Modeling annual precipation extremes from a deterministic model

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Acknowledgements

- Francis Zwiers, Environment Canada

Outline

- **General perspectives**
- **Coupled Global Climate Model (CGCM)**
- **Precipitation extremes**
- **Review** extreme value theory-**shortcomings**
- **Alternate approach:** basic theory
- **Application:** Coupled Global Climate Model
- **Monitoring fields of extremes:**
- **Conclusions**

Retrospective week 1:

- Talks
- Discussion
- New research directions
- Opportunities for the future?

Retrospective week 1:

Oreskes et al. :

> “The primary purpose of models in heuristic...useful for guiding further study but not susceptible to proof... [Any model is] a work of fiction. ... A model, like a novel may resonate with nature, but is not the ‘real thing’.”

Retrospective week 1:

Why Model?

To:

- impute unmeasured responses
  - temporal forecasting
  - spatial prediction, eg of systematically unmeasured responses eg species at certain sites
- integrate physical and statistical models
- integrate “misaligned” response measurements (upscaling and downscaling)
- detect spatial or temporal gradients or trends
- to understand environmental processes (“heuristics”)
  - test model hypotheses, current beliefs

Retrospective week 1:

Why Model?

To:

- optimize location of monitoring stations to be added or deleted
- generate inputs for environmental impact models
- smooth noisy data
  - disease mapping
- facilitate **REGULATION, CONTROL,PREDICTION OF "HOTSPOTS"**
- to examine "what if" scenarios (e.g. climate change)

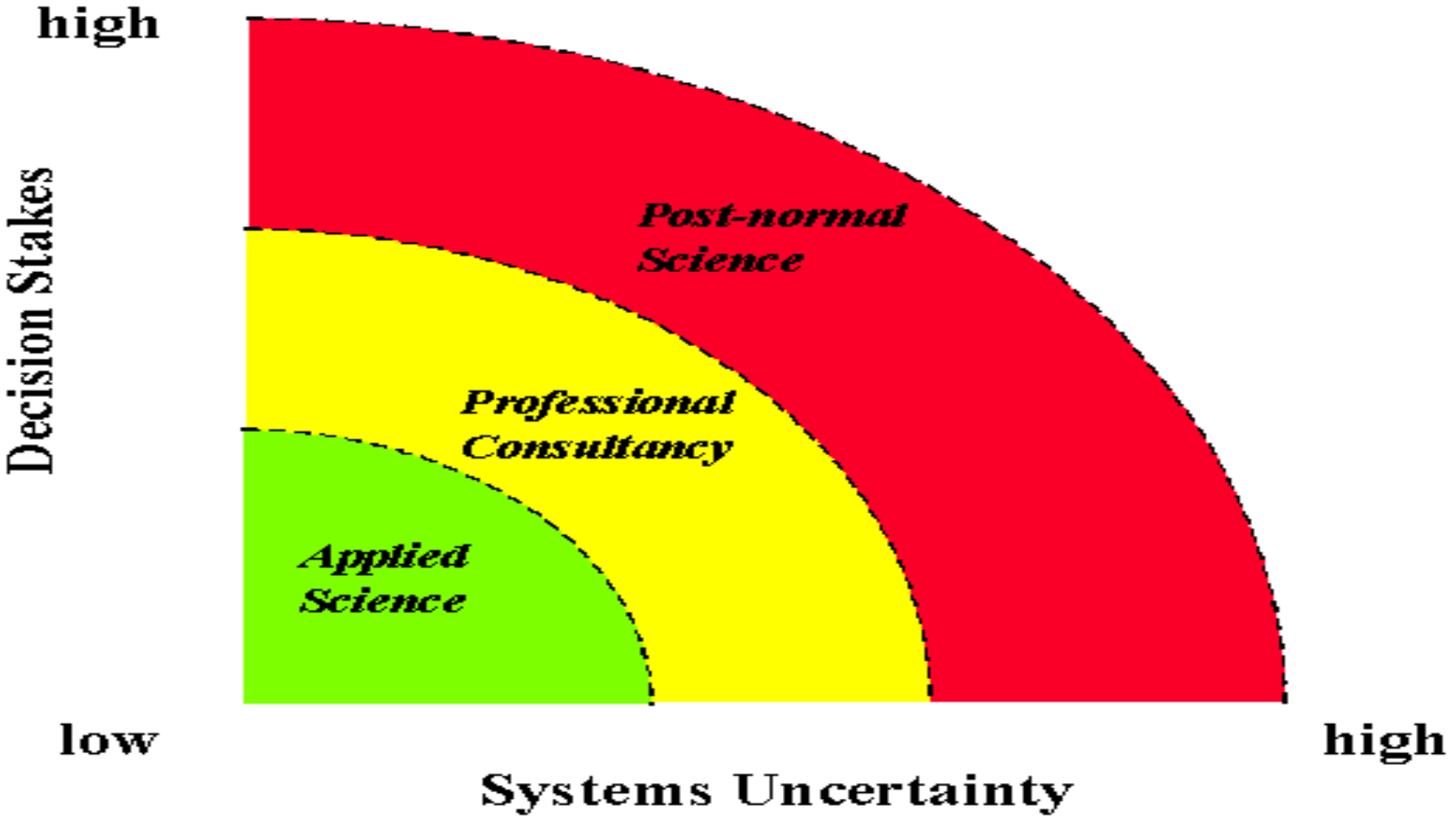
Post-normal science

Funtowicz, Ispra Ravetz (2004?) Nusap.net:

> "...key properties of complex systems, radical uncertainty and plurality of legitimate perspectives....When facts are uncertain, values in dispute, stakes high, and decisions urgent the ...guiding principle of research science, the goal of achievement of truth,...must be modified. In post-normal conditions, such products may be ...an irrelevance."

