Wavy properties and analytical modeling of free-surface flows in the solution of fluid-structure interactions

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We consider one or several bodies floating on the free surface which is extended horizontally to infinity. The major issues of such free-surface hydrodynamics include the fluid viscous effect, the non-linearities and wavy behavior of the free surface. Significant progress has recently been made in Computational Fluid Dynamics (CFD) to model viscous flows and in non-linear computations of potential flows, thanks to the amazing development of powerful computation resources. However, still a reliable and practical method is missing in the modeling of fluid-body interaction like seakeeping of ships advancing in waves. One major reason is associated with the peculiar property of free-surface flows which are oscillatory and dispersive. It is easy to understand that most advanced numerical algorithms in CFD are merely useful to capture wavy flows since the dimension of meshed domain should be large enough to contain several waves while the mesh (cell or finite volume) size should be enough small to capture complex flow of vorticity. Recent research has thus oriented to develop multi-domain methods to combine different approaches to model the viscous effect and non-linearities on one side and to take care of wavy flow on the other side.

Classical works based on Laplace equation and the linearized boundary condition on the free surface have been very successful in giving the elementary solution (often called Green function) associated with an elementary singularity. The elementary solution gives effectively rich information about wavy properties including wave patterns (wave length, wave phase and group velocities, wave crest-lines and cusp angles) and the slow decay rate of wave amplitude to infinity. However, this elementary solution satisfying the linearized boundary condition on the free surface is not directly useful. Its usefulness is embedded in the boundary integral equation which can represent the velocity potential at any point in the domain by only the surface integral on parts of the boundary. Furthermore, the Green function of most problems like seakeeping of ships advancing in waves presents the complex singular and highly oscillatory properties, the numerical evaluation of itself and its integration on the meshed boundary surface are very difficult, if not impossible, to obtain. To overcome above difficulties, we have been developing a new multi-domain method consisting to divide the fluid domain by a control surface surrounding floating bodies and being at some distance. In the external domain from the control surface, the free-surface Green function satisfying the boundary condition on the free surface is used while in the internal domain limited by the control surface, the body hull and portion of free surface in between, the simple Green function ($1/r$) is used. Unlike classical methods, the free-surface Green function is not explicitly evaluated but its integration on the control surface is computed analytically. This is possible if we define a perfect smooth control surface on which unknowns are expressed by expansions composed of base functions. In the case of an infinite vertical cylinder, we can write the velocity potential and its radial derivative by an expansion composed of Fourier series in polar direction and Laguerre function in vertical direction. The influence coefficients in the integral equation are then associated with all elementary Fourier-Laguerre distribution and represented by six-fold integrations including the double integration on the control surface of the Green function (double Fourier integral) and the double integral following Galerkin collocation. The six-fold integrals have been analytically calculated and reduced to single integrals in wavenumber. Furthermore, the single wavenumber integrals could be evaluated semi-analytically and approximated by Tchebychev polynomials. The solution of the external domain gives the relationship of velocity potential and its normal derivative often called the Dirichlet-to-Neumann $[\mathbf{D}^2_{\mathbf{N}}]$ operator.

This $[\mathbf{D}^2_{\mathbf{N}}]$ operator can be directly used in the coupling, through continuous conditions on the control surface, with the internal domain in which we use the simple Green function to solve linear potential problems. The extension could be to couple with a CFD solver of viscous flows or non-linear potential flows in the internal domain. In this case, a transient domain between the external and internal domains is necessary to make a smooth transition from viscous or/and non-linear problem to the linear free-surface wavy problem. In this new multi-domain method, the numerical algorithms used in CFD are well suited for local flows in the vicinity of bodies while the semi-analytical solutions ensure the wavy behavior of free surface flows to be consistent and accurate. The detail of mathematical development and the results of seakeeping of ships advancing in waves as an example will be presented in the workshop.