Market Design for Emission Trading Schemes

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[1]parts are based on joint work with R. Carmona, M. Fehr, A. Pourchet

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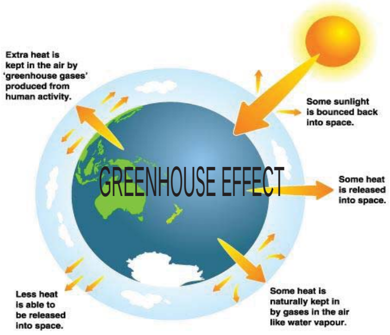
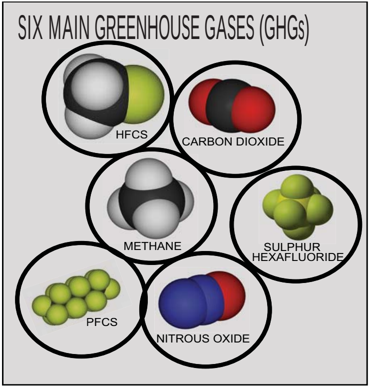
Greenhouse gas effect





Reduction

by cap-and-trade mechanism=emission trading scheme

- central authority
  - allocates credits (allowances) to polluters
  - sets penalty for each unit of pollutant not covered by credits
  - defines compliance dates within a time period
- polluters reduce or avoid penalty by
  - applying abatement measures
    - technological changes
    - replacement of input/output products,
  - trading allowances
    - physically (spot)
    - financially (forwards/futurues)

Example EU ETS Phase I and II credits are called EUA

EUA 2007 has died

Source: European Energy Exchange

EUA 2012 is alive, may reach 100 EURO

Source: European Energy Exchange

Theory

- Market-based mechanisms are the most promising tool to combat global warming
- Reason: allowance trading leads to price discovery, which helps to identify and to exercise cheapest ways of pollution reduction
- By market mechanisms, the reduction resources are allocated optimally

However, there are some problems ....

Misconception 1: Cap-and-trade system is cheap for everyone

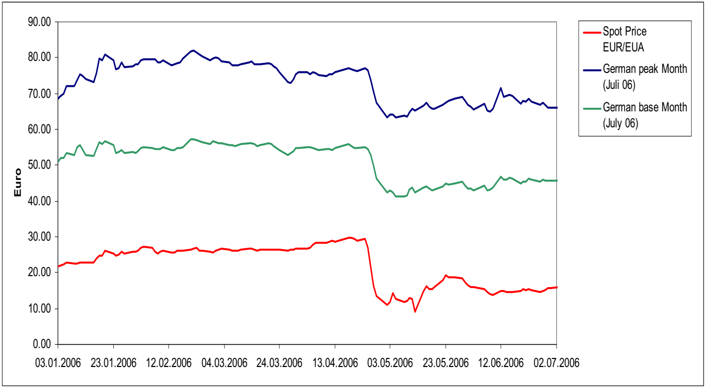
In the generic scheme design, allowance trading may be very costly for consumers.

For some reasons, consumers burden by increased electricity price exceeds by far the true social cost of reduction.

The difference results in huge revenues of energy producers, also known as windfall profits.

Allowance price is passed through on the consumer

There is clear evidence that emission allowance price is added to electricity price.



Explanation for pass-through

are the so-called opportunity costs.

When selling electricity, generators figure out the opportunity to not produce and to sell the effectively saved carbon allowances to the market.

If generator supplies electricity, he wants to be rewarded for the lost profit.

Example: If

- production cost is 30 EURO/MWh
- allowance price is 10 EURO/tonne
- specific emission is 0.4 tonne/Mwh

then the energy is supplied only if the price exceeds

$$30 + 10 \times 0.4 \ \mathrm{EURO/MWh}$$

Apparent reason for pass-through

Allowances are given for free. Generators who charge the consumer behave not fair.

A fair company does not charge for allowances and gains competitive advantage.

