Speaker: Ameera Chowdhury

Title: Inclusion Matrices and the MDS Conjecture

Abstract: Let $\mathbb{F}_q$ be a finite field of order q with characteristic p. An arc is an ordered family of vectors in $\mathbb{F}_q^k$ in which every subfamily of size k is a basis of $\mathbb{F}_q^k$. The MDS conjecture, which was posed by Segre in 1955, states that if $k \leq q$, then an arc in $\mathbb{F}_q^k$ has size at most $q+1$, unless q is even and $k = 3$ or $k = q-1$, in which case it has size at most $q+2$.

We propose a conjecture which would imply that the MDS conjecture is true for almost all values of k when q is odd. We prove our conjecture in two cases and thus give simpler proofs of the MDS conjecture when $k \leq p$, and if q is not prime, for $k \leq 2p-2$. To accomplish this, given an arc $G \subset \mathbb{F}_q^k$ and a nonnegative integer n, we construct a matrix $M_G^{\uparrow n}$, which is related to an inclusion matrix, a well-studied object in combinatorics. Our main results relate algebraic properties of the matrix $M_G^{\uparrow n}$ to properties of the arc G and may provide new tools in the computational classification of large arcs.