The state equation for Petri Nets with Unordered Data

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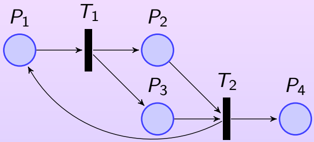
Outline

1. Petri Nets and State equation.
2. Petri Nets with Unordered Data.
3. State equations for *UDPN*.
4. Ideas from the proof.
5. Future work.

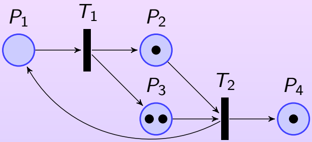
Petri Nets

Petri nets



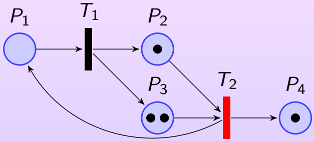
- Places
- Transitions

Petri nets



- Places
- Transitions
- Tokens, a Marking

Petri nets



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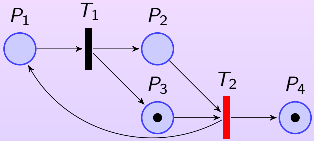

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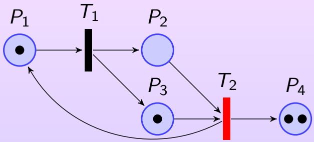

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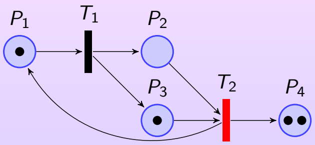

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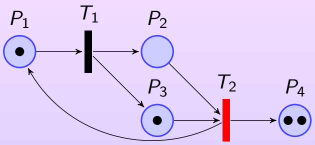
Definition (VAS - equivalent formalism)



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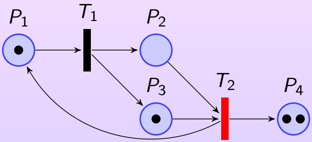

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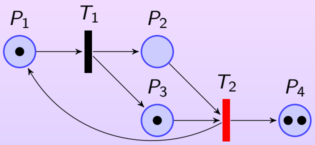

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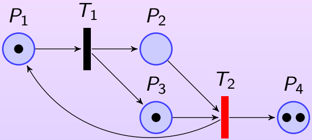

- Places
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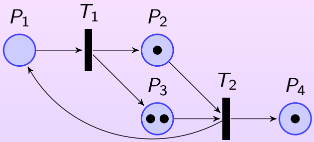
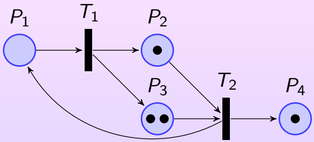
Definition (VAS - equivalent formalism)

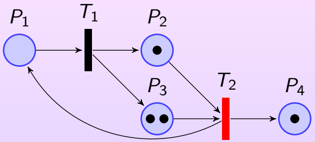
- Places - Dimensions.
- Transitions - Vectors t_i in $\mathbb{Z}^n$.
- the Marking - a Vector m_0 in $\mathbb{N}^n$.
- Firing a transition - Adding a vector to the marking, $m' = m + t_k$ (the effect has to be positive i.e. m' in $\mathbb{N}^n$).



- Dim = 7 (4 places +3 auxiliary)
- $$m_0 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



- $$m_0 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$
- $$T = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$



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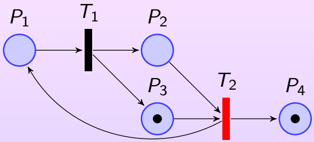

$$\begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

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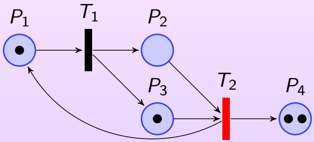

$$\begin{pmatrix} 0\\1\\2\\1\\1\\0\\0 \end{pmatrix} + \begin{pmatrix} 0\\-1\\-1\\0\\-1\\0\\1 \end{pmatrix} + \begin{pmatrix} 1\\0\\0\\1\\1\\0\\-1 \end{pmatrix} = \begin{pmatrix} 1\\0\\1\\2\\1\\0\\0 \end{pmatrix}$$

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Reachability

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$m \longrightarrow m'$ if there is $t \in T$ such that $m' = m + t$.