Thus, pass-through happens due to lack of competition.

Misconception 2: Windfall profits are due to lack of competition

An analysis of equilibrium models shows that pass-through is the correct strategy in a perfectly competitative equilibrium.

Generators must take windfall profits.

Regulator creates the problem giving allowances for free.

If allowances were auctioned, the profits could be returned to the consumers.

Misconception 3: By auctioning, we can return money

Auction is not appropriate.

In equilibrium, allowance price in the upfront auction should come close to the expected allowance price in the continuous trading.

$$\begin{aligned}\text{Auction revenue} &\approx \text{allowance price} \times \text{number of allowances}\\ \text{Windfall profit} &\approx \text{allowance price} \times \text{emission rate} \times\\ &\quad \times \text{number of consumed MWh}\end{aligned}$$

Windfall profits are intrinsic for cap-and-trade mechanism (?)

Misconception 4: There is no way to overcome windfall profits

In an alternative market design (relative scheme), allowances are allocated depending on demand.

For instance, for each produced MWh generator receives allowances for 0.2 tonne CO_2.

With this, allowance price does not strongly affect electricity price.

Analyzing relative scheme, one finds out that by correct parameter choice, one obtains a very cheap and effective way of pollution reduction.

Misconception 5: Relative scheme solves all problems

Unlike classical cap-and-trade mechanisms, relative scheme has a soft cap.

Could winter $\Rightarrow$ high energy demand $\Rightarrow$ high emissions, still compliance

The reduction of relative scheme is not sharp.

Although parameters can be adopted such that the expected reduction is the same or even better than in the classical scheme, the 'unknown' outcome creates problems.

Problems may occur when negotiation with other markets on emission targets is to worked out.

Could we achieve negotiation based on reduction distribution?

Quantitative understanding of emission trading schemes is needed

Illustration: one-step market

Agents $i = 1, \dots, N$ follow (electricity) production and trade allowances

- Today: Trading and production decisions
- Tomorrow: Compliance date

Agent $i \in \{1, \dots, N\}$ decides on

- ξ^i production strategy with
  - $V^i(\xi^i)$ production volume (MWh)
  - $C^i(\xi^i)$ production costs (EURO)
  - $E^i(\xi^i)$ emission (tonne CO_2)
- θ^i change in allowance amount by trade

One-step market

Regulator yields

- γ^i initial allocation for each agent $i = 1, \dots, N$
- π penalty for non-compliance

Market yields

- A allowance price
- P electricity price

This results in the revenue of the agent i

$$L^{A,P,i}(\xi^i, \theta^i) = -A\theta^i - C^i(\xi^i) + PV^i(\xi^i) - \pi(E^i(\xi^i) - \theta^i - \gamma^i)^+$$

One-step equilibrium

Given demand $D \in [0, \infty[$, equilibrium price (A^*, P^*) is characterized by existence of agent's strategies $(\xi^{i*}, \theta^{i*})_{i=1}^N$ with

1) $\sum_{i=1}^N \theta^{i*} = 0$.

2) $\sum_{i=1}^N V^i(\xi^i) = D$

3) the mapping

$$(\xi^i, \theta^i) \mapsto E(U^i(L^{A,P,i}(\xi^i, \theta^i)))$$

is maximized at (ξ^{i*}, θ^{i*}), for each $i = 1, \ldots, N$.

Analyzing the equilibrium, one finds out that the allowance price must be passed through on the consumer.

