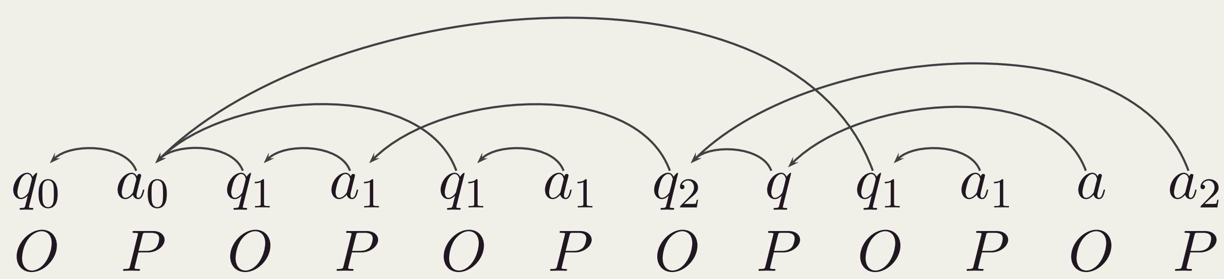




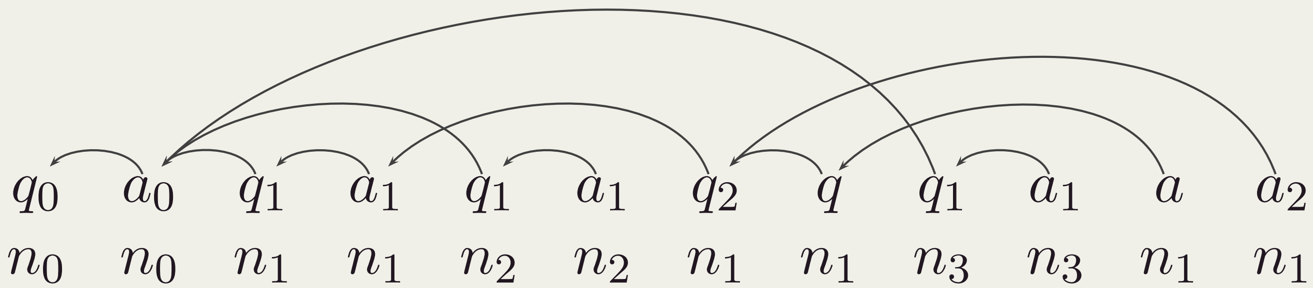
Visibly pushdown languages

$\vdash M$: unit $\to$ (unit $\to$ unit) $\to$ unit





Name-based representation



$$(q_0,n_0)(a_0,n_0)(q_1,n_1)(a_1,n_1)(q_1,n_2)(a_1,n_2)(q_2,n_1)(q,n_1)(q_1,n_3)(a_1,n_3)(a,n_1)(a_2,n_1)$$

$\vdash M : \mathsf{unit} \to (\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}$

$$(q_0,n_0)(a_0,n_0)(q_1,n_1)(a_1,n_1)(q_1,n_2)(a_1,n_2)(q_2,n_1)(q,n_1)(q_1,n_3)(a_1,n_3)(a,n_1)(a_2,n_1)$$

Observations

- communicating visibly pushdown languages
- interleaving subject to global stack discipline
- threads tagged with names (identifiers)
- complete thread no longer implies complete play

Visibly Pushdown Class Memory Automata (VPCMA)

$$(q_0,n_0)(a_0,n_0)(q_1,n_1)(a_1,n_1)(q_1,n_2)(a_1,n_2)(q_2,n_1)(q,n_1)(q_1,n_3)(a_1,n_3)(a,n_1)(a_2,n_1)$$

$$(Q, \Sigma, \Gamma, \delta, F)$$

$$\delta \subseteq (Q \times Q_\perp \times (\Sigma_{\text{push}} \cup \Sigma_{\text{pop}}) \times \Gamma \times Q) \cup (Q \times Q_\perp \times \Sigma_{\text{noop}} \times Q)$$

- finite state
- access to last class state
- pop-name same as push-name

Thm: Emptiness is equivalent to EBVASS reachability (decidability status unknown).