The *reachability* relation is a transitive closure of $\longrightarrow$.

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Can $m_f - m_i$ be expressed as a sum $\sum_i t_i$

where $t_i \in T$?

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It is an invariant of the reachability relation.

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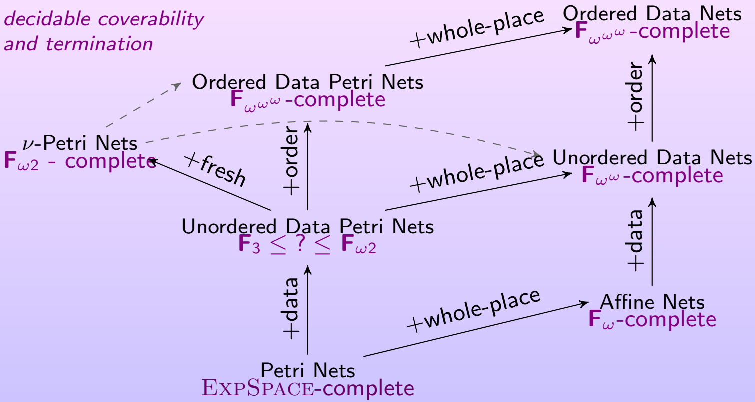
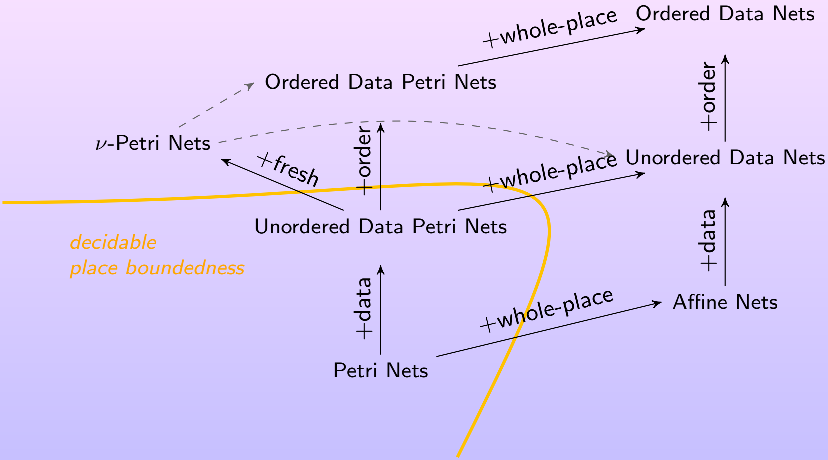
Lemma

If there is no solution for the state equation then m_f is not reachable from m_i.

Nets with Data



Nets with Tokens Which Carry Data, by Ranko Lazic, Thomas Newcomb, Joël Ouaknine, Andrew Roscoe, James Worrell.



Ordered Data Nets
+whole-place
Ordered Data Petri Nets
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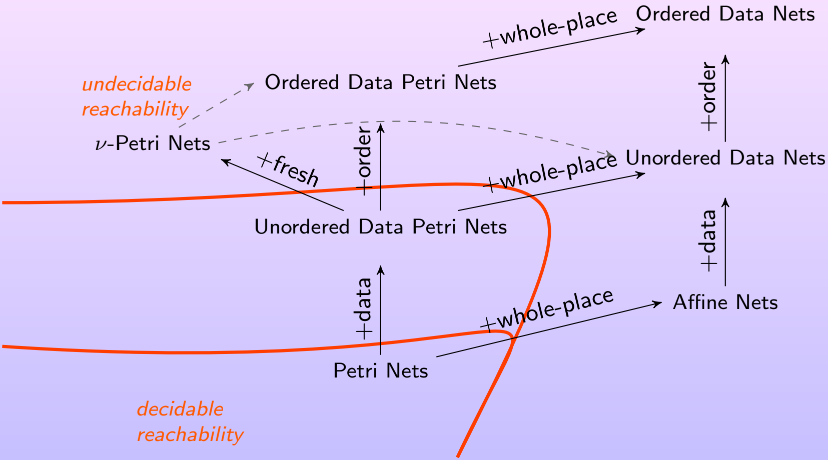
Petri Nets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Let $\mathbb{D}$ be an infinite data domain.

