Hitting Families of Schedules

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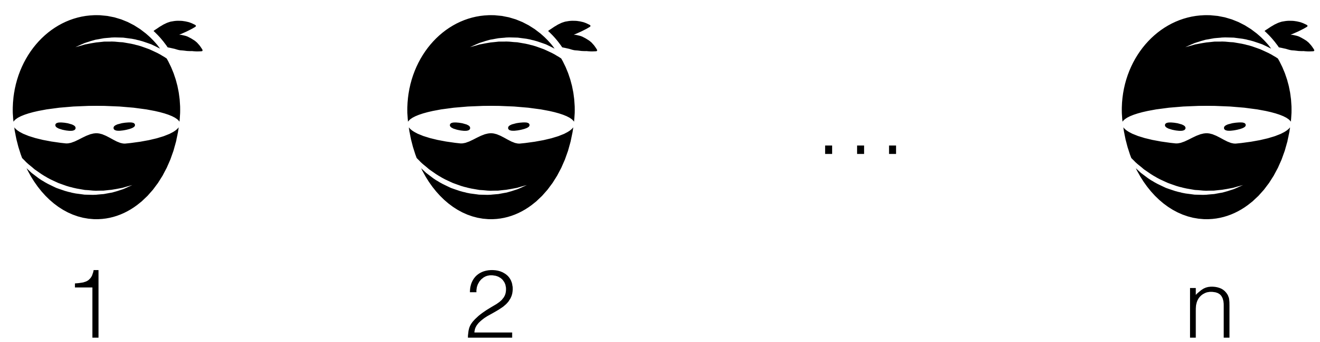
Ninjas at a conference banquet



A banquet is **complete** if for every pair of ninjas (**i**, **j**), there's a course served to ninja **i** before ninja **j**.

How many courses make a banquet complete?

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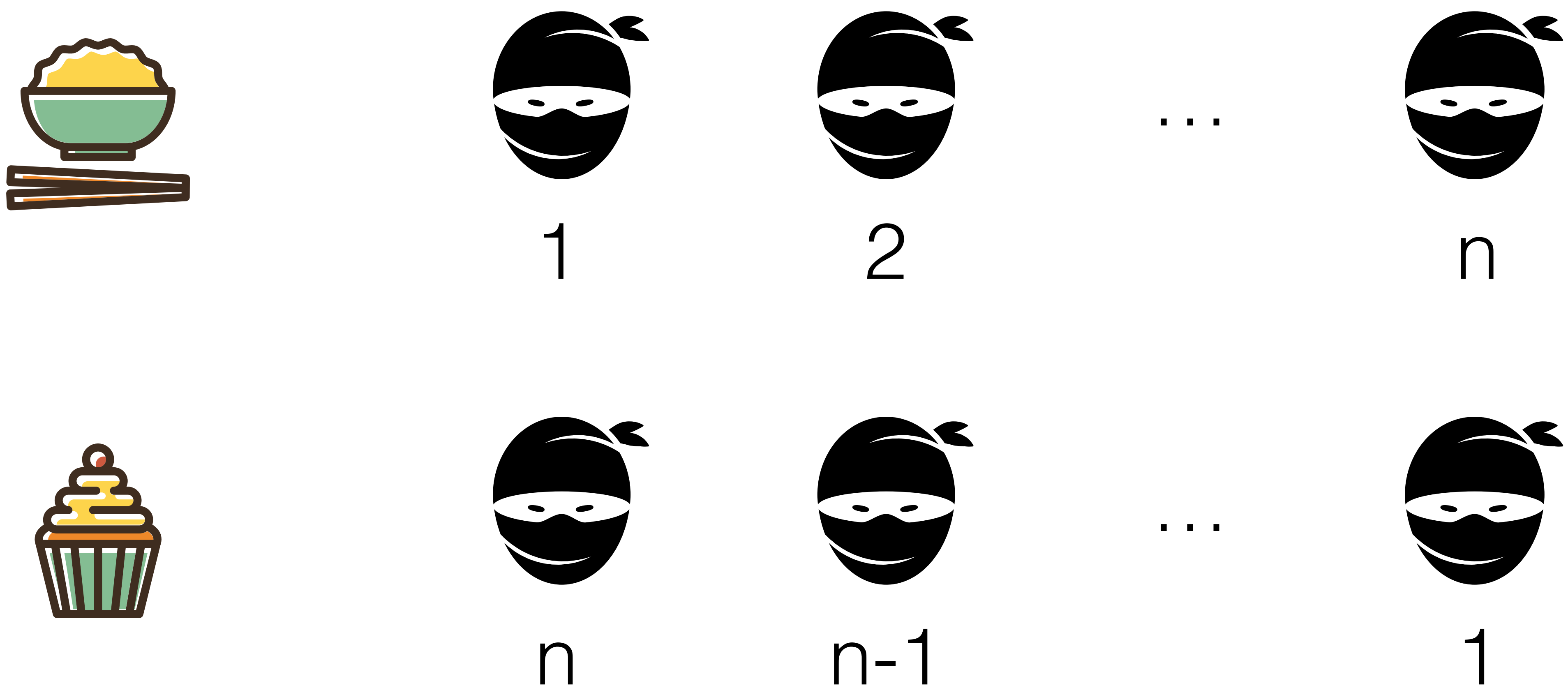
Ninjas at a conference banquet

Two courses suffice:



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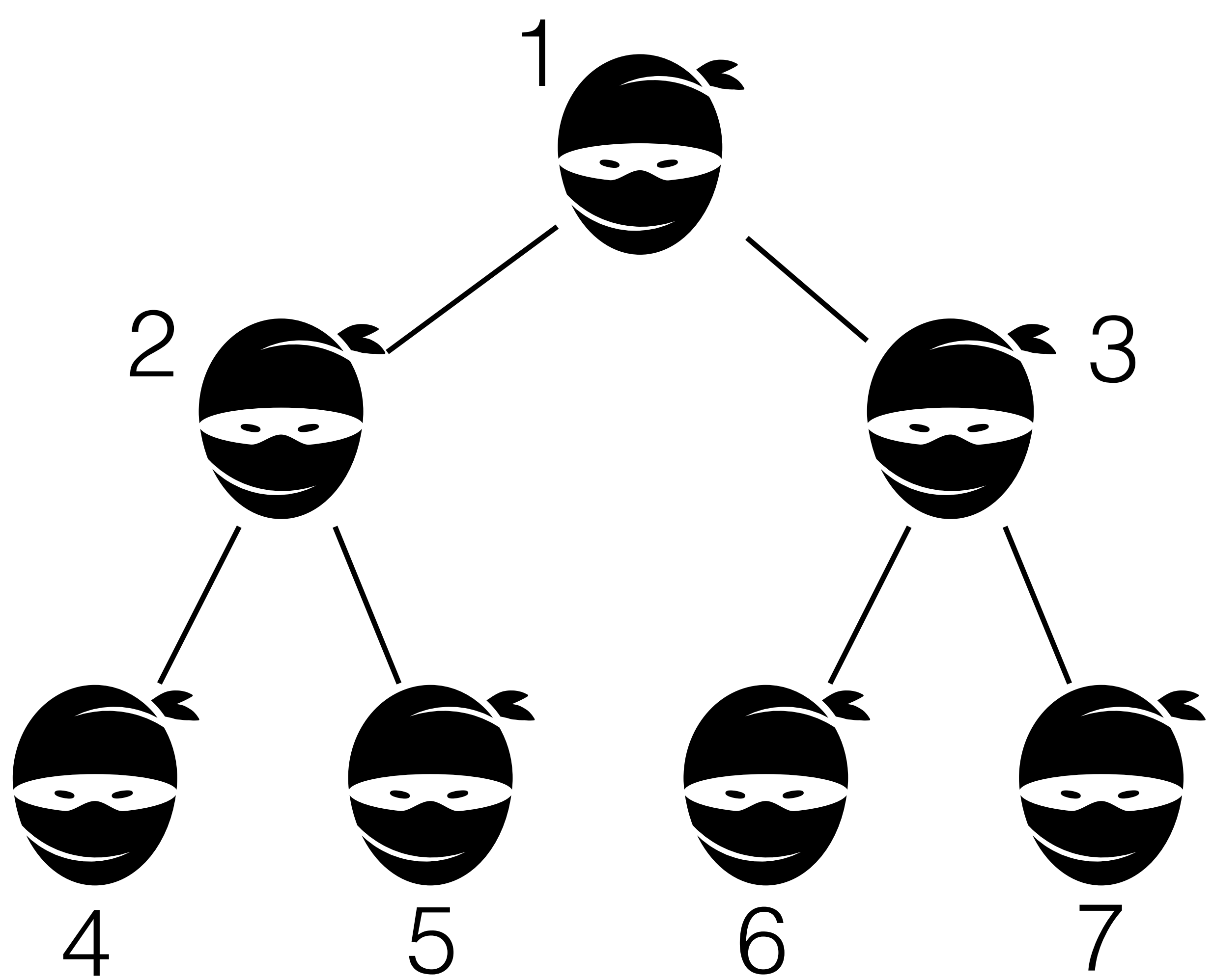
Ninjas at a conference banquet

What if ninjas form a hierarchy?
A **master** is always served **before** their **student**.