Post-normal science

From Funtowicz et al:



Retrospective week 1

Physical - statistical modeling

THEME 1: Statistics can help assess physical (simulation) models (if you must)

- The US EPA says you must!!
- Fuentes, Guttorp, Challenor (2003). NRCSE TR # 076.

Retrospective week 1

Physical - statistical modeling

- **THEME 2**: Physical and statistical models can produce synergistic benefits by ”melding” them.
  - Wikle, Milliff, Nychka, Berliner (2001). JASA.
  - Example: how can simulated (modeled) and real rainfall data be usefully combined?

Retrospective week 1

Physical - statistical modeling

THEME 3: Statistics can help interpret, analyze, understand, exploit outputs of complex physical models.

- Nychka (2003). Workshop presentation
- Example: statistical analysis of CGCM precipitation (precip) extremes gives coherent return values over space for design

Week 2!

Extremes

What's "extreme"?

For dams, hydro electric, water storage or flood control:

1000 year return period

NOTE: Meaning

$$P[\text{Dam failure in a given year}] = 1/1000$$

What's "extreme"?

Highway bridges:

100 year return period

NOTE: Not too extreme - 99th percentile

What's "extreme"?

EPA regulations for particulate pollution ($PM_{2.5}$):

- At each monitoring site, compute daily concentration averages
- Compute 98th percentile of these
- Compute T = 3 year average of these
- Requirement: $T \leq 65$ μg m^{-3} at each site

Precipitation extremes

EG: The 100 year rain!

- **return values** for annual max precipitation levels important - but Canada little monitored
- **solution**: simulate precipitation extreme fields using CGCM: 312 Canadian grid cells.
- **Required:**
  - spatially coherent cell return values!
  - joint 312 dimensional distribution to
    - enable prediction of T = number of 312 return value exceedances with E(T), SD(T), etc

Coupled Global Climate Model

- ocean and atmosphere models run separately
  - over centuries
  - then coupled thru 14 yr “integration” periods
- output forced by input of greenhouse gas scenarios
  - eg as observed up to 1990 and 1% per yr increase in CO_2 to 2100

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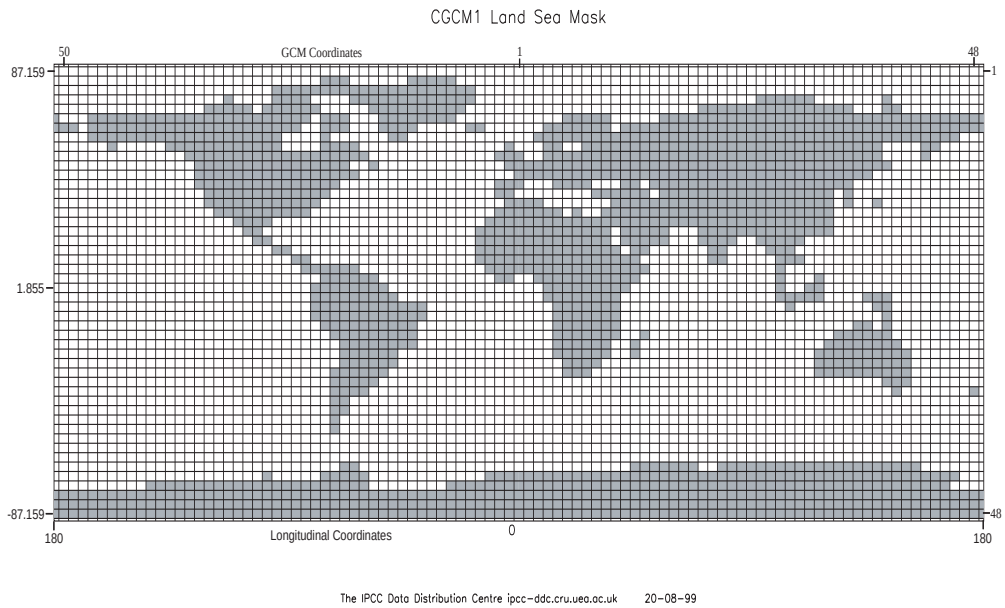
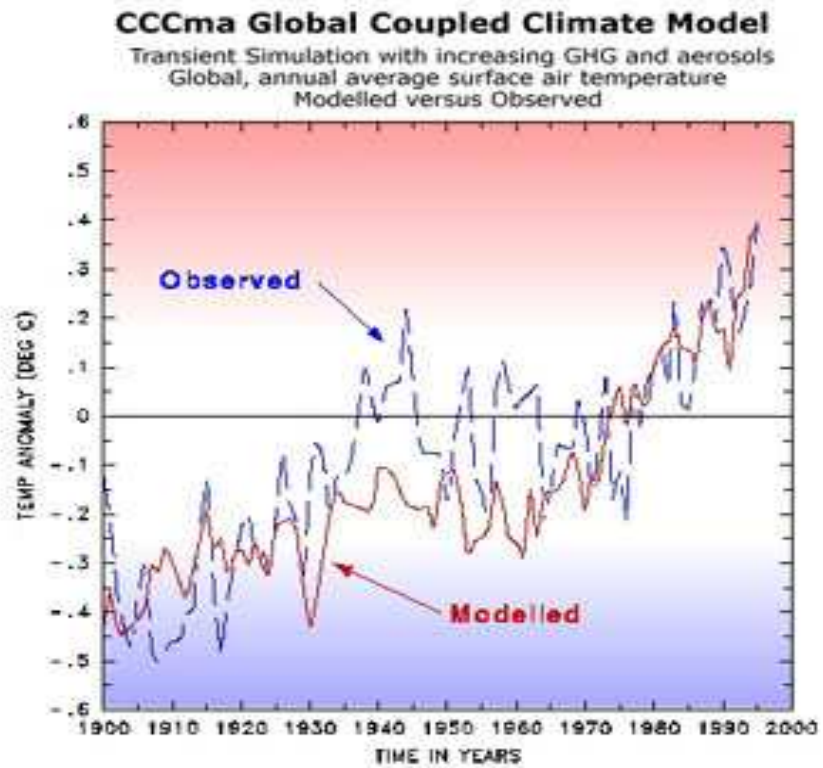


