THE IMMERSED INTERFACE METHOD FOR THE AXIS-SYMMETRIC NAVIER-STOKES EQUATION IN CYLINDRICAL COORDINATES

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Abstract

Many physical application problems are axis-symmetric. Using axis-symmetric properties, many three dimensional problems can be solved efficiently using two dimensional axis-symmetric coordinates. In this paper, the immersed interface method (IIM) in axis-symmetric Cylindrical coordinates is developed for the Navier Stokes equation with interfaces. A projection method is developed in order to separate the problem into two Helmholtz equations for the velocity and one Poisson equation for the pressure. The effect of the interface is compensated through correction terms for the derivatives that are added to the right hand side of previous equations. A staggered grid is used to overcome the pole singularity. Some of the challenges that must be overcome to solve the proposed problem are: derive the jump relations in axis-symmetric cylindrical coordinates, designing the numerical algorithm when the interface is close to the pole ($r = 0$), computing interface quantities such as the normal and tangential directions, surface derivatives, curvature information, etc. The numerical algorithm is based on a finite difference discretization and uniform grid in the axis-symmetric coordinates. The finite difference scheme is the standard one away from the interface but is modified at grid points near and on the interface. The method is shown to be second order accurate in the infinity norm. The developed new IIM is applied to the solution of some related problems.