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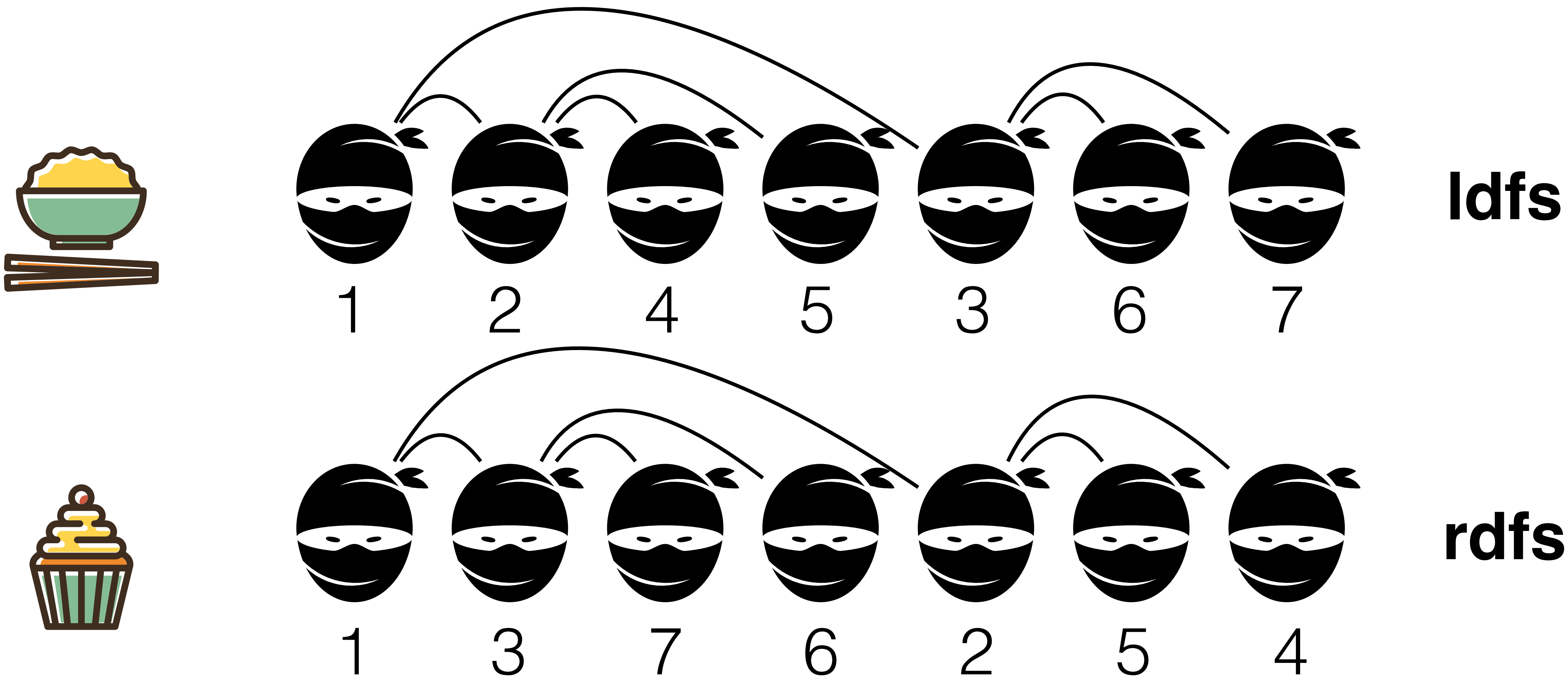
Ninjas at a conference banquet

Again, **two** courses suffice:



Ninjas at a conference banquet

What if instead of pairs we consider **triplets** of ninjas?

A banquet is **3-complete** if for every triplet of ninjas (**i**, **j**, **k**), there's a course served to ninja **i** before **j**, and **j** before **k**.