Coupled Global Climate Model

- ocean and atmosphere models run separately
  - over centuries
  - then coupled thru 14 yr “integration” periods
- output forced by input of greenhouse gas scenarios
  - eg as observed up to 1990 and 1% per yr increase in CO_2 to 2100
- precipitation & latent heat released when local rel humidity hi enough
  - liquid water falls to the surface as precipitation

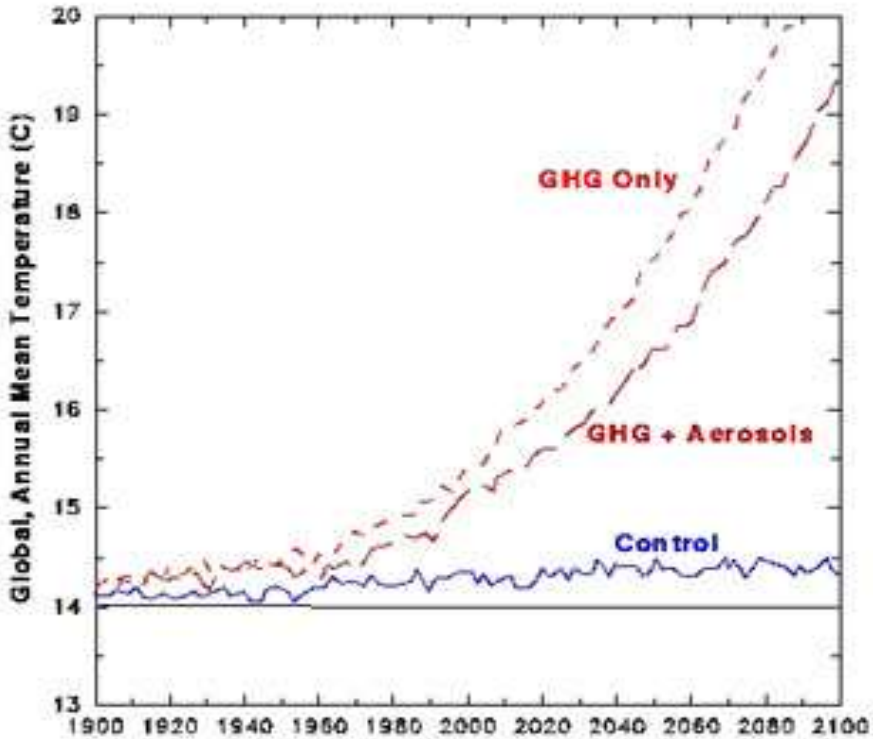
Coupled Global Climate Model

"Confirmation" Run: modelled & observed global annual average surface temperature, 1900 - 1990. Scenario: like that above.



Coupled Global Climate Model

Looking ahead under various scenarios



CGMC Data

- 3 independent simulation runs of hourly precipitation (mm/day)
  - in 21-year windows (to look for trends)
  - 1975-1995 2040-2060 2080-2100
- 26×12 grid covers Canada, cell size = $(3.75°)^2$
- gives $21 \times 3 = 63$ annual precipitation maxima per cell $\times$ time window

Modelling extreme fields

E.G.: annual precip maxima. Assume no year-to-year correlation in sequence

- Approach 1: multivariate extreme value theory. (Why it fails!)
- Approach 2: use hierarchical Bayes (HB) (Why it works!)

Tutorial on HB

For Bayesians:

- ***uncertainty*** = ***probability***.
- ***knowledge*** = ***belief***
- ***data*** = ***information***; alters degrees of uncertainty
- ***prior knowledge*** expressed through prior probability distribution
- information in data expressed through the ***likelihood function***
- change in ***state of uncertainty*** expressed through the ***posterior probability distribution***

Tutorial on HB

More precisely:

- ***data*** = $y = (y_1, \ldots, y_n)$
- degree of prior belief in hypothesis i.e. model parameters $\theta = (\theta_1, ;\theta_2)$, = $\pi(\theta|\alpha)$ = prior probability given hyperparameter α
- information expressed by the likelihood of y if θ were correct = $L(\theta)$
- the ***posterior probability distribution*** is given by application of Bayes rule:

$$\begin{aligned} \pi(\theta|y,\alpha) &= \text{posterior prob of } \theta \text{ given y and } \alpha \\ &\doteq \frac{L(\theta) \times \pi(\theta|\alpha)}{\int L(\theta') \times \pi(\theta'|\alpha) d\theta'} \end{aligned}$$

Tutorial on HB Cont'd

But what is the hyperparameter α is also unknown? Answer: it also have a prior probability $\pi_1(\alpha)$. The ***unconditional*** prior probability of θ becomes $\pi(\theta) = \int \pi(\theta|\alpha)\pi_1(\alpha)d\alpha$ and the posterior for θ is

$$\begin{aligned} \pi(\theta|y) &= \text{posterior prob of } \theta \\ &\doteq \frac{L(\theta) \times \pi(\theta)}{\int L(\theta') \times \pi(\theta')d\theta'} \end{aligned}$$

