DEFORMING IRREGULAR SINGULAR POINTS

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The geometric data classifying an irregular singularity of a (vector valued) o.d.e. is well known: a formal normal form, and Stokes data, consisting in the generic case basically of a family of $n-1$ upper triangular matrices and $n-1$ lower triangular matrices, the Stokes data, as well as a formal normal form at the singularity. Likewise, generically, for regular singularities one has the monodromy data , as well as formal normal forms at the singularity. These data, while similar, are not the same, and one would like some picture which holds relatively uniformly in the family. More precisely, let us consider, on a neighbourhood of the origin in $\mathbb{C}$,

$$y' = \frac{A(\epsilon, x)}{p_\epsilon(x)} \cdot y. \tag{0.1}$$

where

$$p_\epsilon(x) = x^{k+1} + \epsilon_{k-1}x^{k-1} + \cdots + \epsilon_1 x + \epsilon_0, \tag{0.2}$$

is a polynomial, and ϵ_i are small complex parameters, with $\epsilon = 0$ corresponding to an irregular singularity. The $n \times n$ matrix $A(\epsilon, x)$ is holomorphic in x, ϵ, on a neighbourhood of the origin. Its leading order term A_0 is supposed to be diagonalisable, with distinct eigenvalues, which, changing variables if necessary, we can assume have distinct real part. One would like monodromy data for the family. The answer starts with the introduction of an extra variable t:

$$\dot{y} = A(\epsilon, x) \cdot y \tag{0.3}$$

$$\dot{x} = p_\epsilon(x) \tag{0.4}$$

and proceeds from there. Joint work with Christiane Rousseau.