Ninjas at a conference banquet

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Naive approach with **2n** courses:

```
for each i@{1,…,n}:
  serve ancestry line to i; ldfs the rest
  serve ancestry line to i; rdfs the rest
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Ninjas at a conference banquet

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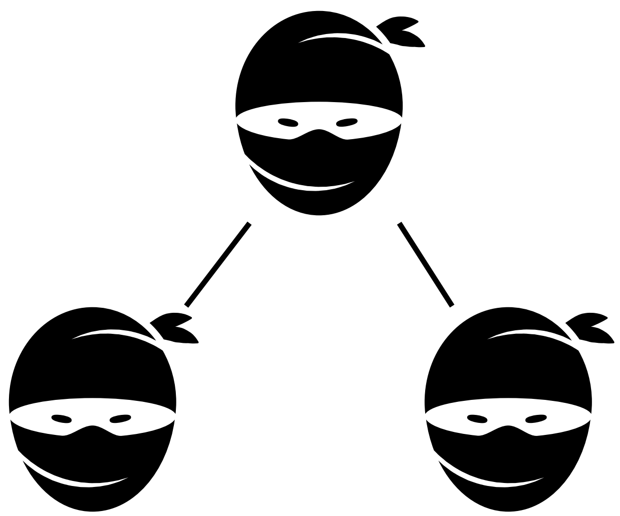
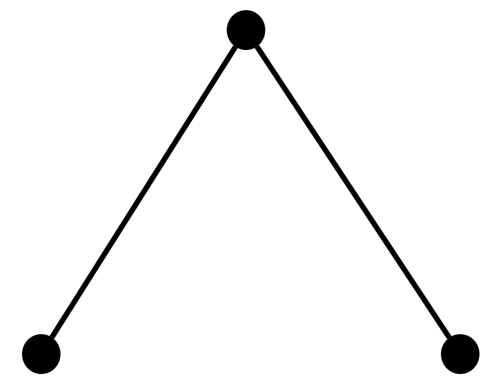
Can be done with **O(log n)** courses!

From ninjas to concurrent systems





ninjas	events
hierarchy	partial order
courses	schedules
d-complete banquet	**d-hitting family of schedules**

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d-hitting families of schedules

Given a poset of events, a schedule **hits** a d-tuple of events $\mathbf{(e_1,\ldots,e_d)}$ if it executes the events in the order $\mathbf{e_1<\ldots<e_d}$.