Tutorial on HB Cont'd

Notes

- The hierarchical Bayes model allows uncertainty to be modeled in layers, making specifying uncertainty easier
- The posterior is an updated prior updated by "Bayes rule" given new information in y and it shows the impact of the information on our uncertainty about θ.
- The posterior of α given y can also be found. (**Exercise!**)

Theory: Single cell (site)

Assume $X_1, X_2, \cdots, X_n$ iid. Let $M_n = max\{X_1, X_2, \cdots, X_n\}$. Fisher-Tippett (1928)showed:

$$P(\frac{M_n - b_n}{a_n} \leq x) \quad \rightarrow \quad H(x), \quad \text{as n} \rightarrow \infty$$

where H has **GEV** distribution

$$H(x) = \begin{cases} \exp\left[-\left\{1+\xi\left(\frac{x-\mu}{\sigma}\right)\right\}^{-1/\xi}\right], & 1+\xi\left(\frac{x-\mu}{\sigma}\right) > 0, \xi \neq 0 \\ \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right] & \xi = 0 \end{cases}.$$

Theory: Single cell (site)

Alternatives: Generalized Pareto (GPD) model: For large x:

$$P(X > x) \simeq \lambda \left[1 + \xi \left(\frac{x-u}{\sigma}\right)\right]_+^{-1/\xi}, \quad x > u$$

for parameters, $\lambda > 0,\ \sigma > 0,\ \text{and}\ \xi \in (-\infty, \infty)$.

NOTES: If number of exceedances over time is Poisson you get for $x > u$ the Poisson–GPD model:

$$P(max_{1 \le i \le N} X_i \le x) = \exp\left\{-\lambda \left[1 + \xi \left(\frac{x-u}{\sigma}\right)\right]^{-1/\xi}\right\}$$

Theory: Single cell (site)

Alternatives:

Peak over Threshold (POT) model:

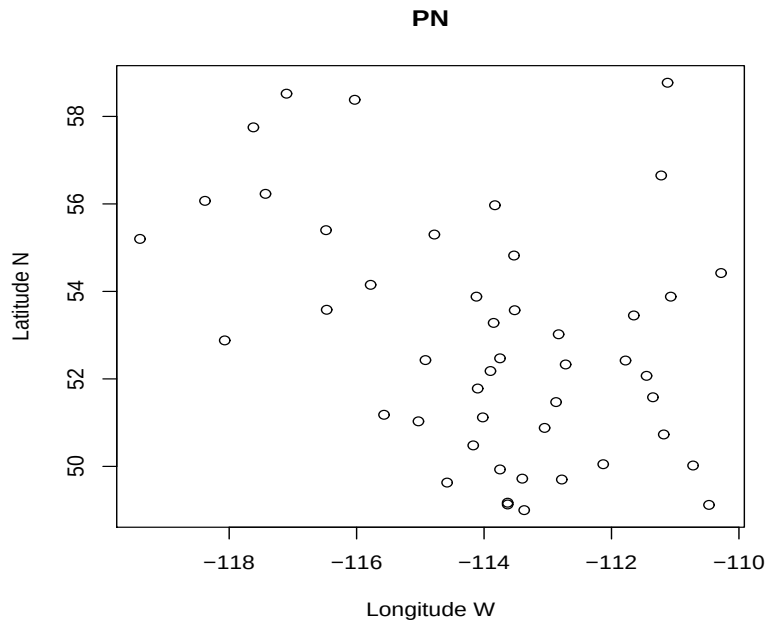
Model only values above a "threshold" u :

$$P[X > x + u \mid X > u] = \left[1 + \xi \left(\frac{x-u}{\sigma}\right)\right]_+^{-1/\xi}, \quad x > u$$

- Good idea since only extremes of interest
- Small ξ gives
  $P[X > x + u \mid X > u] \simeq \exp\{-\left(\frac{x-u}{\sigma}\right)\}, \quad x > u$

Theory: Single cell (site)