To understand pass-through

Introduce opportunity merit order costs

$$\mathcal{C}^A(D) = \min\{\sum_{i=1}^{N}(C^i(\xi^i) + AE^i(\xi^i)) \, : \, \xi^1, \ldots, \xi^N, \sum_{i=1}^{N} V^i(\xi^i) \geq D\}$$

Proposition (under natural assumptions)

If (A^*, P^*) is an equilibrium price then

i) Production is scheduled in opportunity merit order

$$\sum_{i=1}^{N}(C^i(\xi^{i*}) + A^* E^i(\xi^*)) = \mathcal{C}^{A^*}(\sum_{i=1}^{N} V^i(\xi^{i*}))$$

ii) Electricity price is an opportunity merit order price

$$(\xi^{i*})_{i=1}^N \text{ maximizes } (\xi^i)_{i=1}^N \mapsto -\mathcal{C}^{A^*}(\sum_{i=1}^{N} V^i(\xi^i)) + P^* \sum_{i=1}^{N} V^i(\xi^i)$$

Example

If there is only one technology, then the allowance price must be just added to the business-as-usual electricity price at the specific emission rate e.

$$(\xi^{i*})_{i=1}^N \text{ is a maximizer to}$$

$$(\xi^i)_{i=1}^N \quad \mapsto \quad \underbrace{-\mathcal{C}^{A^*}\Big(\sum_{i=1}^N V^i(\xi^i)\Big) + P^* \sum_{i=1}^N V^i(\xi^i)}_{\overbrace{-\mathcal{C}^0\Big(\sum_{i=1}^N V^i(\xi^i)\Big) + (P^* - A^* e) \sum_{i=1}^N V^i(\xi^i)}}$$

What we see from one-period model

In equilibrium,

- allowance price changes merit order of production units
- demand is covered according to changed merit order
- emission abatement happens automatically, trigged by allowance price
- this is true in general: in equilibrium, allowance price triggers abatement measures

Dynamical model

- compliance date T
- action times $t = 0, \ldots, T$
- all processes on $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t=0}^{T})$ are adapted
- finite number of agent $i \in I$
- interest rate zero, for simplicity

Model ingredients

Revenue of agent i for (ξ^i, ϑ^i), given prices $A = (A_t)_{t=0}^T$

$$L^{A,i}(\vartheta^i, \xi^i) = -\sum_{t=0}^{T}(\vartheta_t^i A_t + C^i(\xi_t^i)) - \underbrace{\pi}_{\text{penalty}}(\mathcal{E}_T^i - \sum_{t=0}^{T}(\xi_t^i + \vartheta_t^i))^+$$

- $\mathcal{E}_T^i$ are Business-as-usual emissions less allocated allowances of the agents $i \in I$
- Abatement policy $\xi^i = (\xi_t^i)_{t=0}^T$ of the agent $i \in I$
- Costs of abatement policy $(\xi_t^i)_{t=0}^T$ are $\sum_{t=0}^T C^i(\xi_t^i)$
- ϑ_t^i change of allowance number by trade at time t
- $\sum_{t=0}^T A_t \vartheta_t^i$ costs of trading for allowance prices $(A_t)_{t=0}^T$

Equilibrium state

Definition

$A^* = (A^*_t)_{t=0}^T$ is an equilibrium allowance price process, if there exist agent's policies $(\vartheta^{*i}, \xi^{*i})$, $i \in I$ such that:

(i) Each agent $i \in I$ is satisfied by the own policy

$$(\vartheta^{*i}, \xi^{*i}) \text{ is maximizer to} (\vartheta^i, \xi^i) \mapsto \frac{\ln E(-e^{-\lambda^i L^{A^*,i}(\vartheta^i, \xi^i)})}{\lambda^i}$$

(ii) Changes in allowance positions are in zero net supply

$$\sum_{i \in I} \vartheta_t^{*i} = 0, \text{ for all } t = 0, \ldots, T.$$

Three equilibrium properties (under additional assumptions)

It turns out that in the equilibrium:

a) No arbitrage opportunities for allowance trading
b) Allowance price instantaneously triggers all abatement measures whose costs are below allowance price
c) There are merely two final outcomes for allowance price
    - $A_T^* = 0$ in the case of allowance excess
    - $A_T^* = \pi$ in the case of allowance shortage