Given a poset of events, a **family of schedules F** is **d-hitting** if for every admissible d-tuple of events there is a schedule in **F** that hits it.

Why d?

Empirically: Many bugs involve small number of events—**bug depth d**

[Lu et al. ASPLOS ’08] [Burckhardt et al. ASPLOS ’10] [Jensen et al. OOPSLA ’15] [Qadeer et al. TACAS ’05]

- d = 2: order violation
- d = 3: atomicity violation

A d-hitting family of schedules provides a notion of **coverage**:
it hits **any** bug of depth d.

Moreover, for certain kinds of partial orders we can **explicitly construct small d-hitting families**.

Contributions

- The notion of d-hitting families of schedules
- For antichains with n elements, existence of hitting families of size **$O(\exp(d) \cdot \log n)$**
- For trees of height h:
    - d = 3: explicit construction of hitting families of size **4h** (optimal)
    - d > 3: explicit construction of hitting families of size **$O(\exp(d) \cdot h^{d-1})$**

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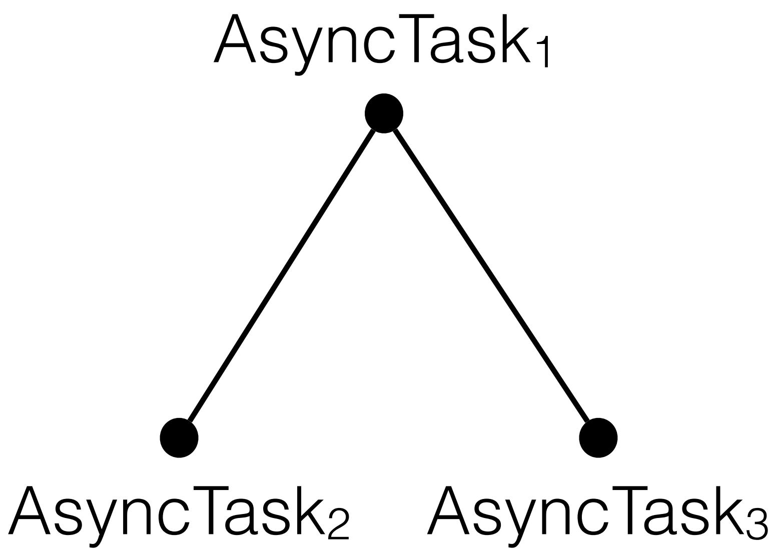
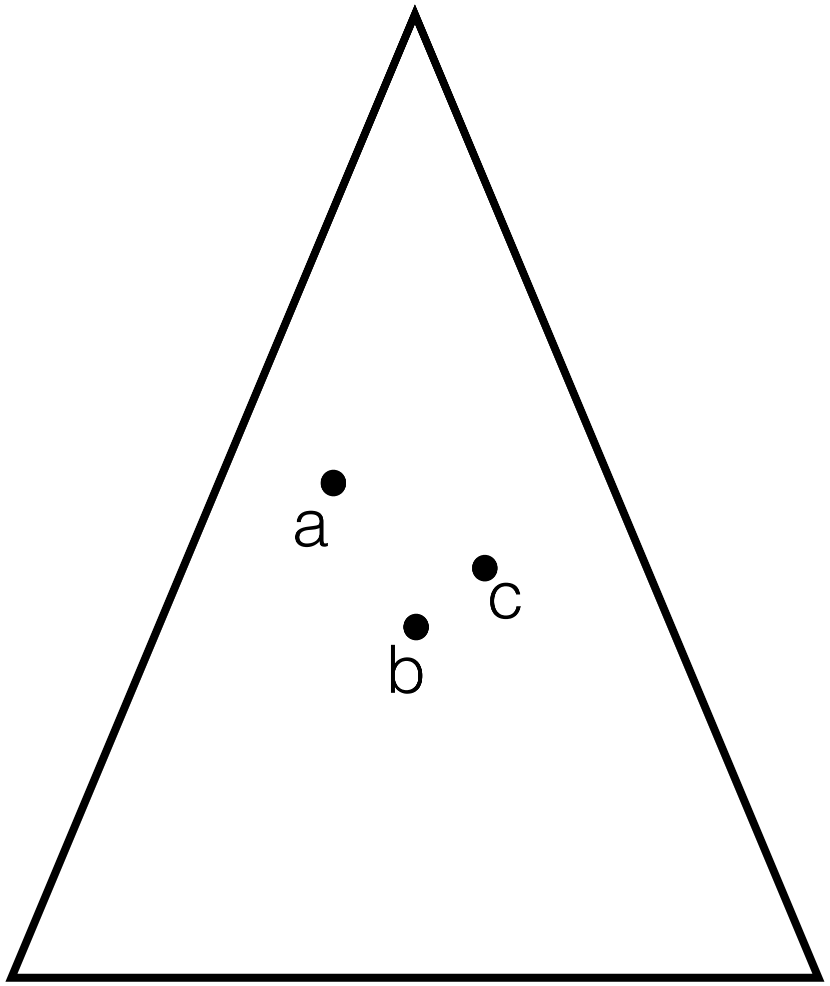
Why trees?



Trees arise from a simple **fire-and-forget** model of **asynchronous programs**.

3-hitting families for trees

admissible (a,b,c)



height h

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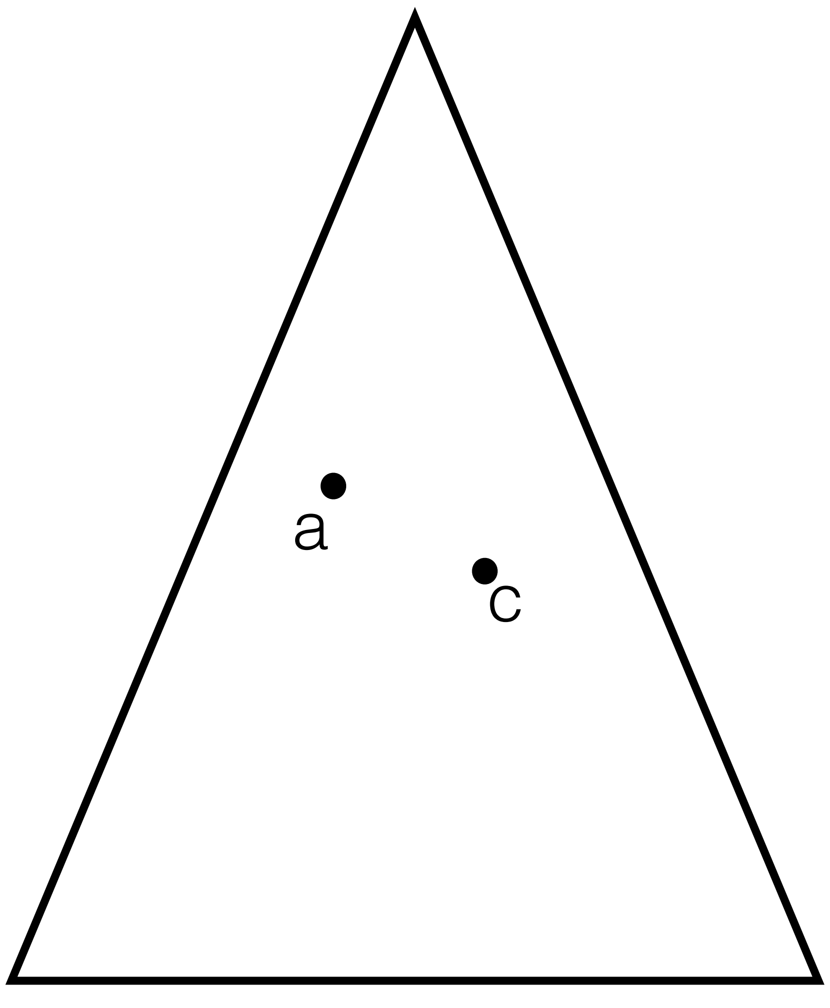

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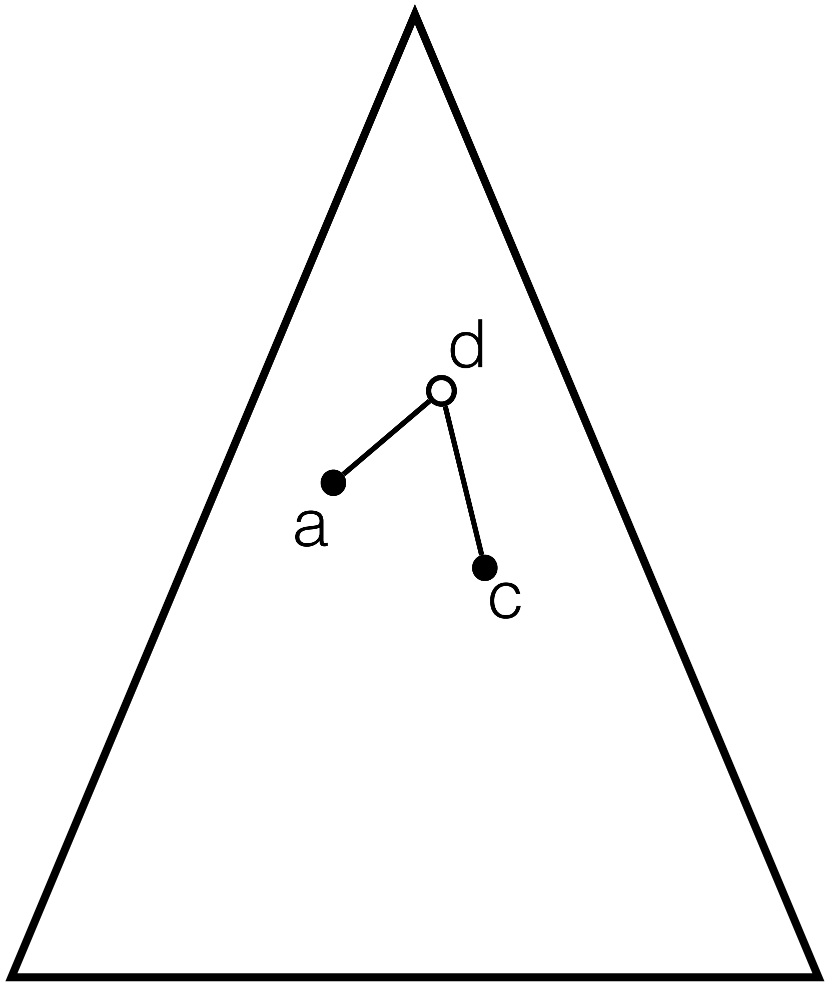

height h

admissible (a,b,c)

d = lca(a,c) (could be a itself)

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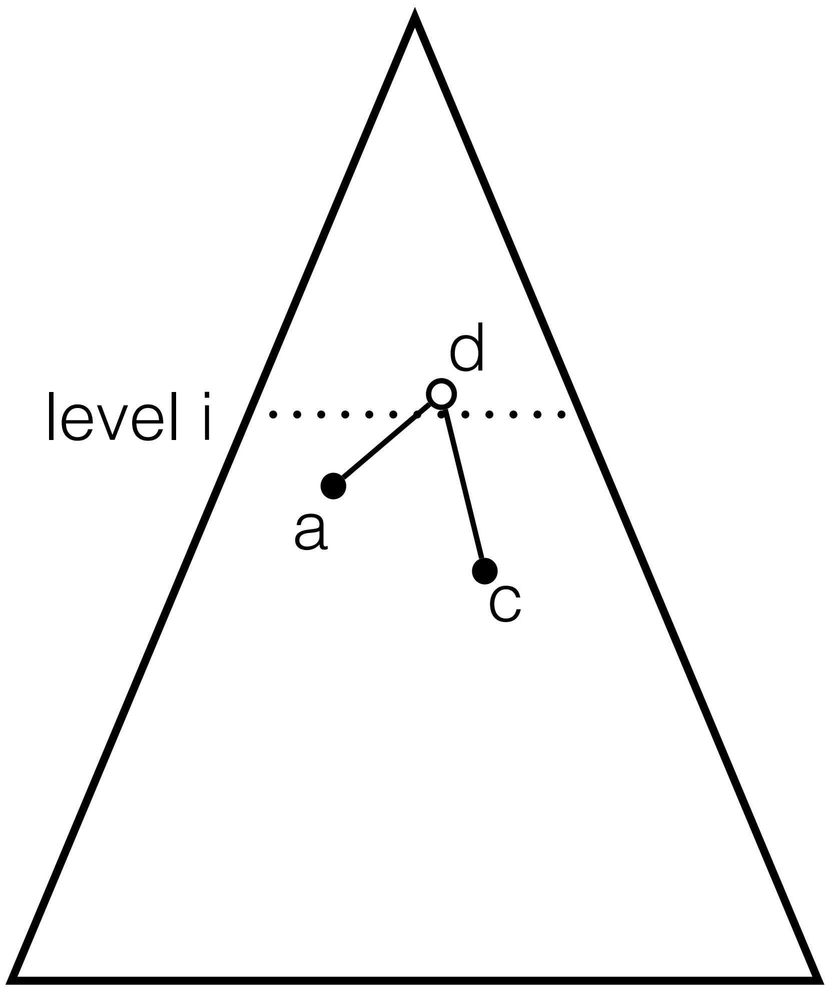

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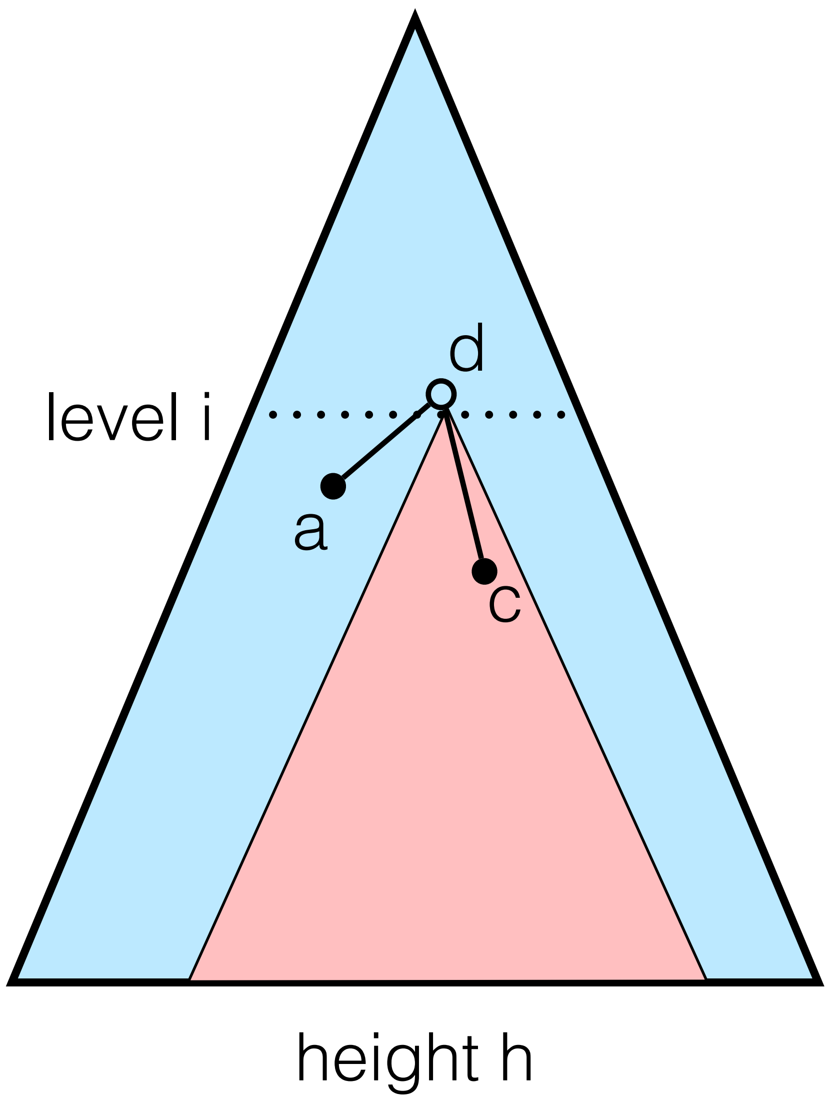

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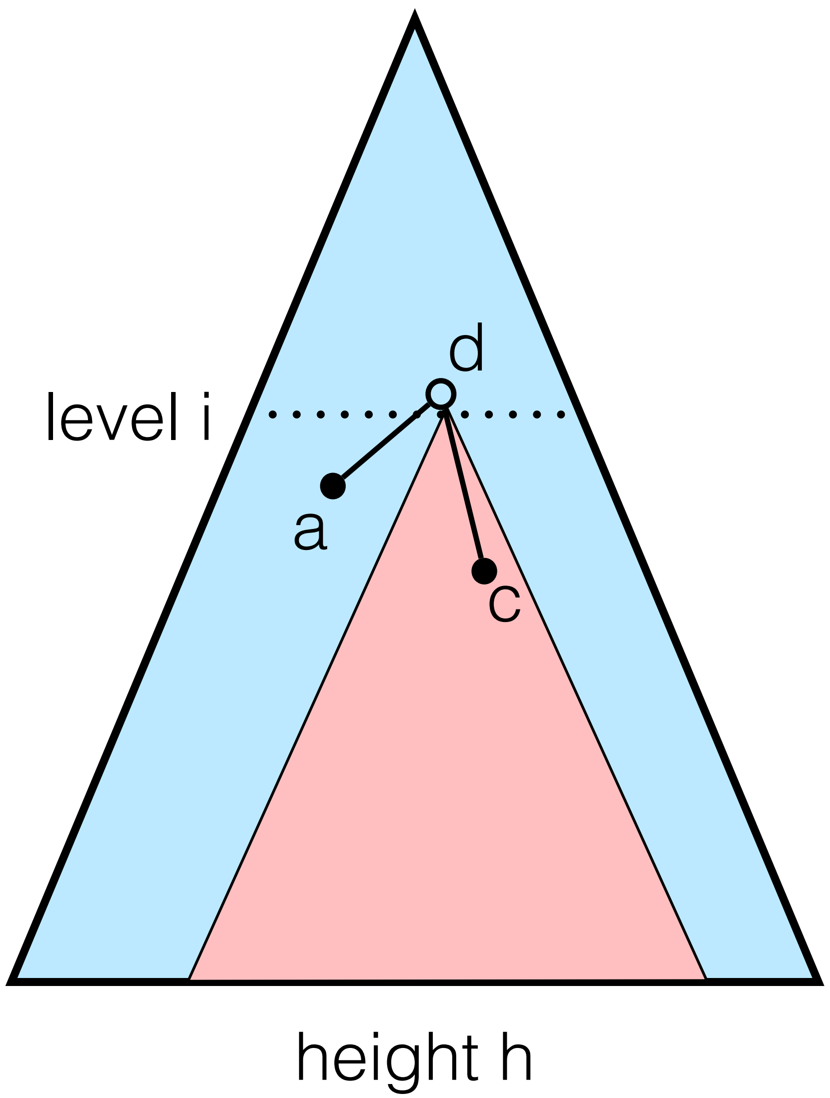