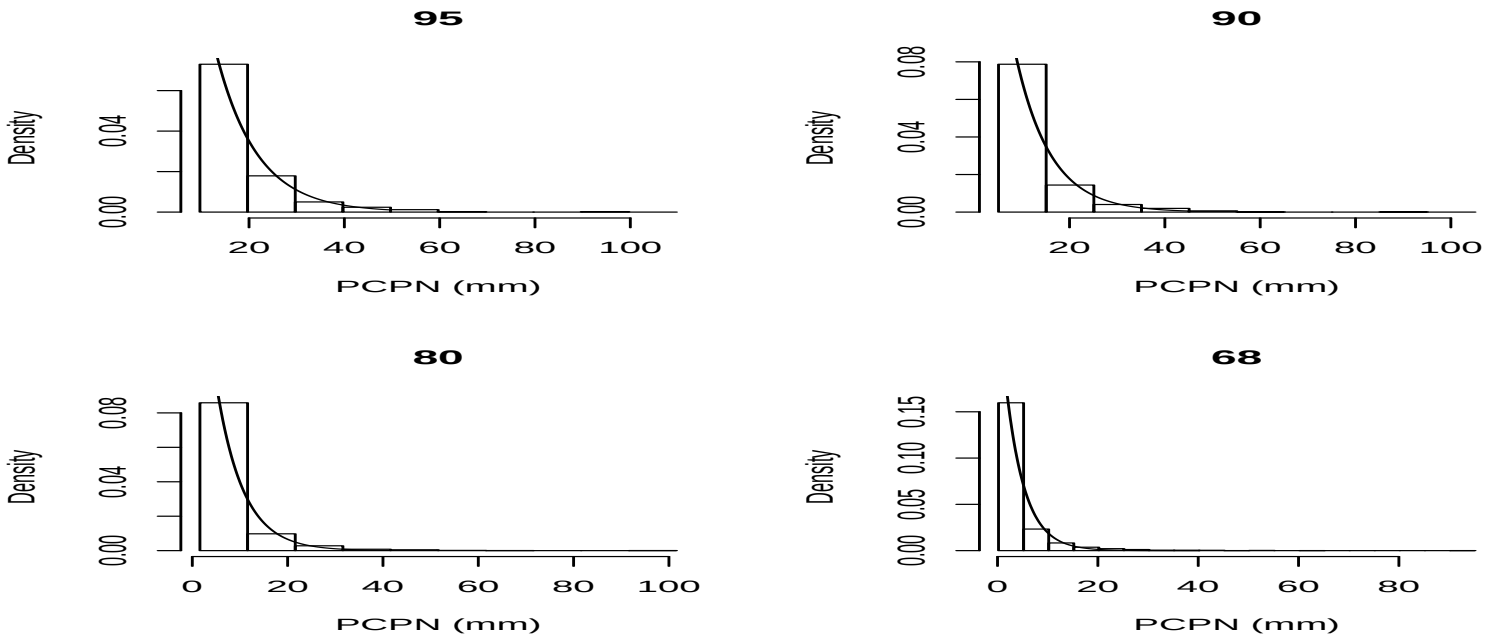
Alberta Climate Example



Locations of precipitation monitoring sites in the Province of Alberta.

Theory: Single cell (site)

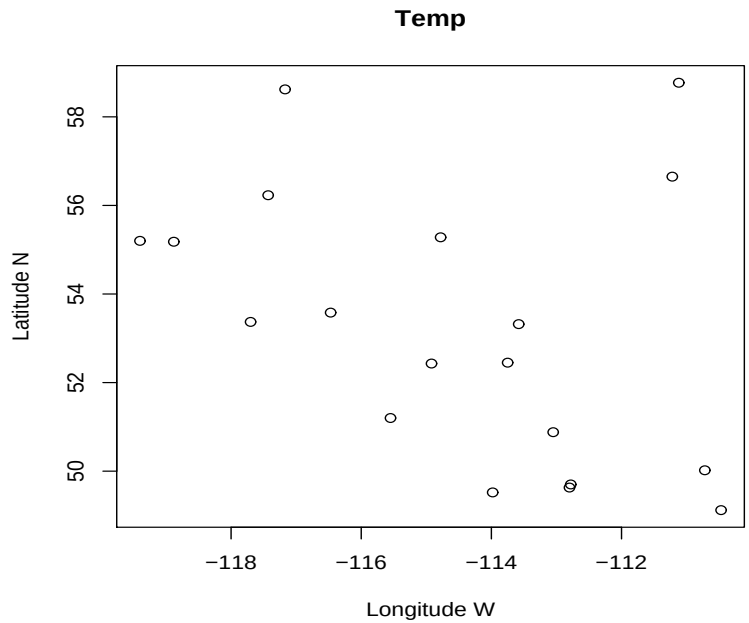
Alberta Climate Example



Distributions of daily rainfall totals (mm) exceeding each of 4 thresholds corresponding to sample %-iles for daily rainfall (for days with rain exceeding 2 mm.

Theory: Single cell (site)

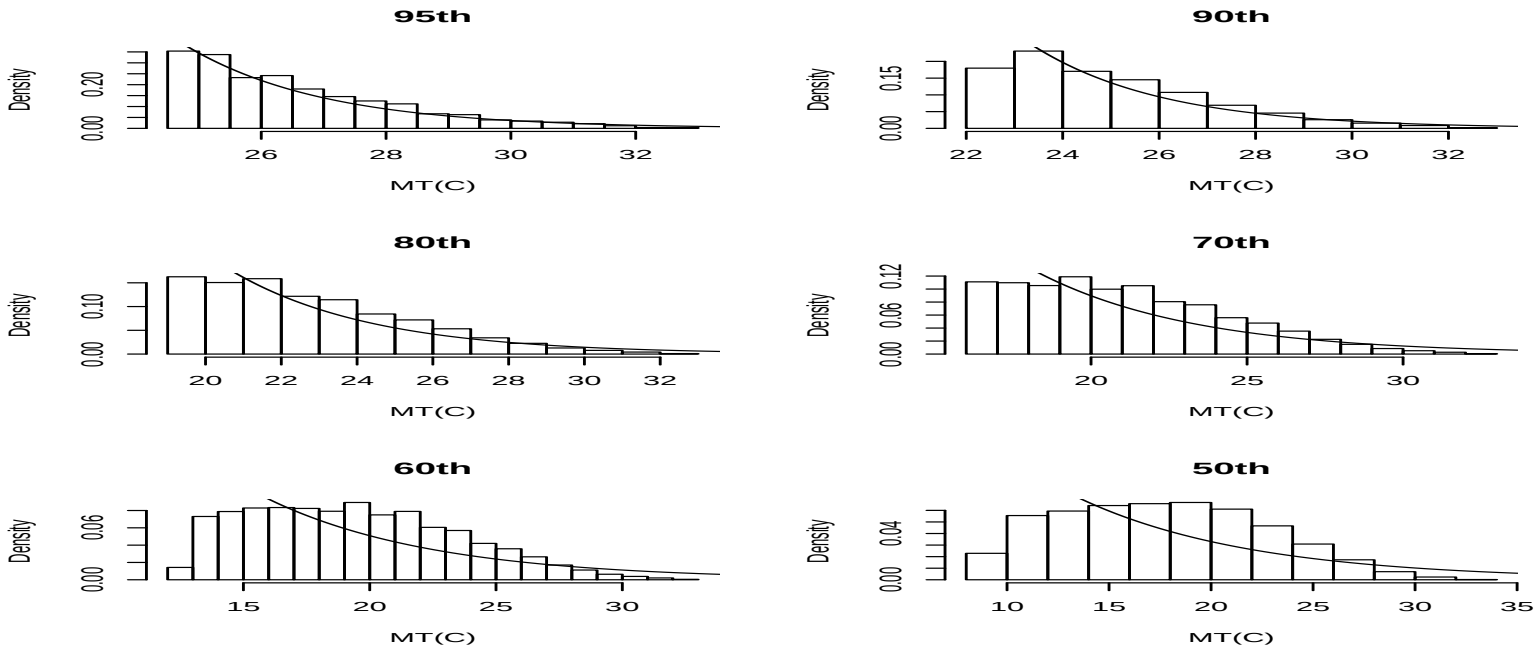
Alberta Climate Example



Locations of temperature monitoring sites in the Province of Alberta.

