

Analytical and Computational Challenges in Kinetic Models of Dilute Polymers

John W. Barrett

Department of Mathematics, Imperial College London, UK.

David J. Knezevic

Massachusetts Institute of Technology, School of Engineering, Boston MA, USA.

and

Endre Süli

Centre for Nonlinear PDE, Mathematical Institute, University of Oxford, UK.

We review recent analytical and computational results for macroscopic-microscopic bead-spring models that arise from the kinetic theory of dilute solutions of incompressible polymeric fluids with noninteracting polymer chains, involving the coupling of the unsteady Navier–Stokes system in a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2$ or 3 , with an elastic extra-stress tensor as right-hand side in the momentum equation, and a (possibly degenerate) Fokker–Planck equation over the $(2d+1)$ -dimensional region $\Omega \times D \times [0, T]$, where $D \subset \mathbb{R}^d$ is the configuration domain and $[0, T]$ is the temporal domain. The Fokker–Planck equation arises from a system of (Itô) stochastic differential equations, which models the evolution of a $2d$ -component vectorial stochastic process comprised by the d -component centre-of-mass vector and the d -component orientation (or configuration) vector of the polymer chain. We show the existence of global-in-time weak solutions to the coupled Navier–Stokes–Fokker–Planck system with microscopic cut-off for a general class of spring potentials including the widely used finitely extensible nonlinear elastic (FENE) potential; cf. [1, 2, 3]. The numerical approximation of this high-dimensional coupled system is a formidable computational challenge, complicated by the fact that for practically relevant spring potentials, such as the FENE potential, the drift term in the Fokker–Planck equation is unbounded on ∂D ; cf. [4, 5, 6, 7, 8].

References

1. *JW Barrett, C Schwab and E Süli*. Existence of global weak solutions for some polymeric flow models. *M3AS*, 6(15), pp. 939–983, 2005.
2. *JW Barrett and E Süli*. Existence of global weak solutions to some regularized kinetic models of dilute polymers. *SIAM MMS*, 6(2), pp. 506–546, 2007.
3. *JW Barrett and E Süli*. Existence of global weak solutions to dumbbell models for dilute polymers with microscopic cut-off. *M3AS*, 18(6), pp. 935–971, 2008.
4. *JW Barrett and E Süli*. Numerical approximation of corotational dumbbell models for dilute polymers. *IMAJNA*, Advance Access: Publ. 18/9/2008.
5. *JW Barrett and E Süli*. Finite element approximation of kinetic dilute polymer models with microscopic cut-off. *M2AN* (Submitted, November 2008).
6. *DJ Knezevic and E Süli*. Spectral Galerkin approximation of Fokker–Planck equations with unbounded drift. *M2AN* (Accepted, October 2008).
7. *DJ Knezevic and E Süli*. A heterogeneous alternating-direction method for a micro-macro dilute polymeric fluid model. *M2AN* (Accepted, June 2009).
8. *C Schwab, E Süli, and R-A Todor*. Sparse finite element approximation of high-dimensional transport-dominated diffusion problems. *M2AN*, 42(5), 2008, 777–820.