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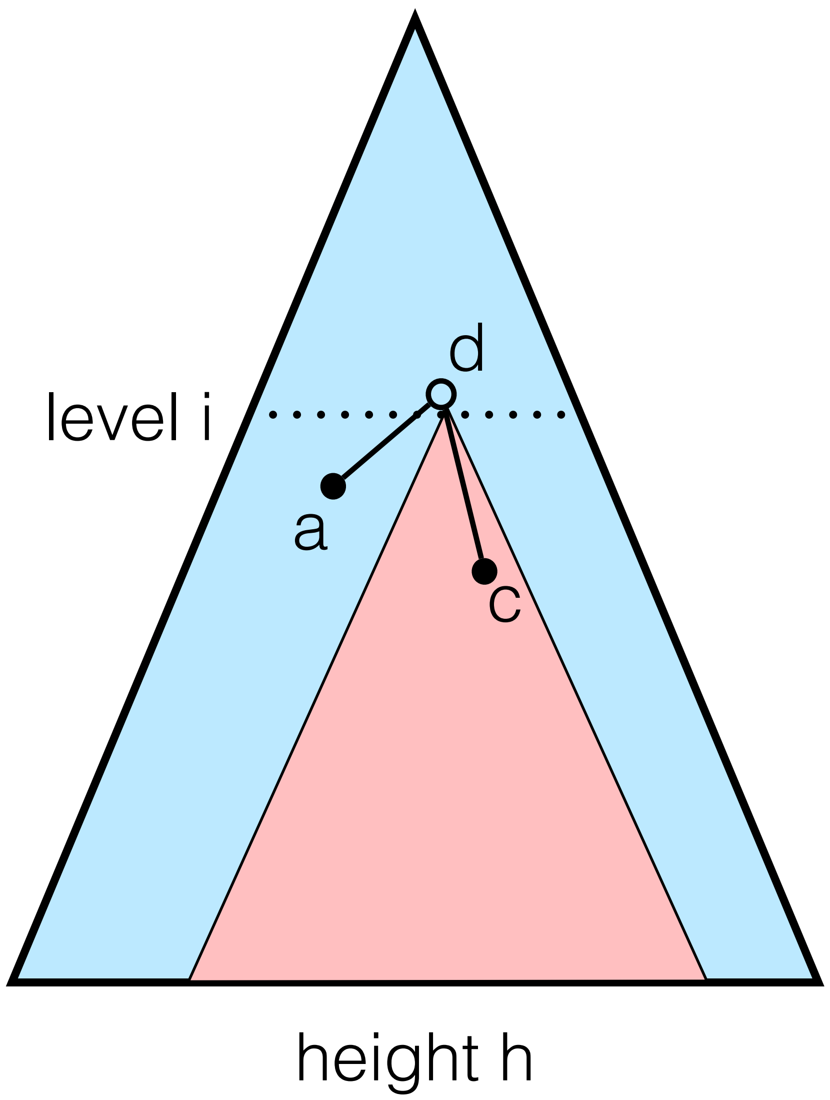
```
dfs blocking right@i; dfs the rest
```

3-hitting families for trees



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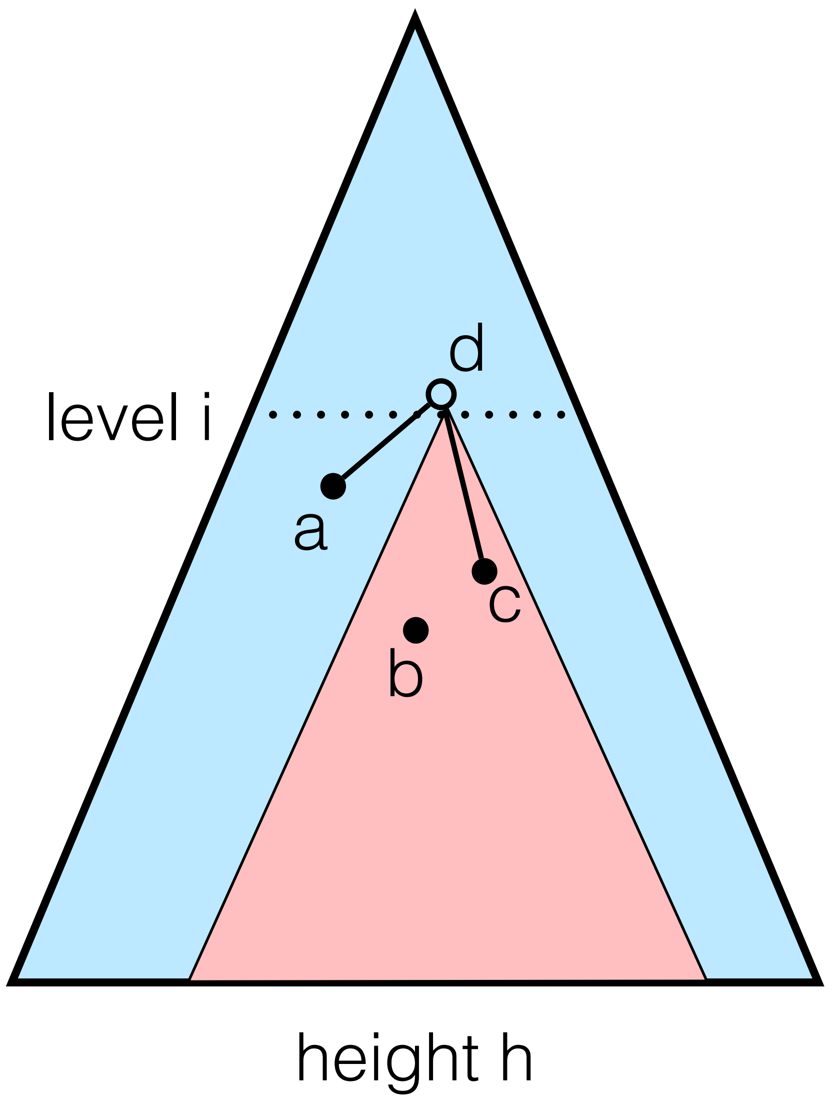

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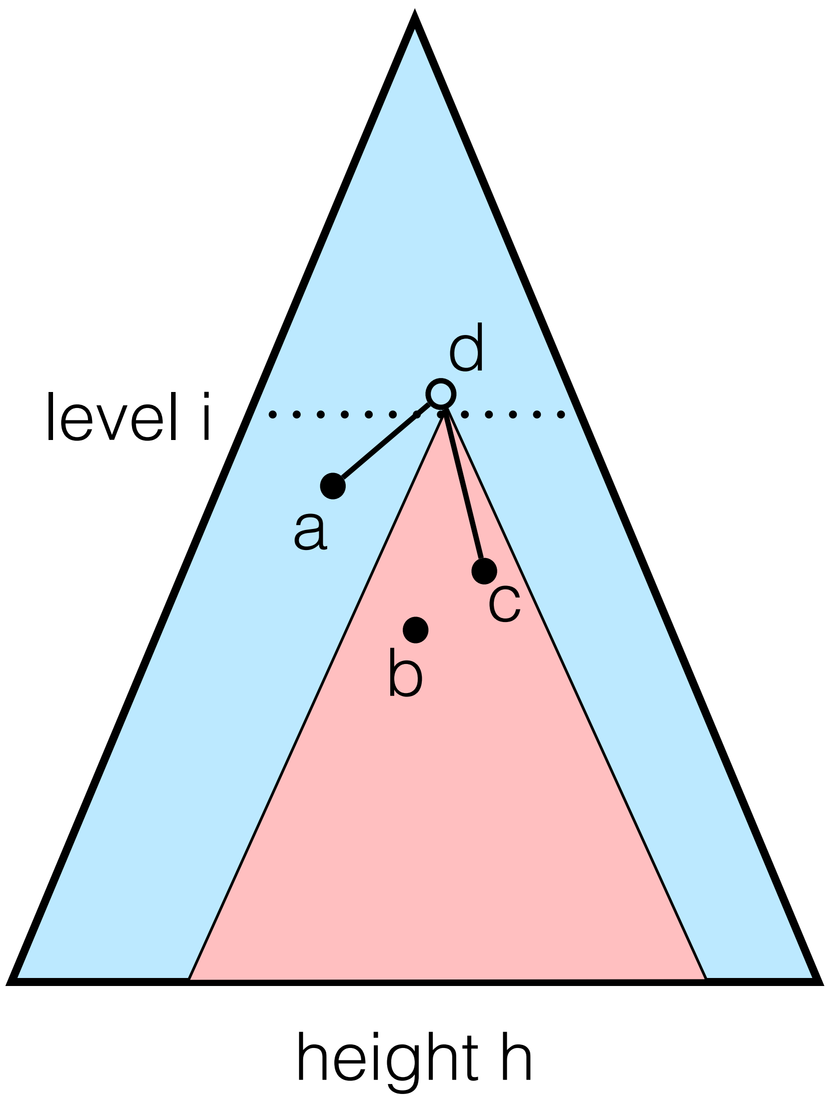

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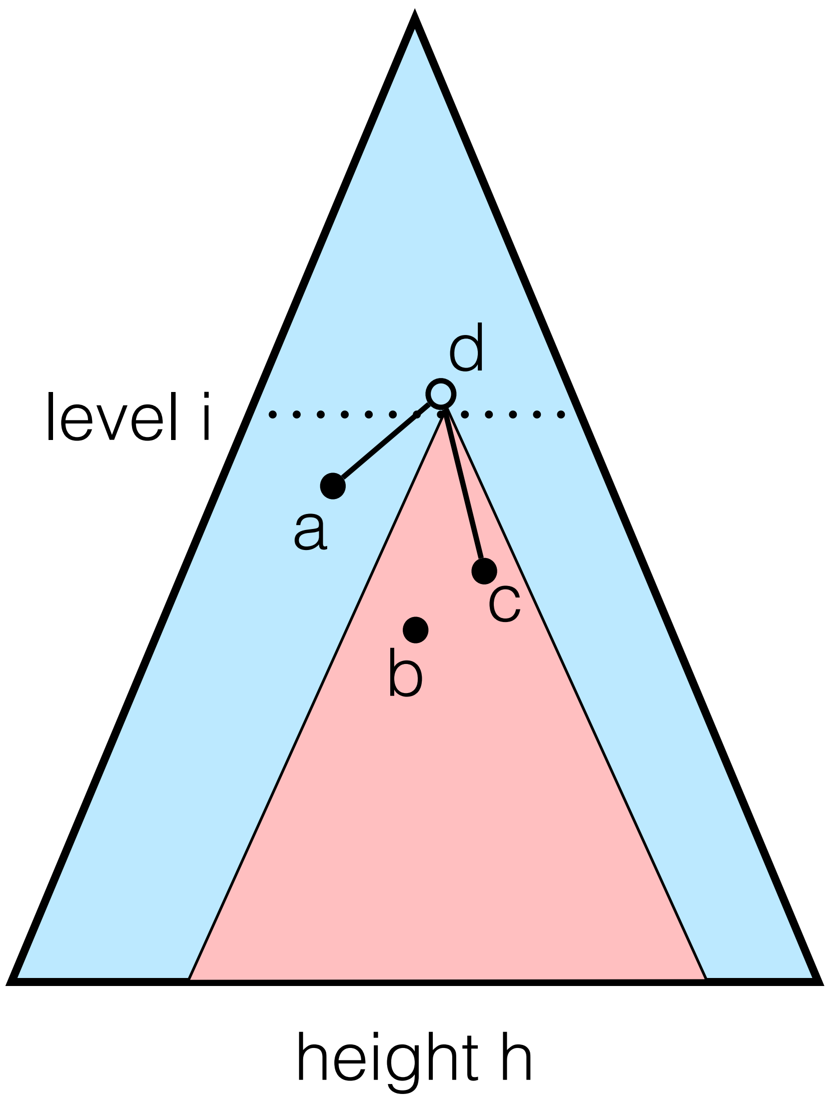

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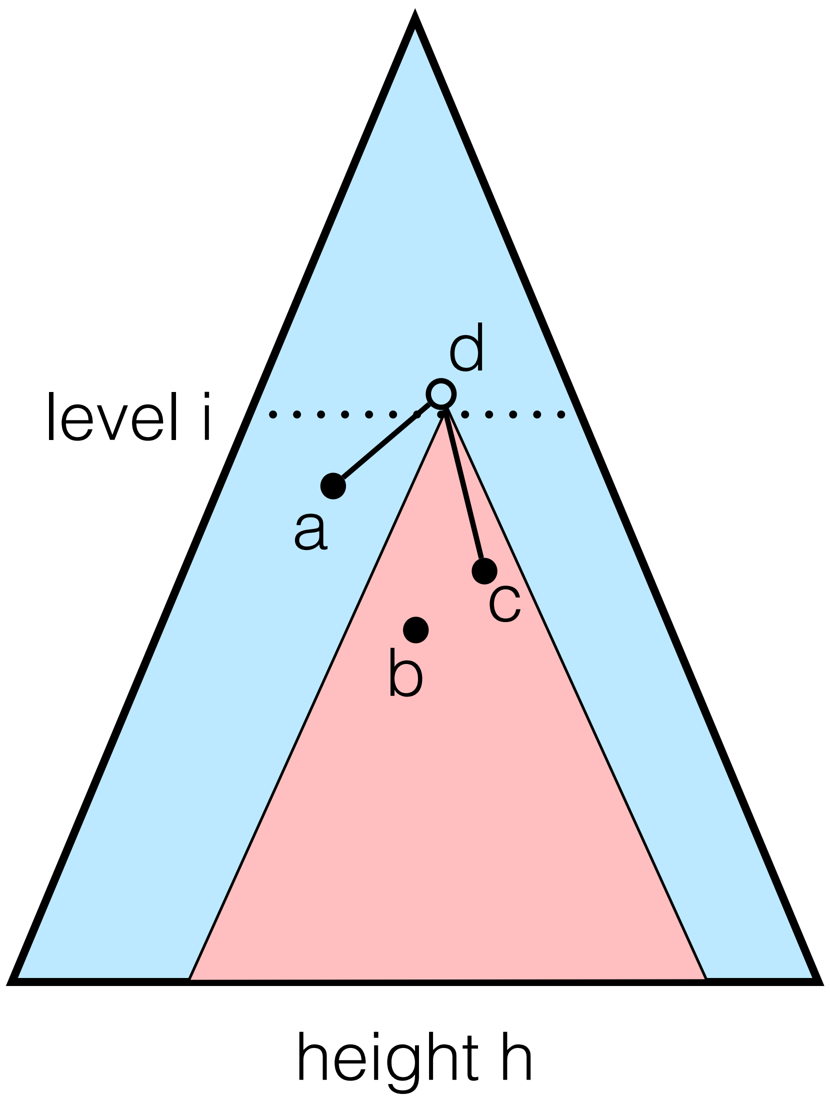

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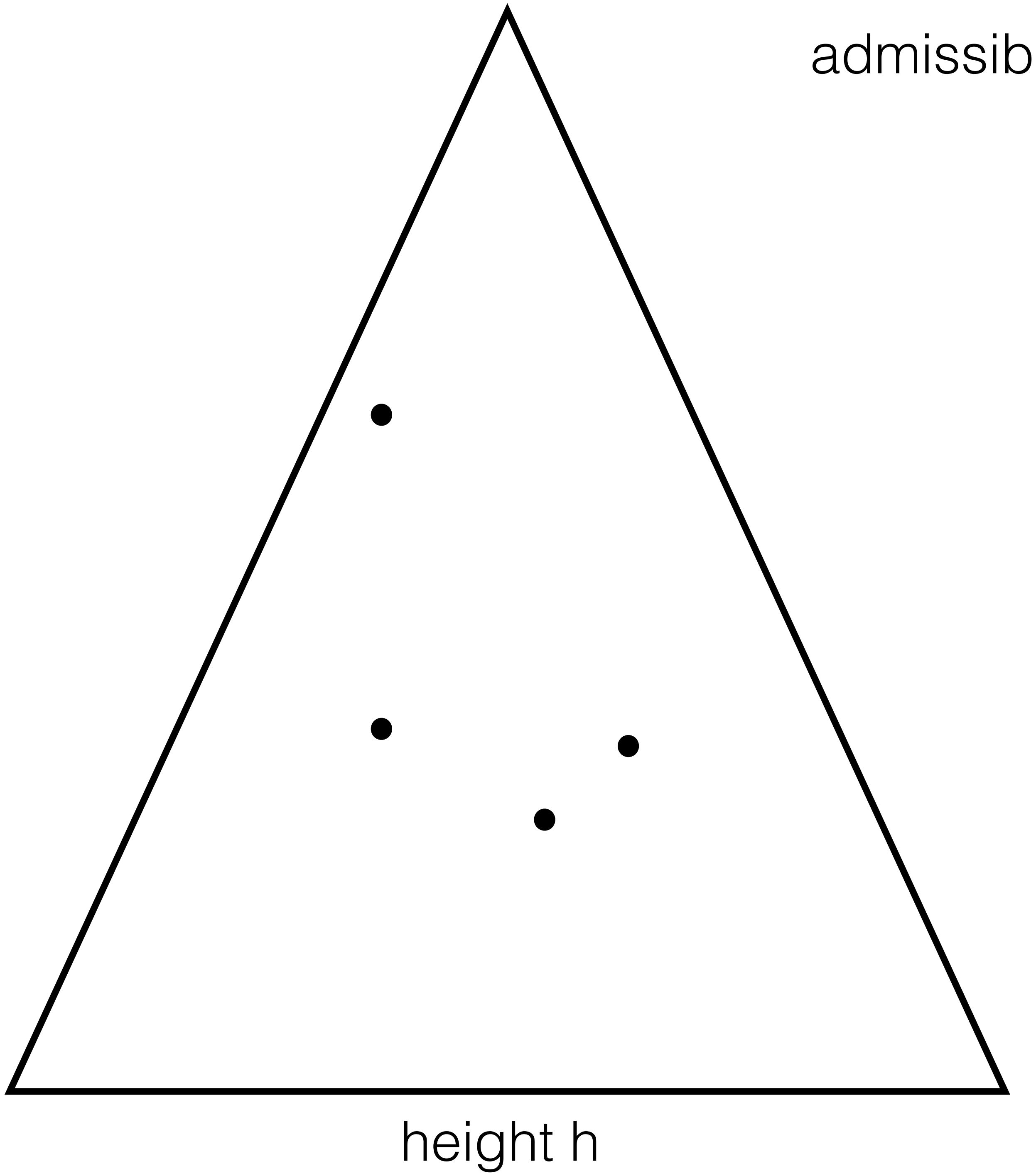
Total: **4h** schedules
(**4·log n** for a balanced tree)

d-hitting families for d≥4

admissible $(x_1,...,x_d)$



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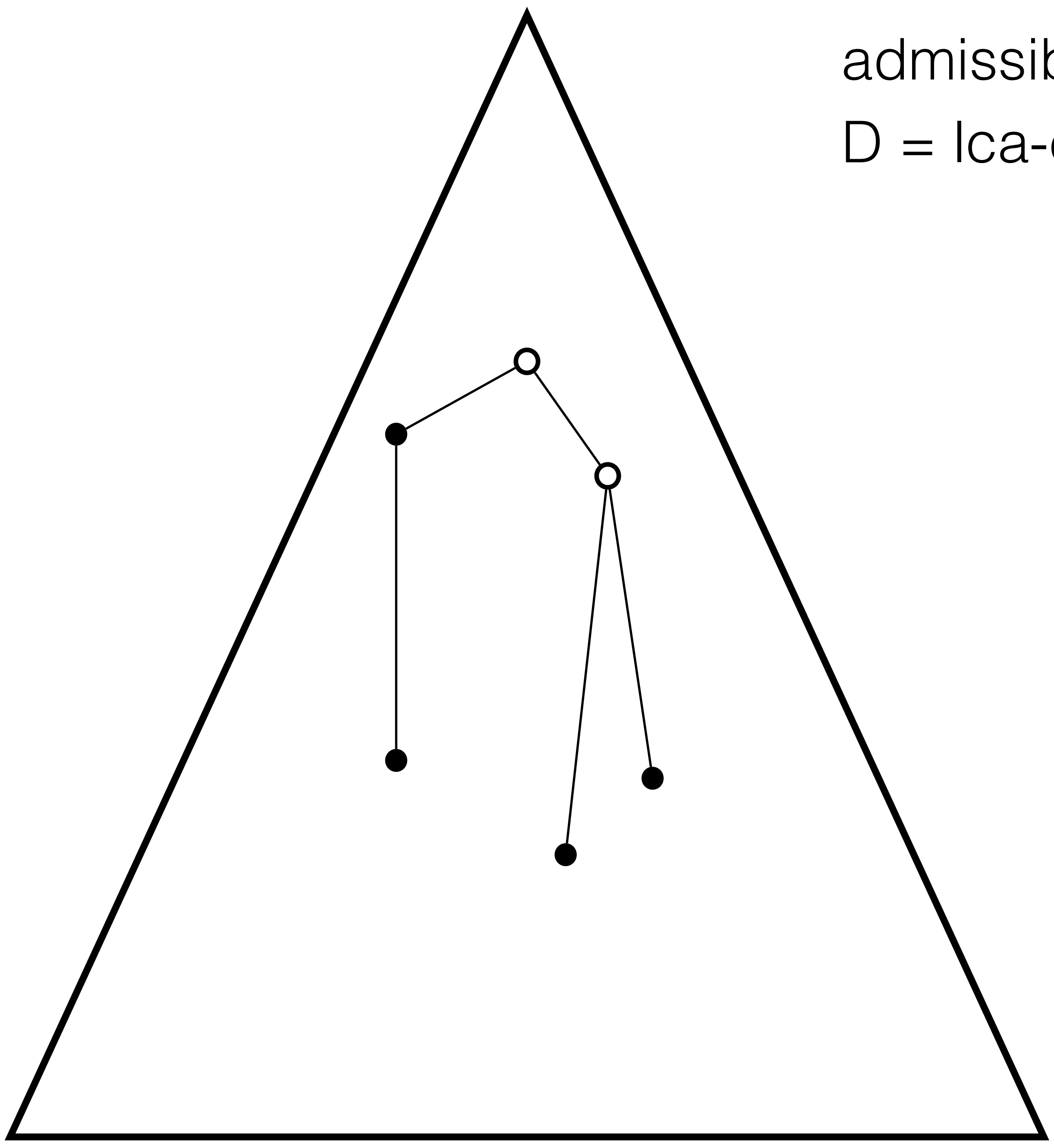
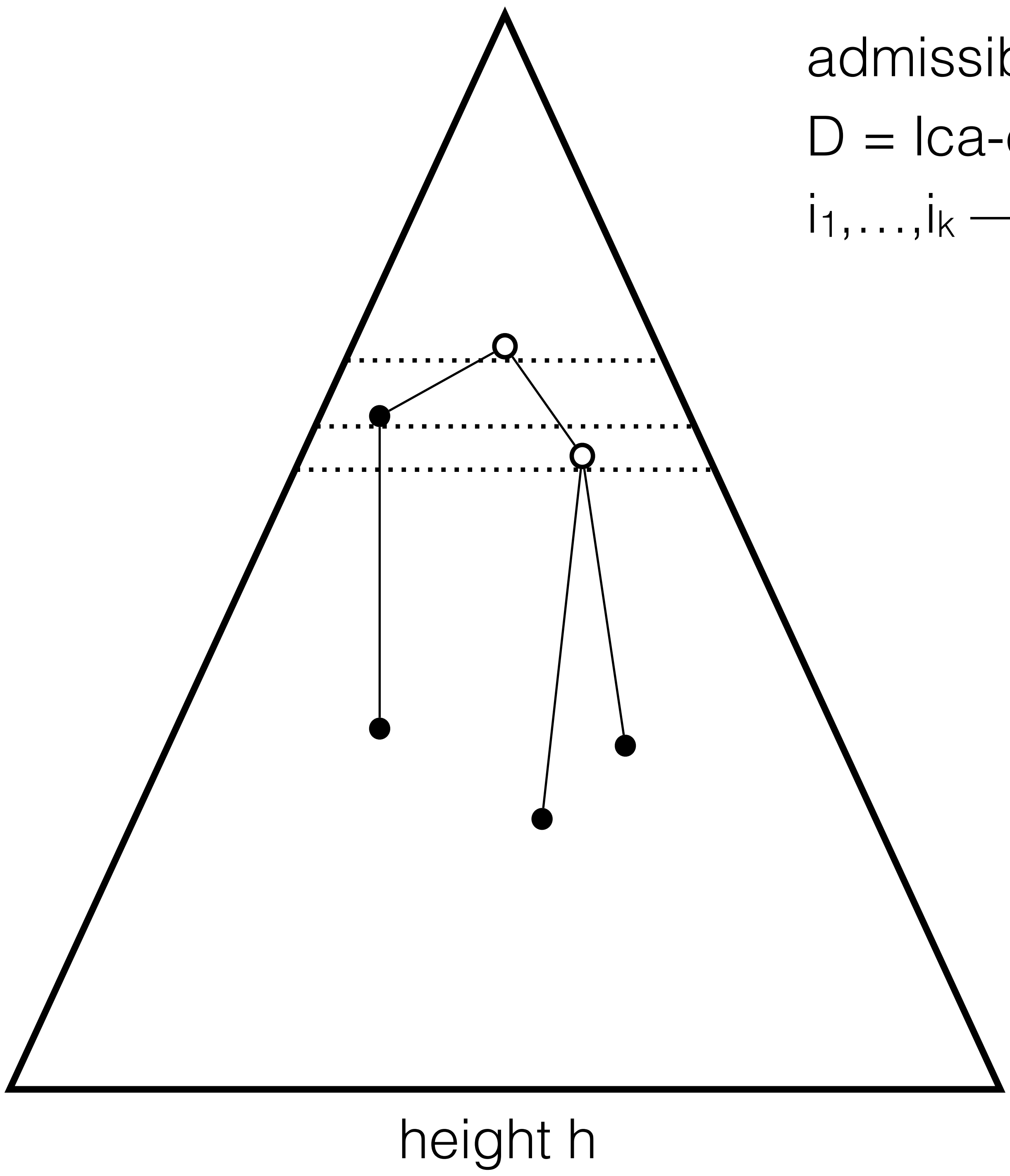


height h

admissible $(x_1,\ldots,x_d)$