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Definition (Data VAS)

- Places - Dimensions n.
- Marking - a finitely supported function from $\mathbb{D}$ to $\mathbb{N}^n$.
- Transitions - a set of finitely supported functions from $\mathbb{D}$ to $\mathbb{Z}^n$ which is closed under data permutation.

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- Marking - a finitely supported function from $\mathbb{D}$ to $\mathbb{N}^n$.
- Transitions representation T - a finite set of finitely supported functions from $\mathbb{D}$ to $\mathbb{Z}^n$.
- Firing an abstract transition $t \in T$.
  1. Instantiate a transition - $t' = t \circ \pi$.
  2. $m' = m + t'$ and m' has to be a proper marking.

Example

Let $n = 3$

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$$m_0 = \begin{pmatrix} 2\alpha + 3\beta \\ 1\gamma + 2\alpha \\ 4\beta \end{pmatrix} \quad t_1 = \begin{pmatrix} -1\alpha \\ 1\alpha \\ 0 \end{pmatrix} \quad t_2 = \begin{pmatrix} \beta \\ \alpha \\ -2\gamma \end{pmatrix}$$

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Different instantiations: $t_1 \circ \pi_1 = \begin{pmatrix} -1\delta \\ 1\delta \\ 0 \end{pmatrix} t_1 \circ \pi_2 = \begin{pmatrix} -1\beta \\ 1\beta \\ 0 \end{pmatrix}$

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BAD

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Let $n = 3$

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$$m_0 + t_1 \circ \pi_2 = \begin{pmatrix} 2\alpha + 3\beta \\ 1\gamma + 2\alpha \\ 4\beta \end{pmatrix} + \begin{pmatrix} -1\beta \\ 1\beta \\ 0 \end{pmatrix} = \begin{pmatrix} 2\alpha + 2\beta \\ 1\gamma + 2\alpha + \beta \\ 4\beta \end{pmatrix}$$

GOOD

Example

Let $n = 3$

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$$m_0 + t_2 \circ \pi_3 = \begin{pmatrix} 2\alpha + 3\beta \\ 1\gamma + 2\alpha \\ 4\beta \end{pmatrix} + \begin{pmatrix} 1\alpha \\ 1\delta \\ -2\beta \end{pmatrix} = \begin{pmatrix} 3\alpha + 3\beta \\ 1\gamma + 2\alpha + \delta \\ 2\beta \end{pmatrix}$$

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$$m_f - m_i = \sum_i \mathbf{t_i} \circ \pi_i$$

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OUTPUT	If $\mathbf{m}$ can be expressed as $\sum_i \mathbf{t_i} \circ \pi_i$?

Theorem

State equation problem for UDNP is in NP.

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State equation problem for UDNP is in NP.

Exactly the same like the state equation for Petri Nets.

Problems and Ideas

Idea: Problems.

Definition (Support)

Let $\mathit{supp}\,(\mathbf{x}) \stackrel{\text{def}}{=} \{\alpha : \alpha \in \mathbb{D} \text{ and } \mathbf{x}(\alpha) \neq 0\}$.

Problem 1- unbounded number of data.

There is no bound on the number of data involved in the sum.

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Suppose $\mathbf{m} = \sum_{i=1}^{n} \mathbf{t_i} \circ \pi_i$, and $\mathbb{S} = \bigcup_{0<i<n} supp\,(\mathbf{t_i} \circ \pi_i)$.

What is the bound on $|\mathbb{S}|$?

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$|\mathbb{S}|$ *can be bounded by polynomially.*

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Suppose $|\mathbb{S}|$ is already bounded by a polynomial and we reduce problem to the state equation for a Petri Net. What will be the size of the Petri Net?

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Exponential!

The main combinatorial insight.