EBVASS (Jacquemard, Segoufin, Dimino; LMCS 2016)

BVASS

$$\frac{(q, \vec{v_1} + \vec{v_2})}{(q_1, \vec{v_1}) \qquad (q_2, \vec{v_2})}$$

EBVASS

$$C = \{c_1, \cdots, c_m\} \qquad\qquad c_i = (l_i, r_i, p_i) \in [1, k]^3$$

$$\frac{(q, \vec{v_1} + \vec{v_2} + \sum_i n_i \cdot \overline{e_{\pi_3(c_i)}})}{(q_1, \vec{v_1} + \sum_i n_i \cdot \overline{e_{\pi_1(c_i)}}) \qquad (q_2, \vec{v_2} + \sum_i n_i \cdot \overline{e_{\pi_2(c_i)}})}$$

Status

EBVASS-equivalent

$$\mathsf{unit} \to (\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}$$

$$\mathsf{unit} \to (\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$

Undecidable

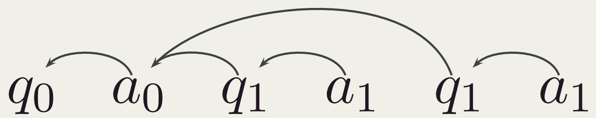
$$(\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit} \to \mathsf{unit}$$

$$\mathsf{unit} \to \mathsf{unit} \to (\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}$$

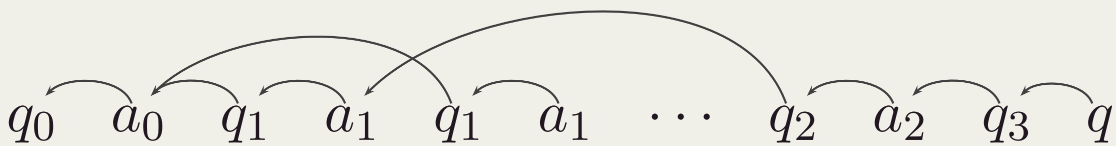
$$((\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}) \to \mathsf{unit}$$

rVASS reachability at unit → unit → (unit → unit) → unit

- START

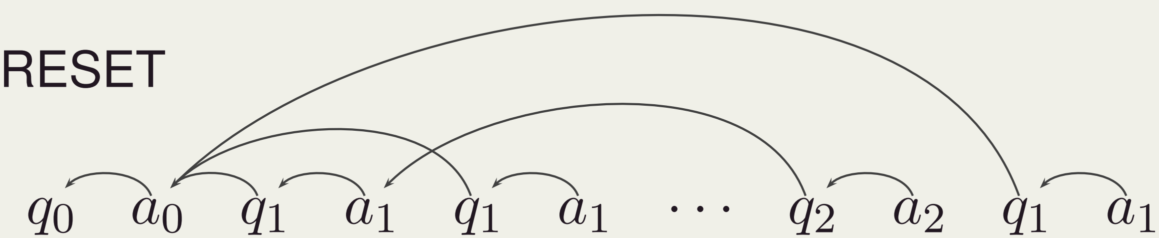


- INC

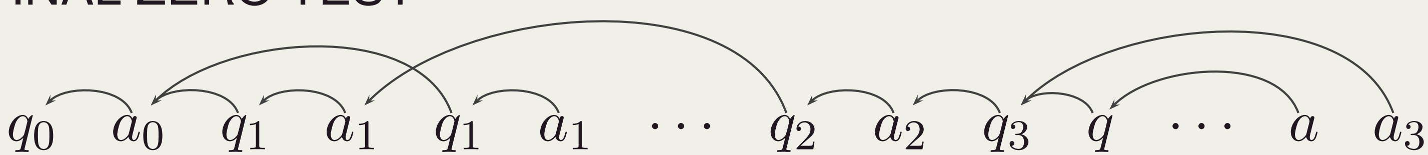


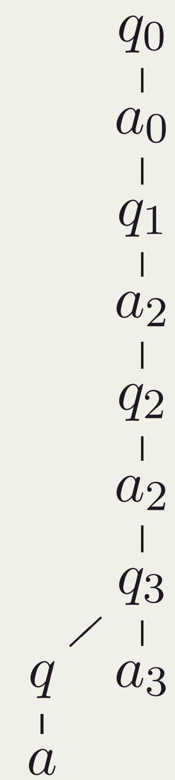
- DEC

- RESET



- FINAL ZERO TEST





Summary

- decidable

$$\mathsf{unit} \to \cdots \to \mathsf{unit}$$
$$(\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$

- EBVASS-equivalent

$$\mathsf{unit} \to (\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$

- undecidable

$$(\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit} \to \mathsf{unit}$$
$$\mathsf{unit} \to \mathsf{unit} \to (\mathsf{unit} \to \cdots \to \mathsf{unit}) \to \mathsf{unit}$$
$$((\mathsf{unit} \to \mathsf{unit}) \to \mathsf{unit}) \to \mathsf{unit}$$