Proof of a Conjecture on Monomial Graphs

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Abstract

Let e be a positive integer, p be an odd prime, $q = p^e$, and $\mathbb{F}_q$ be the finite field of q elements. Let $f, g \in \mathbb{F}_q[X, Y]$. The graph $G_q(f, g)$ is a bipartite graph with vertex partitions $P = \mathbb{F}_q^3$ and $L = \mathbb{F}_q^3$, and edges defined as follows: a vertex $(p) = (p_1, p_2, p_3) \in P$ is adjacent to a vertex $[l] = [l_1, l_2, l_3] \in L$ if and only if $p_2 + l_2 = f(p_1, l_1)$ and $p_3 + l_3 = g(p_1, l_1)$. If $f = XY$ and $g = XY^2$, the graph $G_q(XY, XY^2)$ contains no cycles of length less than eight and is edge-transitive. Motivated by certain questions in extremal graph theory and finite geometry, people search for examples of graphs $G_q(f, g)$ containing no cycles of length less than eight and not isomorphic to the graph $G_q(XY, XY^2)$, even without requiring them to be edge-transitive. So far, no such graphs $G_q(f, g)$ have been found. It was conjectured that if both f and g are monomials, then no such graphs exist. In this paper we prove the conjecture.

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