Integrable deformations of local analytic fibrations with singularities

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Abstract

In this talk we shall study analytic integrable deformations of the germ of a holomorphic foliation given by $df = 0$ at the origin $0 \in \mathbb{C}^n, n \geq 3$. We consider the case where f is a germ of an irreducible and reduced holomorphic function. Our central hypotheses is that, *outside of a dimension* $\leq n-3$ *analytic subset* $Y \subset X$, *the analytic hypersurface* $X_f : (f = 0)$ *has only normal crossings singularities*. We then prove that, as germs, such deformations also exhibit a holomorphic first integral, depending analytically on the parameter of the deformation. This applies to the study of integrable germs writing as $\omega = df + f\eta$ where f is quasi-homogeneous. Under the same hypotheses for $X_f : (f = 0)$ we prove that ω also admits a holomorphic first integral. Finally, we conclude that an integrable germ $\omega = adf + f\eta$ admits a holomorphic first integral provided that: (i) $X_f : (f = 0)$ is irreducible with an isolated singularity at the origin $0 \in \mathbb{C}^n, n \geq 3$; (ii) the algebraic multiplicities of ω and f at the origin satisfy $\nu(\omega) = \nu(df)$. In the case of an isolated singularity for $(f = 0)$ the writing $\omega = adf + f\eta$ is always assured so that we conclude the existence of a holomorphic first integral. Some questions related to Relative Cohomology are naturally considered and not all of them answered.

References

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