

A REMARK ON GENERALIZED JACOBIAN CONJECTURE FOR \mathbb{A}^2/G

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Abstract

Let G be a small finite subgroup of $\mathrm{GL}(2, \mathbb{C})$ and let $X = \mathbb{A}^2/G$. Let $\varphi : X \rightarrow X$ be a quasi-étale endomorphism which is, by definition, étale on the smooth part X° of X . We consider the generalized Jacobian conjecture for X which asserts that φ is an automorphism. The quotient morphism $\pi : \mathbb{A}^2 \rightarrow X$ is a quasi-universal covering. Then φ lifts to a (G, χ) -equivariant étale endomorphism $\tilde{\varphi} : \mathbb{A}^2 \rightarrow \mathbb{A}^2$, i.e., $\tilde{\varphi}(O) = O$ and $\tilde{\varphi}(gx) = \chi(g)\tilde{\varphi}(x)$ for $x \in \mathbb{A}^2$ and $g \in G$, where $\chi : G \rightarrow G$ is a group automorphism. If $\chi = \mathrm{id}$, then we say that $\tilde{\varphi}$ is G -equivariant. Let \hat{X} be the minimal resolution of singularity of X . Then φ extends to an étale endomorphism $\hat{\varphi} : \hat{X} \rightarrow \hat{X}$ such that $\hat{\varphi}$ restricted onto the exceptional loci is an automorphism provided $\tilde{\varphi}$ is G -equivariant. On the other hand, the smooth part X° has the standard \mathbb{A}_*^1 -fibration $p : X^\circ \rightarrow \mathbb{P}^1$. If φ preserves the standard \mathbb{A}_*^1 -fibration p then both φ and $\tilde{\varphi}$ are automorphisms. We look for sufficient conditions with which φ preserves the standard \mathbb{A}_*^1 -fibration or its substitute.

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