"Holomorphic factorization of maps into the special linear group"

Abstract:

It is standard material in a Linear Algebra course that the group $\mathrm{SL}_m(\mathbb{C})$ is generated by elementary matrices $E + \alpha e_{ij}$ $i \neq j$, i.e., matrices with 1's on the diagonal and all entries outside the diagonal are zero, except one entry.

The same question for matrices in $\mathrm{SL}_m(R)$ where R is a commutative ring instead of the field $\mathbb{C}$ is much more delicate, interesting is the case that R is the ring of complex valued functions (continuous, smooth, algebraic or holomorphic) from a space X.

For $m \geq 3$ (and any n) it is a deep result of Suslin that any matrix in $\mathrm{SL}_m(\mathbb{C}[\mathbb{C}^n])$ decomposes as a finite product of unipotent (and equivalently elementary) matrices.

In the case of continuous complex valued functions on a topological space X the problem was studied and solved by Thurston and Vaserstein. For rings of holomorphic functions on Stein spaces, in particular on $\mathbb{C}^n$, this problem was explicitly posed as the **Vaserstein problem** by Gromov in the 1980's.

In the talk we explain a complete solution to Gromov's *Vaserstein Problem* from a joint work with B. Ivarsson. The proof uses a very advanced version of the Oka-principle proposed by Gromov and proved in recent years by Forstnerič: An elliptic stratified submersion over a Stein space admits a holomorphic section iff it admits a continuous section.