Formal characterization

Theorem

If $(A_t^*)_{t=0}^T$ is an equilibrium price and $(\xi_t^{i*})_{t=0}^T$ for $i \in I$ are corresponding abatement policies, then

(a) $(A_t^*)_{t=0}^T$ *is a martingale with respect to some $Q \sim P$*

(b) *For each $i \in I$ holds*

$$\xi_t^{i*} = c^i(A_t^*), \quad t = 0, \dots, T-1,$$

with abatement volume function

$$c^i(a) = \mathrm{argmax}(x \mapsto -C^i(x) + ax)$$

(c) *The terminal allowance price is given by*

$$A_T^* = \pi 1_{\{\sum_{i \in I}(\mathcal{E}_T^i - \sum_{t=0}^T \xi_t^{i*}) \geq 0\}}$$

From risk-neutral perspective, allowance price

is a Q-martingale, whose terminal value

$$\boxed{A^*_T = \pi 1_{\{\mathcal{E}_T - \sum_{t=1}^{T} c(A^*_{t-1}) \geq 0\}}}$$

depends on the intermediate values through

B.A.U. allowance demand

$$\mathcal{E}_T = \sum_{i \in I} \mathcal{E}^i_T$$

and market abatement volume function

$$c(a) := \sum_{i \in I} c^i(a)$$

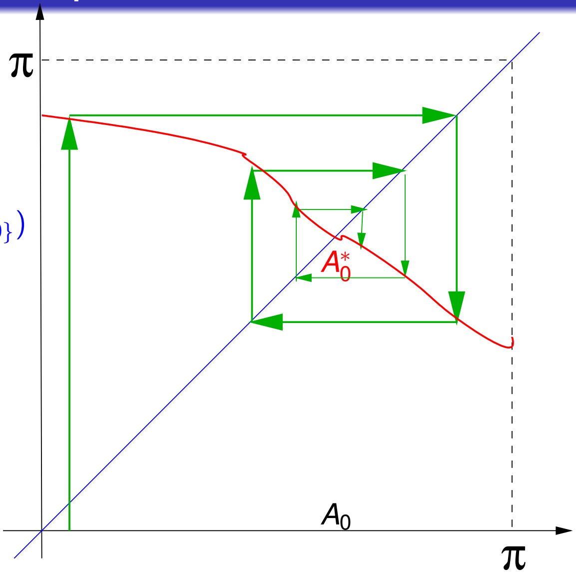
Reduced form model

Given Q, $\mathcal{E}_T$, c solve fixed point equation

$$A^*_t = E^Q(\pi 1_{\{\mathcal{E}_T - \sum_{s=1}^T c(A^*_{s-1}) \geq 0\}} \,|\, \mathcal{F}_t), \qquad t = 0, \ldots, T$$

Illustration for one time step from 0 to $T = 1$

$A_0^* = \pi \mathbb{E}_0^Q(1_{\{\mathcal{E}_T - c(A_0^*) \geq 0\}})$



Follow the intuition that the allowance price is a function of

$$A_t^*(\omega) = \alpha_t(G_t(\omega))(\omega)$$

- recent time t
- current situation ω
- reduction demand $G_t = \underbrace{E_t^Q(\mathcal{E}_T)}_{\mathcal{E}_t} - \sum_{s=1}^t c(A_{s-1}^*)$

Guess a recursion from martingale property

Idea

$$\alpha_t(g)(\omega) = \mathbb{E}^{\mathrm{Q}}_t(\alpha_{t+1}(g - c(\alpha_t(g)(\omega)) + \varepsilon_{t+1}))(\omega),$$