Theory: Single cell (site)

Alberta Climate Example



Distributions of daily max temperatures exceeding each of 4 thresholds corresponding to sample %-iles for max temperature.

Theory: Single cell (site)

- **However:**
  - for large threshold "u" little data
  - for small "u" poor tail approximation
- tail model parameters depend on "u"
- results sensitive to the choice of "u"

Theory: Single cell (site)

Alternatives:

Probability Weighted Moment (PWM) model: unduly complex for certain purposes

Theory: Multiple cells

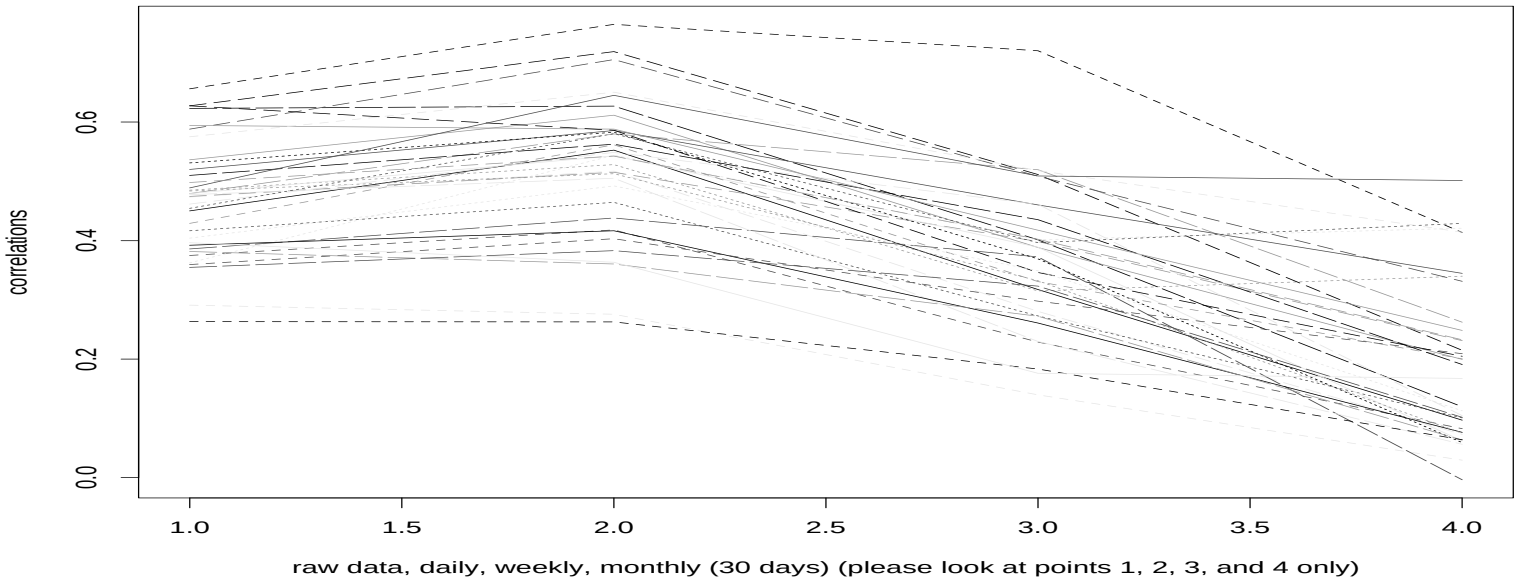
Approaches:

Extend Fisher-Tippett

Problems: leads to a big class of possible limit distributions. Moreover, extremes must be asymptotically dependent for large return periods.

Theory: Multiple cells

An example: Small inter - site correlations *Inter- site dependence declines with increasing extreme's"range" for many (not all!) site pairs [London and Vancouver analyses]*



Inter-site correlations for Vancouver's PM_{10} decline with max range.

Theory: Multiple cells

Approaches:

Specify individual cell (parametric) distributions. Put a joint distribution over their parameters

Problems: *ah hoc* and complex. No compelling dependence structure

Theory: Multiple cells

Point process (PP) approach:
Space–time points where threshold exceedance occurs is non-homogeneous Poisson process, intensity function:

$$\Lambda(A) = (t_2 - t_1)\Psi(y; \mu, \psi, \xi)$$

where

$$A = (t_1, t_2) \times (y, \infty)$$

$$\Psi(y; \mu, \psi, \xi) = \left[1 + \xi\left(\frac{y-\mu}{\psi}\right)\right]^{-1/\xi}, \; 1 + \xi\left(\frac{y-\mu}{\psi}\right) > 0$$

Problems: Complicated distribution; unclear how to extend to multivariate responses; what about fixed site monitors?

Theory: Multiple cells

Approaches:

Our hierarchical Bayesian method:

- Approximate transformed cell max precip data by joint multivariate t distribution
- Very flexible & lots of available theory

Problems:

- may not work for "extreme extremes".
- asymptotically independent sites

Our method: Assumptions

$\mathbf{Y}_j : p \times 1$ annual precipitation maxima; p cells, years $j = 1, \ldots, n$.

