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Lieb's concavity theorem, matrix geometric means, and semidefinite optimization

A celebrated result of Lieb is that the map $(A, B) \mapsto \mathrm{tr}\left[K^* A^{1-t} K B^t\right]$ is jointly concave in the pair (A, B) of positive definite matrices if K is any fixed $m \times n$ matrix and $t \in [0, 1]$. We show that if $t = p/q \in [0, 1]$ is rational then the hypograph of Lieb's function has an explicit semidefinite description of size $O(mn \log(q))$. This allows us to give semidefinite formulations for a number of other jointly concave functions arising in quantum information. Our constructions all follow from a semidefinite formulation of the weighted matrix geometric mean.

Based on joint work with Hamza Fawzi.