for all $g \in \mathbb{R}$, $\omega \in \Omega$

$$\begin{aligned}
\alpha_t(G_t(\omega))(\omega) = A^*_t(\omega) = \mathbb{E}^{\mathrm{Q}}_t(A^*_{t+1})(\omega) &= \mathbb{E}^{\mathrm{Q}}_t(\alpha_{t+1}(G_{t+1}))(\omega) \\
&= \mathbb{E}^{\mathrm{Q}}_t(\alpha_{t+1}(G_t - c(A^*_t) + \varepsilon_{t+1}))(\omega) \qquad \varepsilon_{t+1} = \mathcal{E}_{t+1} - \mathcal{E}_t \\
&= \int_\Omega \alpha_{t+1}(G_t(\omega') - c(A^*_t(\omega')) + \varepsilon_{t+1}(\omega'))(\omega')\mathrm{Q}_t(d\omega')(\omega) \\
&= \int_\Omega \alpha_{t+1}(G_t(\omega) - c(A^*_t(\omega)) + \varepsilon_{t+1}(\omega'))(\omega)\mathrm{Q}_t(d\omega')(\omega) \\
&= \mathbb{E}^{\mathrm{Q}}_t(\alpha_{t+1}(G_t(\omega) - c(A^*_t(\omega)) + \varepsilon_{t+1}))(\omega) \\
&= \mathbb{E}^{\mathrm{Q}}_t(\alpha_{t+1}(G_t(\omega) - c(\alpha_t(G_t(\omega))(\omega)) + \varepsilon_{t+1}))(\omega)
\end{aligned}$$

Recursion for $(\alpha_t)_{t=0}^T$

Idea

$$\alpha_t(g)(\omega) = \mathbb{E}_t^{\mathrm{Q}}(\alpha_{t+1}(g - c(\alpha_t(g)(\omega)) + \varepsilon_{t+1}))(\omega),$$
$$\text{for all } g \in \mathbb{R},\ \omega \in \Omega$$

- start with $\alpha_T(g) = \pi 1_{[0,\infty[}(g), \quad$ for all $g \in \mathbb{R}$
- proceed recursively for $t = T-1, \ldots, 1$, determining $\alpha_t(g)(\omega)$ as the unique solution to the fix point equation

$$a = \mathbb{E}_t^{\mathrm{Q}}(\alpha_{t+1}(g - c(a) + \varepsilon_{t+1}))(\omega)$$

Formal result

Theorem

i) *Given measure* $\mathrm{Q} \sim P$ *there exist functionals*

$$\alpha_t : \mathbb{R} \times \Omega \to [0, \pi], \quad \mathcal{B}(\mathbb{R}) \otimes \mathcal{F}_t\text{-measurable, for } t = 0, \dots T$$

which fulfill for all $g \in \mathbb{R}$

$$\begin{aligned} \alpha_T(g) &= \pi 1_{[0,\infty[}(g), \\ \alpha_t(g) &= \mathbb{E}_t^{\mathrm{Q}}(\alpha_{t+1}(g - c(\alpha_t(g)) + \varepsilon_{t+1})), \quad t = 0, .., T-1 \end{aligned}$$

ii) *There exists a* Q*–martingale* $(A_t^*)_{t=0}^T$ *which satisfies*

$$\begin{aligned} A_T^* &= \pi 1_{\{\mathcal{E}_t - \sum_{t=1}^T c(A_{t-1}^*) \geq 0\}} \\ A_t^* &:= \alpha_t(\mathcal{E}_t - \sum_{s=1}^t c(A_{s-1}^*)), \quad t = 0, .., T-1 \end{aligned}$$

A numerical example

Suppose that

$$\varepsilon_{t+1} \text{ and } \mathcal{F}_t \text{ are independent under } Q \text{ for all } t = 0, \dots, T-1.$$

which makes calculations easier, since the randomness enters allowance price through the present up-to-day emissions only. More precisely one verifies that

$$\omega \mapsto \alpha_t(g)(\omega) = \alpha_t(g) \quad \text{is constant on } \Omega.$$