Definition (n-histogram)

Suppose $\mathbb{S} \subset \mathbb{D}$ is a finite set. A function $H : \mathbb{S} \times \mathbb{D} \longrightarrow \mathbb{N}$ is a n-histogram if:

1. $\sum_{\beta \in \mathbb{D}} H(\alpha, \beta) = n$ for all $\alpha \in \mathbb{S}$,
2. $\sum_{\alpha \in \mathbb{S}} H(\alpha, \beta) \leq n$ for all $\beta \in \mathbb{D}$.

Histograms over the same set $\mathbb{S}$ can be added pointwise.

$$\begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 0 \\ \hline 0 & 0 \\ \hline 0 & 1 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline \end{array} + \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 1 & 0 \\ \hline 0 & 0 \\ \hline 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} + \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} = \begin{array}{|c|c|}\hline 0 & 0 \\ \hline 1 & 0 \\ \hline 0 & 0 \\ \hline 1 & 1 \\ \hline 2 & 0 \\ \hline 0 & 3 \\ \hline \end{array}$$

A relation between 1-histogram and $t_i \circ \pi_i$.

Definition

For a given π and $\mathbf{t}$ we define 1-histogram $H_{\pi}^{supp(\mathbf{t})} : supp(\mathbf{t}) \times \mathbb{D} \longrightarrow \mathbb{N}$ such that

$$H_{\pi}(\alpha, \beta) = 1 \iff \pi(\beta) = \alpha.$$

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Let

$$t = \begin{pmatrix} 2\alpha + 3\beta \\ 2\alpha \\ 4\beta \end{pmatrix} \qquad \pi(\phi) \stackrel{\text{def}}{=} \begin{cases} \beta & \text{if } \phi = \alpha \\ \delta & \text{if } \phi = \beta \\ \alpha & \text{if } \phi = \delta \\ \phi & \text{otherwise} \end{cases}$$

$H_\pi^{supp(\mathbf{t})} =$

α	β	
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0	0	β
0	0	γ
1	0	δ

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$$t \circ \pi = \begin{pmatrix} 2\delta + 3\alpha \\ 2\delta \\ 4\alpha \end{pmatrix}$$

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Observe:

$$\mathbf{t} \circ \pi(\phi) = \sum_{\eta \in supp(\mathbf{t})} \mathbf{t}(\eta) \cdot H_\pi(\eta, \phi) \text{ for all } \phi \in \mathbb{D}.$$

Histograms - Theorem

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Histograms - Theorem

Definition: Homomorphism *Eval*

$$Eval(\mathbf{t}, H)(\phi) \stackrel{\text{def}}{=} \sum_{\eta \in supp(\mathbf{t})} \mathbf{t}(\eta) \cdot H(\eta, \phi) \qquad \text{for all } \phi \in \mathbb{D}.$$

Histograms - Theorem

Definition: Homomorphism *Eval*

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Theorem

A function $H : \mathbb{S} \times \mathbb{D} \to \mathbb{N}$ is an n-histogram if, and only if, H is the sum of n 1-histograms over $\mathbb{S}$.

Lemma

$\mathbf{m_t}$ *can be expressed as* $\sum_{i=1}^{n} \mathbf{t} \circ \pi_i$ *iff*

there is a n-histogram H such that $\mathbf{m_t} = Eval(\mathbf{t}, H)$.

Histograms - Theorem

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$\mathbf{m}$ can be expressed as $\sum_{\mathbf{t} \in T} \sum_{i=1}^{n_{\mathbf{t}}} \mathbf{t} \circ \pi_{\mathbf{t},i}$ iff

there is a sequence of $n_{\mathbf{t}}$-histogram $H_{\mathbf{t}}$ for any $\mathbf{t} \in T$ such that

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The set of data that appear in the solution, $\mathbb{T} \stackrel{def}{=} |\bigcup_{\mathbf{t}\in T, i\in\mathbb{N}} supp\,(\mathbf{t}\circ\pi_{\mathbf{t},i})|$*, can be bounded by polynomially.*

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- Guess values and check the sum. Incorrect as we don't have bound on numbers.

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- We need to build a system of linear equations which incorporates conditions for being a histogram and proper *Eval*-uation

Future.

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Thank You.