D = lca-closure$(x_1,\ldots,x_d)$ (an ordered tree)

d-hitting families for d≥4



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$i_1,\ldots,i_k$ — levels of D's internal nodes

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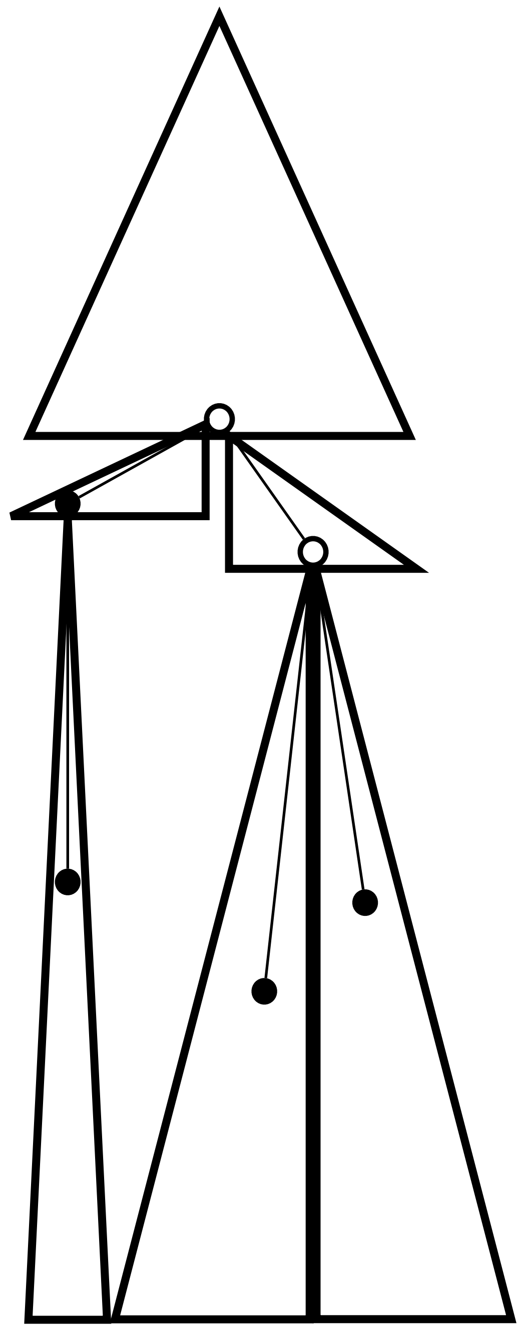

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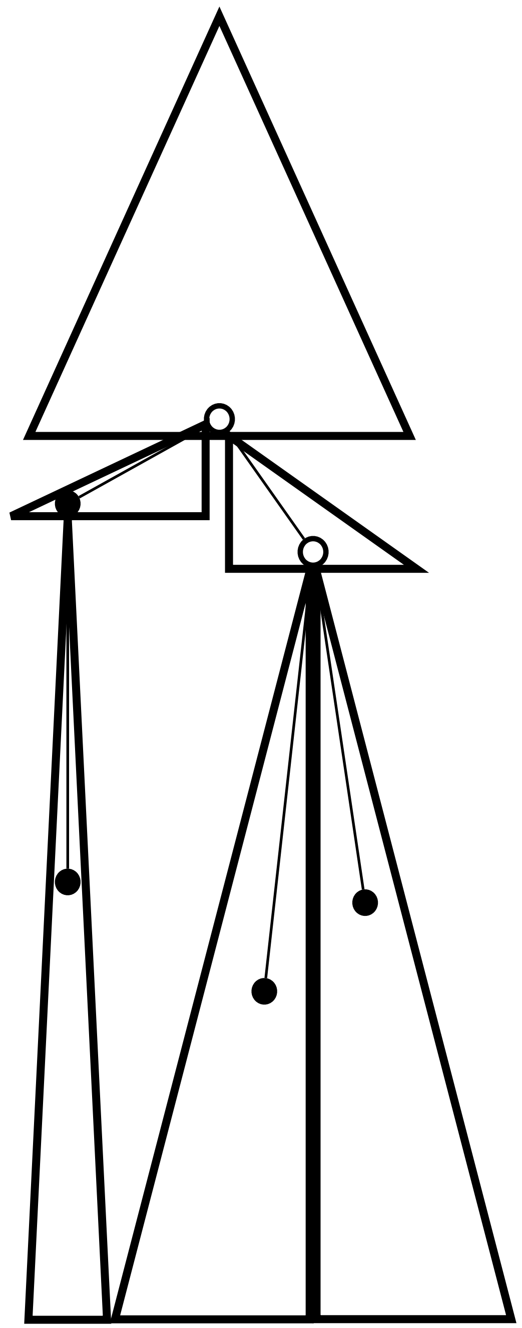

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π — schedule of D that hits $(x_1,\ldots,x_d)$

$(D, i_1,\ldots,i_k, \pi)$ is a **pattern**:

- determines a partition of the tree
- by scheduling parts according to π, determines a schedule that hits $(x_1,\ldots,x_d)$

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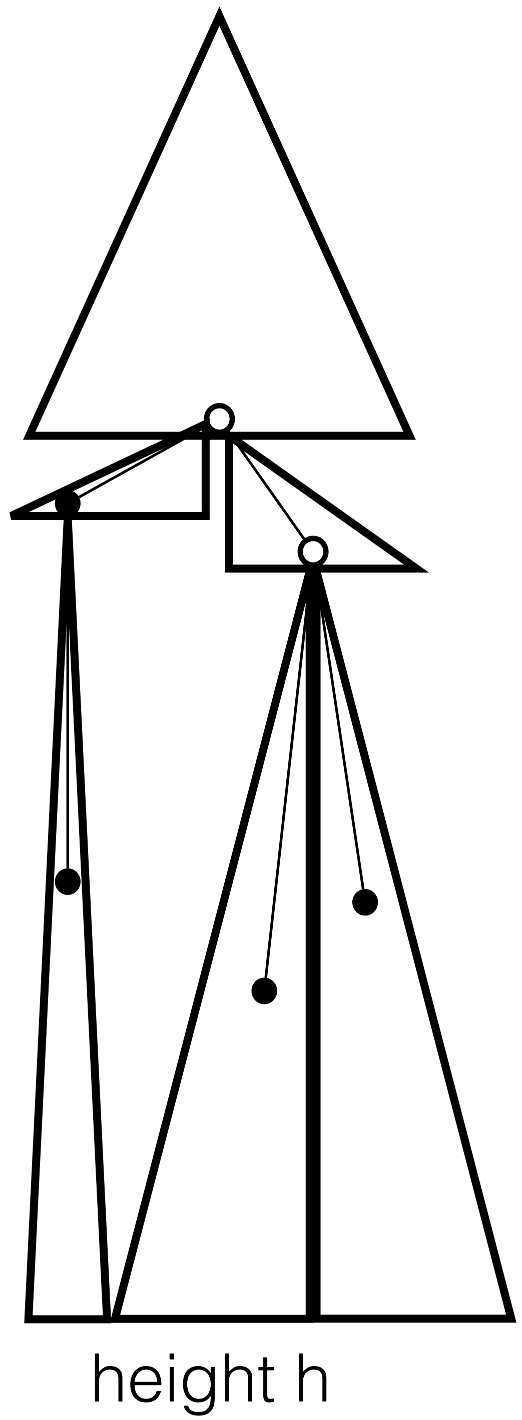

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```
for each pattern:
  schedule according to pattern
```

d-hitting families for $d \geq 4$

Claim. For any nodes $x_1,\ldots,x_d$, $|D| \leq 2d-1$. Moreover, D has at most d-1 internal nodes.

Accounting:

- at most **exp(d)** ordered trees with 2d-1 nodes
- at most $\mathbf{h^{d-1}}$ choices for levels $i_1,\ldots,i_{d-1}$
- at most **d!** schedules π

Total: at most $\mathbf{exp(d) \cdot d! \cdot h^{d-1}}$ patterns

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Note: For d=3, this is $O(h^2)$ instead of $O(h)$ schedules

From hitting families to systematic testing

Posets of events need not be static

- Use on-the-fly constructions as a heuristic

Beyond trees

- Our results extend to series-parallel graphs
- In general, even the case of d=2 is difficult (order dimension [Dushnik & Miller, '41])

Unbalanced trees

- Height h can be close to number of nodes n
- Use domain-specific properties to first reduce the poset

Summary

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http://www.mpi-sws.org/~fniksic/