Hence, allowance price A^*_{t+1} is just Borel function of the present up-to-day emission G_{t+1} and the condition $\mathcal{F}_t$ can be replaced by the condition $\sigma(G_t)$:

$$\alpha_t(G_t) = \mathbb{E}^Q(\alpha_{t+1}(G_t - c(\alpha_t(G_t)) + \varepsilon_{t+1}) \,|\, \sigma(G_t)).$$

A numerical example

Given the fixed point equation for Borel measurable function α_t

$$\alpha_t(G_t) = \mathbb{E}^Q(\alpha_{t+1}(G_t - c(\alpha_t(G_t)) + \varepsilon_{t+1}) \,|\, \sigma(G_t)),$$

try to obtain a solution as limit $\alpha_t = \lim_{n\to\infty} \alpha_t^n$ of iterations

$$\alpha_t^{n+1}(G_t) = E^Q(\alpha_{t+1}(G_t - c(\alpha_t^n(G_t)) + \varepsilon_{t+1}) \,|\, \sigma(G_t)), \quad n \in \mathbb{N}$$

started at $\alpha_t^0 = \alpha_{t+1}$.

For numerical calculations, we suggest to use the least-square Monte-Carlo method. The idea here is to consider functions within a linear space spanned by basis functions and to replace the integration by a sum over finite sample.

A numerical example – least-square Monte-Carlo method

1. *Initialization:* Given sample $S = (e_k, g_k)_{k=1}^K \subset \mathbb{R}^2$ and a set of basis functions $\Psi = (\psi_i)_{j=1}^J$ on $\mathbb{R}$, define

$$M = \left(\psi_j(g_k)\right)_{k=1,j=1}^{K,J}$$

Set $\alpha_T(g) = 1_{[0,\infty]}(g)$ for all $g \in \mathbb{R}$, and proceed in the next step with $t := T - 1$.

2. *Iteration:* Define $\alpha_t^0 = \alpha_t$, and proceed in the next step with $n := 0$.

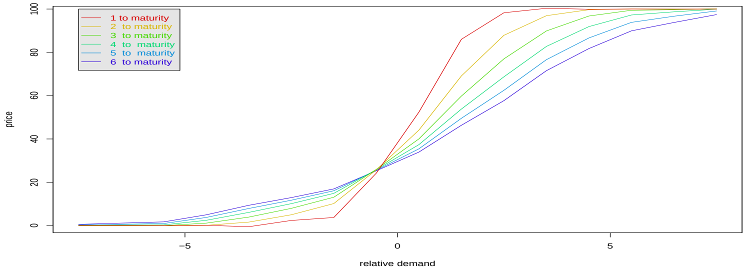
2a) Calculate $\phi^{n+1}(S) := (\alpha_{t+1}(g_k - c(\alpha_t^n(g_k)) + e_k))_{k=1}^K$

2b) Determine a solution $q^{n+1} \in \mathbb{R}^J$ to $M^\top M q^{n+1} = M^\top \phi^{n+1}(S)$.

2c) Define $\alpha_t^{n+1} := \sum_{j=1}^J q_j^{n+1} \psi_j$.

2d) If $\max_{k=1}^K |\alpha_t^{n+1}(g_k) - \alpha_t^n(g_k)| \geq \varepsilon$, then put $n := n + 1$ and continue with the step 2a).
If $\max_{k=1}^K |\alpha_t^{n+1}(g_k) - \alpha_t^n(g_k)| < \varepsilon$ then set $t := t - 1$. If $t > 0$, go to the step 2, otherwise finish.