Likelihood construction:

Conditional on parameters $(\boldsymbol{\mu}, \Sigma)$

$\mathbf{X}_j \doteq \log \mathbf{Y}_j \overset{iid}{\sim} MVN_p(\boldsymbol{\mu}, \Sigma)$

Our method: Assumptions

$\mathbf{Y}_j : p \times 1$ annual precipitation maxima; p cells, years $j = 1, \ldots, n$.

Prior distribution:

- $\boldsymbol{\mu}|\Sigma \sim MVN(\boldsymbol{\nu}, F^{-1}\Sigma)$
- $\Sigma \sim W^{-1}(\Sigma|\Psi, m)$
- Ψ and m are the *hyperparameters*
- F^{-1} re-scales Σ to mean's level of uncertainty

Implications

Posterior distribution (of precip max field):

- $\mathbf{X}_{p\times 1}|D \sim t\left(\tilde{\mathbf{x}}, \tilde{\Sigma}, l\right)$ with
- $\tilde{\mathbf{x}} = \boldsymbol{\nu} + (\bar{\mathbf{x}} - \boldsymbol{\nu})\hat{E}$
- $\tilde{\Sigma} = \frac{1+nF^{-1}-n\hat{E}F^{-1}}{l}\hat{\Psi}$ and
- $l = m + n - p + 1$,
  $\hat{\Psi} = \Psi + (n-1)S + (\bar{\mathbf{x}} - \boldsymbol{\nu})(n^{-1} + F^{-1})^{-1}(\bar{\mathbf{x}} - \boldsymbol{\nu})'$

The Hyperparameters

- ν: estimated by **smoothing spline** over all cells
- $\Psi = c \times \Phi$, Φ a **covariance matrix** estimated by *semivariogram*
- $\Phi_{ij} = Cov(X_i, X_j) = \sigma^2 - \gamma(h_{ij})$
  - σ^2 = common sample variance
  - h_{ij} = Euclidean distance between sites i, j
  - $\gamma(h)$ = isotropic semivariogram model fitted to data
- **EM algorithm** estimates c & degrees of freedom, m
- F^{-1} estimated by method of moments

The Hyperparameters

Justifying empirical Bayes:

- Posterior must be well-calibrated w.r.t. real max precip fields. Hence prior must be *fitted* to enable good match
- simplicity
- equates with using diffuse prior

Diagnostic tool

Validating **joint normality** assumption

Method
For any given year:

- delete data from selected sites
- predict them by $\hat{\mathbf{x}}_{\mathbf{u}} = \boldsymbol{\nu}_u + (\hat{\mathbf{x}}_g - \boldsymbol{\nu}_g)' \Psi_{gg}^{-1} \Psi_{gu}$ where
  - u means *ungauged* (missing) and g, *gauged*
  - ($\boldsymbol{\nu}_u$,$\boldsymbol{\nu}_g$) partitioned *prior mean* conformably
  - likewise for Ψ_{gg} and Ψ_{gu}

Diagnostic tool

Validating **joint normality** assumption

Method

Now see if vector of missing values falls into 95% credibility interval:

- $\{\mathbf{X_u} : (\mathbf{X_u} - \hat{\mathbf{x}}_\mathbf{u})' \Psi_{u|g}^{-1} (\mathbf{X_u} - \hat{\mathbf{x}}_\mathbf{u}) < b\}$ where
  - $b = (u \times P_{u|g} \times F_{1-\alpha,u,m-u+1}) \times (m-u+1)^{-1}$
  - $\Psi_{u|g} = \Psi_{uu} - \Psi_{ug}\Psi_{gg}^{-1}\Psi_{gu}$ and
  - $P_{u|g} = 1 + F^{-1} + (\mathbf{x_g} - \boldsymbol{\nu}_g)' \Psi_{gg}^{-1} (\mathbf{x_g} - \boldsymbol{\nu}_g)$

Diagnostic tool

Validating **joint normality** assumption

Method

- do this repeatedly. Randomly remove subsets of sites of fixed size
- compute the relative coverage frequency - it should be around 95%!! Offers check on the validity of the model.

Estimating Return Values

- Definition: T year return value, X_T

  $P(X > X_T) = \frac{1}{T}$

- can be estimated for each cell from the joint posterior t distribution
- approximation: use log normal instead of log t distribution, $x_{1-T,i} = \tilde{x}_i + \Phi(1-T) \times \tilde{\sigma}_i$

