

The local Gan-Gross-Prasad conjectures (Bessel case) give a precise description of some branching-laws between orthogonal or unitary groups over local fields in terms of some arithmetic invariants. In this talk, I shall mainly focus on the case of unitary groups. The simplest form of the conjecture is as follows. Let  $E/F$  be a quadratic extension of local fields,  $V$  a finite dimensional hermitian space over  $E$  and  $W$  a nondegenerate hyperplane of  $V$ . Set  $G = U(W) \times U(V)$  and  $H = U(W)$ . We may view  $H$  as a subgroup of  $G$  via the diagonal embedding. For  $\pi$  a smooth irreducible representation of  $G(F)$ , denote by  $m(\pi)$  the space of  $H(F)$ -invariant linear forms on  $\pi$  (with the further requirement that we only consider continuous linear forms in the archimedean case). By multiplicity one theorems of Aizenbud-Gourevitch-Rallis-Schiffmann and Jiang-Sun-Zhu, we know that  $m(\pi)$  is always less or equal to one. The Gan-Gross-Prasad conjecture predicts that this result continue to hold if we replace  $\pi$  by a tempered  $L$ -packet. Namely, if  $\Pi$  is such an  $L$ -packet then

$$\sum_{\pi \in \Pi} m(\pi) \leq 1$$

Moreover, this should become an equality if we replace  $\Pi$  by the corresponding extended Vogan  $L$ -packet (restricted to the pure inner forms of  $G$  that are "relevant"). A more precise version of the conjecture even pins down the unique representation  $\pi$  in the extended  $L$ -packet having multiplicity one. This refinement involves certain local root numbers associated to the Langlands parameter of  $\pi$ . In a pioneering work, Waldspurger and Mœglin-Waldspurger have proved the conjecture and its refinement for orthogonal groups over  $p$ -adic fields. In this talk, I shall present a proof of the conjecture for unitary groups which has the advantage of working equally well over any local field of characteristic 0. If time permits, I will also discuss further development implying the refined conjecture (this is work in progress).