Illustration



Parameters

- penalty $\pi = 100$,
- martingale increments $(\varepsilon_t)_{t=1}^T$ i.i.d, $\varepsilon_t = \mathcal{N}(0.5, 1)$, $K = 1000$
- basis functions $(\Psi_j)_{j=1}^J$ piecewise linear, $J = 16$
- abatement volume function $c : \mathbb{R} \to \mathbb{R}, a \mapsto 0.1\sqrt{(a)^+}$

Outlook: Transformation to continuous time

On the filtered probability space $(\Omega, \mathcal{F}, P, (\mathcal{F}_t)_{t\in[0,T]})$ allowance price dynamics $(A^*_t)_{t\in[0,T]}$ must be a solution to

$$\boxed{A^*_t = \pi E^{Q}(1_{\{\mathcal{E}_T > \int_0^T c(A^*_s)ds\}} \mid \mathcal{F}_t)}, \qquad t \in [0, T].$$

Define martingale

$$\mathcal{E}_t = E^{Q}(\mathcal{E}_T \mid \mathcal{F}_t), \qquad t \in [0, T].$$

Remembering the discrete-time case, assume that

the increments $(\mathcal{E}_t = E^{Q}_t(\mathcal{E}_T))_{t\in[0,T]}$ are independent.

Then search for a solution in form

$$A_t = \alpha(t, \underbrace{\mathcal{E}_t - \int_0^t c(A_s)ds}_{G_t}), \qquad t \in [0, T]$$

Supposing sufficiently smooth α, try

$$dA^*_t = \partial_{(1,0)}\alpha(t, G_t)dt + \partial_{(0,1)}\alpha(t, G_t)\overbrace{dG_t}^{\underbrace{d\mathcal{E}_t - c(\alpha(t, G_t))dt}} + \tfrac{1}{2}\partial_{(0,2)}\alpha(t, G_t)d[G]_t$$

$$=$$

$$\partial_{(0,1)}\alpha(t, G_t)d\mathcal{E}_t$$

$$+$$

$$\underbrace{\partial_{(1,0)}\alpha(t, G_t)dt - \partial_{(0,1)}\alpha(t, G_t)c(\alpha(t, G_t))dt + \frac{1}{2}\partial_{(0,2)}\alpha(t, G_t)d[\mathcal{E}]_t}_{=0}$$

For instance, if

$$d\mathcal{E}_t = \sigma_t dW_t, \qquad t \in [0,T], \quad (\sigma_t)_{t\in[0,T]} \text{ deterministic,}$$

this leads to PDE

$$\partial_{(1,0)}\alpha(t,g) - \partial_{(0,1)}\alpha(t,g)c(\alpha(t,g)) + \frac{1}{2}\partial_{(0,2)}\alpha(t,g)\sigma_t^2 = 0$$
$$\text{with boundary condition } \alpha(T,g) = 1_{[0,\infty[}(g) \text{ for } g \in \mathbb{R},$$

whose solution should give allowance price dynamics as

$$A_t = \alpha(t, G_t) \qquad t \in [0,T].$$

where $(G_t)_{t\in[0,T]}$ is solution to SDE

$$dG_t = d\mathcal{E}_t - c(\alpha(t,G_t))dt, \qquad G_0 = \mathcal{E}_0.$$

If this is OK, option pricing is straight-forward

Say, European Call with payoff

$$(A_\tau - K)^+ = (\alpha(\tau, G_\tau) - K)^+ \qquad \text{at maturity } \tau \in [0, T]$$

is priced at $t \in [0, \tau]$ by

$$E((\alpha(\tau, G_\tau) - K)^+ \mid \mathcal{F}_t) = f^\tau(t, G_t)$$

where the function f^τ is a solution

$$\partial_{(1,0)} f^\tau(t,g) - \partial_{(0,1)} f^\tau(t,g) c(\alpha(t,g)) + \frac{1}{2} f^\tau_{(0,2)}(t,g)\sigma_t^2 = 0$$

with boundary condition $f^\tau(\tau, g) = (\alpha(\tau, g) - K)^+$ for $g \in \mathbb{R}$.

Work to do

- Extension to stochastic abatement costs

  describing dependence of
  opportunity merit order
  on gas and oil prices.

  Here commodity price modeling enters the discussion...

- Quantitative comparison of different market designs (PDEs for windfall profit and total reduction distributions)

Thank you!