Categorical Plucker formula and Homological Projective Duality

In the classical theory of topology of a projective algebraic variety X, on one hand, we have Lefschetz theory for linear sections, which states the cohomology of a hyperplane section almost all comes from the cohomology of the ambient space X except for middle dimension, where new interesting contributions emerge. On the other hand, we have Plucker formula for non-linear sections, which states the Euler characteristics of the transverse intersection X ∩ T of X by another subvariety T can be expressed in terms of the ones of the intersection of their projective dual varieties Xv ∩ Tv.

For the theory of bounded derived category of coherent sheaves, on one hand, Kuznetsov introduced the concept of Homological Projective Duality (HPD) to study linear sections, and showed the HPD-theorem, which states when the HPD of X exists as a variety Y, not only we have Lefschetz type decompositions for the derived categories of linear sections XL⊥, but also descriptions of interesting parts in terms of derived categories of the dual linear sections YL of Y.

For non-linear sections, we show that HPD theorem still holds for non-linear sections of the pair (X,Y) by another HP-dual pair (S,T), as long as if they intersect properly. This enables us to produce examples of decomposition of derived category of varieties as well as give descriptions of their interesting parts. When taking Euler characteristic of Hochschild homology of our formal, we obtain Plucker type formula for HPD. If we take (S,T) to be dual linear projective subspaces, our method gives a more direct and categorical proof of Kuznetsov's original HPD theorem without referring concrete geometries of Grassmannians.

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