Another algebraic decomposition of $\mathbf{R}$

Liu Yiqun

January 4, 2016

In this talk, I will present a result joint with Yang Yue from National University of Singapore.

We study another algebraic decomposition of $\mathbf{R}$ where $\mathbf{R}$ denotes the class of computably enumerable degrees. The first important result of this kind is by Ambos-Spies et al. (1984) that $\mathbf{R}$ can be decomposed as the disjoint union of an ideal $\mathbf{M}$ and a filter $\mathbf{PS}$ where $\mathbf{M}$ denotes the class of $\mathbf{0}$ and the cappable degrees and $\mathbf{PS}$ denotes the class of the promptly simple degrees.

We first define another subclass of $\mathbf{R}$, denoted as $\mathbf{STB}$, which consists of $\mathbf{0}$ together with all degrees containing the base (set) in some Slaman triple. Given any Slaman triple (A, B, C), we say the c.e. set A is the base set for such triple.

We will show $\mathbf{R} = \mathbf{STB} \cup \mathbf{PS}$ in the sense that given any c.e. set A, either there exist c.e. sets B and C such that (A, B, C) is a Slaman triple or A has promptly simple degree. Combined with the classical decomposition, they imply that $\mathbf{M} = \mathbf{STB}$ and in particular, any c.e. set which is part of some c.e. minimal pair is also the base for some Slaman triple.