CGCM Analysis

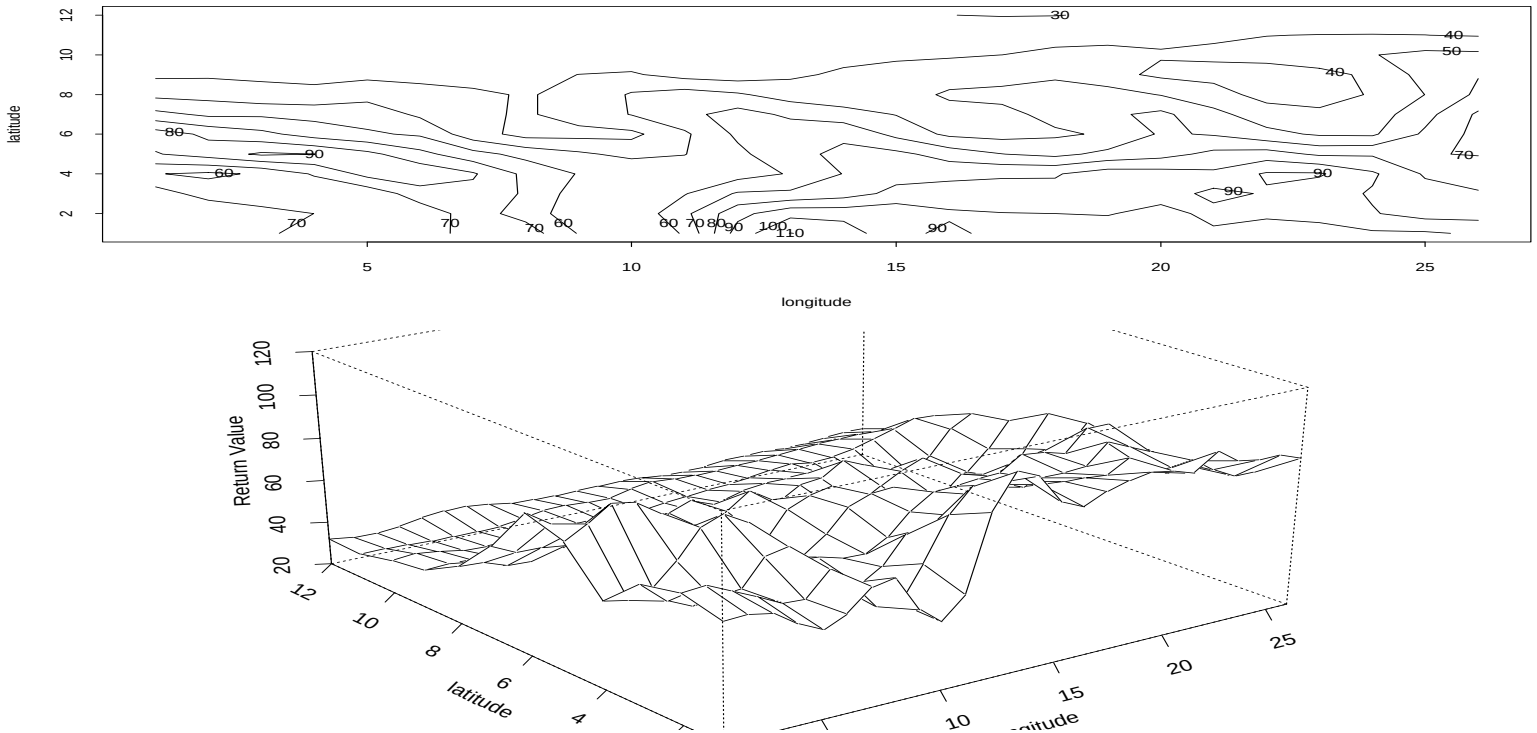
- BEFORE ANALYSIS: log - transform, de-trend
- RESIDUALS:
  - symmetric empirical marginal distribution
  - slightly heavier than normal tails
  - no significant autocorrelation
- AFTER ANALYSIS: re-trend, antilog-transform

CGCM Statistical Model

- **HIERARCHICAL BAYES**: Normal - Inverted Wishart model for residuals
  - estimated variogram for 312*312 dimensional hypercovariance
- **RESULTING POSTERIOR**: 312 dimens'l, multivariate t-distribution, $m = 355$, $c = 49$ (for Ψ estimate)
  Posterior:
  - yields estimates of 312 marginal return values
  - enables simulation of 312 dim'l annual max precip field and
    - distribution of *stat's* computed from it
      - EG: T = # of (312) cells above return values, $E(T)$, predictive interval for T

Results

Contour, perspective plots of estimated 10-year return values (mm/day).



CGGM Stats Model Assessment

CROSS VALIDATION:

- randomly omit 30 of 312 cells repeatedly
- predict their values from rest from the joint t distribution.
- CONCLUSION: The joint t distribution fits the simulated data quite well

Credibility Level	Mean	Median
30%	35	35
95%	96	97
99.9%	99.9	99.9

Table 1: SUMMARY: cred'y ellips'd coverage probs

Discussion

- Joint distribution fits well. Also worked in another study on real air pollution data.
- Allows answers to complex question like chances of say 10 simultaneous exceedances of cell return values
- Suggests model could be used to design extreme precip monitoring networks
- the return value index reveals reasonable joint fit but needs further study

Concluding Remarks

- multivariate t distribution promising model for extreme space-time fields. Data needs to be transformable. But wealth of existing theory for multinormal makes pursuit worth it!
- empirical checking/diagnostics vital before using the method
- general theory allows extension to multivariate responses in each site & covariates too!
- Can posterior be trusted for extreme-extremes? Can any distribution?

Other PIMS CRG events in 2008

- **Banff International Research Station for Mathematical Innovation and Discovery (BIRS): The Climate Change Impacts on Ecology and the Environment** May 4 -9, 2008.
- The 2008 annual **International Environmetrics Society Conference:**
  Quantitative Methods for Environmental Sustainability
  June 8-13, 2008, Kelowna, Canada.
  http://people.ok.ubc.ca/zhrdlick/ties08/call.htm

Other PIMS CRG events in 2008

- The **PIMS International Graduate Institute's Summer School**
  Computation in environmental statistics
  Tentatively: Jul 28-Aug 1, 2008, National Center for Atmospheric Research (NCAR), Boulder Colorado
- **Workshop on extreme climate events**
  Winter, 2008, Lund University

References & Contact Info

- email: jim@stat.ubc.ca
- internet: http://www.stat.ubc.ca/<LINK Faculty Members>
- tech reports: http://www.stat.ubc.ca/<LINK Research Activities>

- **R-based software:** http://enviro.stat.ubc.ca
- Companion to Nhu D Le & James V Zidek (2006) *Statistics analysis of environmental space–time processes*. Springer. To Appear May 12.

NOTE: Chap 14 gives a tutorial on its use.