# Distance sets on circles and Kneser's addition theorem

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Let us consider the following problem: For a set $X$ of $n$ points on the unit circle of $\mathbb{R}^2$ such that exactly $k$ distances occur between two distinct points in $X$, can you say anything about its structure?

We can show that if the number $k$ of distances is sufficiently small relative to the number $n$ of points, then $X$ lies on a regular polygon. More precisely, if $k < 3t$ or $3t - 2$ according to whether $n = 4t, 4t - 1$ or $n = 4t - 2, 4t - 3$, respectively, then $X$ lies on a regular $2k$ or $(2k + 1)$-sided polygon. Furthermore, this bound can not be further improved.

The well-known addition theorem of Kneser is behind our result. The talk will be based on the paper "K. Momihara, M. Shinohara, Distance sets on circles, to appear in *Amer. Math. Monthly*".