Functional CLT for large dimension random matrices

There are two basic problems concerning the Empirical Spectral Distribution of LDRM, which are the Limiting Spectral Distribution and the convergence rate of the ESD. In this talk, I tend to focus on the convergence rate of the ESD of Wigner matrices.

Numerical simulation results conjecture that the convergence rate is of the order O(1/n) for Wigner matrices. It thus seems natural to consider the asymptotics of the empirical process $G_n(x) = n(F_n(x)-F(x))$. However, there are plenty of evidences showing that the process $G_n(x)$ can not converge in any metric space. Thus, we turn to consider the linear functional of the process $G_n(x)$. In Bai and Yao (2005), they have considered the convergence of the Wigner matrix's empirical process indexed by a set of functions. These functions are required to be analytic on an open domain of the complex plane，including the support of the semicircle law. The assumption that the functions are analytic seems to be too restricted. It is natural to ask whether it is possible to relax this stringent condition. Jointly with my supervisors Prof. Bai Zhidong and Dr. Zhou Wang, we considered the empirical process of Wigner matrices, which is indexed by a set of functions with continuous fourth-order derivatives on an open interval of the real line including the support of the semicircle law. We proved that this process converges to a Gaussian process. This result not only gives us deeper understanding on the convergence rates of ESD of Winger matrices, but also enables us to make hypothesis tests of hypothesis in multivariate analysis.