

A THEOREM OF RADO ON MONOCHROATIC PATHS

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Let $c : [\omega]^2 \rightarrow r$. A *monochroatic path* of color j is a listing (possibly empty) of integers $\{a_0, a_1, a_2 \dots\}$ such that, for all $i \geq 0$, if a_{i+1} exists then $c(\{a_i, a_{i+1}\}) = j$. A empty listing can be a path of any color. A singleton can be a path of any color. Paths might be finite or infinite. The color is determined for paths of more than one node. Improving on a result of Edrós, in 1978, Rado published a theorem which implies:

Theorem 1 (Rado). *Let $c : [\omega]^2 \rightarrow r$. Then, for each $j < r$, there is a monochroatic path of color j such that these r paths (as sets) partition ω (so they are pairwise disjoint sets and their union is everything).*

We will provide some results and proofs which allow us to *partially* analyze the effective content of this theorem. These results and proofs are work with Greg Igusa, Ludovic Patey, and Mariya Soskova.