Tutorial
BEC and quantized vortices in superfluidity and superconductivity
6-7 December 2007
Institute for Mathematical Sciences
National University of Singapore

Vortices in superconductors: V. 3D mesoscopic structures

François Peeters

Sphere

Expansion method
Consider only the first GL-equation (i.e. κ>>1)

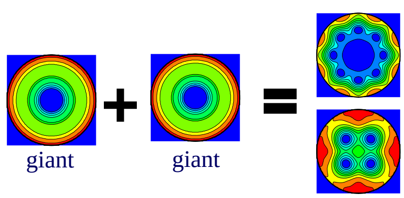
$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\varphi^2}-2is\frac{\partial\Psi}{\partial\varphi}-s^2\rho^2\Psi+\frac{\partial^2\Psi}{\partial z^2}=-\left(1-\frac{T}{T_c}\right)\Psi$$

$$\Psi(\rho,\varphi,z)=e^{iL\varphi}\psi(\rho,z)$$

$$\left(-i\hbar\vec{\nabla}-\frac{2e}{c}\vec{A}\right)_n\Psi=0$$

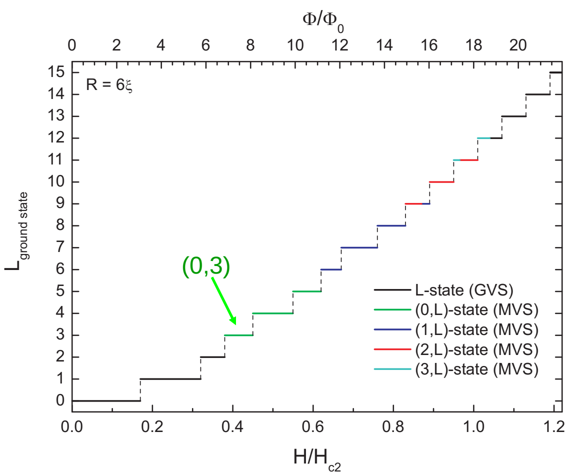
⇒ 2D differential equation

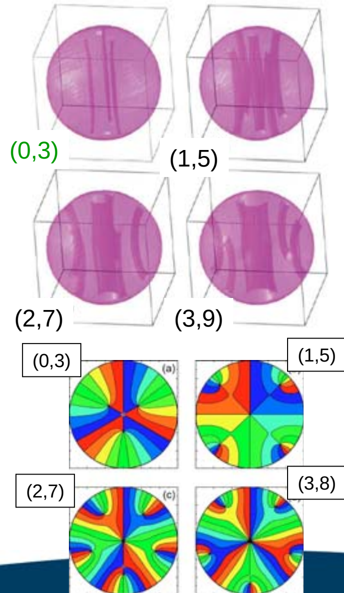
$$\psi(r,\varphi)=\sum_{L_j=0}^{L}\sum_{n}C_{n,L_j}f_{n,L_j}(r)\exp(iL_j\varphi)$$



B.J. Baelus, D. Sun and F.M. Peeters, Phys. Rev. B **75**, 174523 (2007)

Ground state configurations



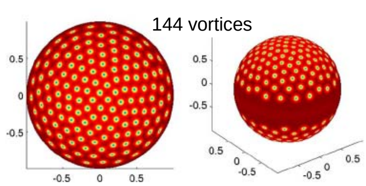


Hollow sphere

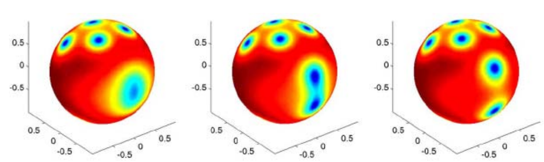
Numerical simulations of the quantized vortices on a thin superconducting hollow sphere
Qiang Du, Lili Ju

Journal of Computational Physics 201 (2004) 511–530

⇒ Numerical solution of the first GL equation (thin sphere shell approximation)



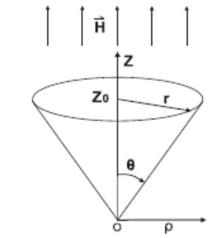
Top view 3D view



Nucleation and splitting of vortex pair near the equator

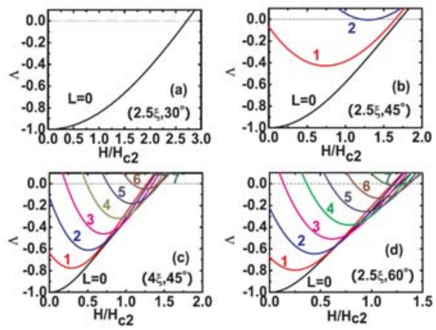
Cone

Y. Chen (UA)
M.M. Doria (Rio de Janeiro)

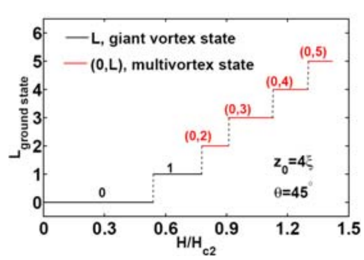


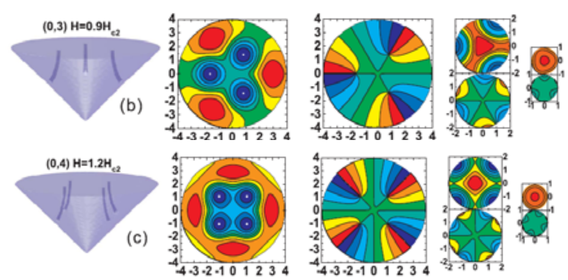
$$\Psi(\rho,\varphi,z)=\sum_{L_j=0}^{L} C_{L_j}\psi_{L_j}(\rho,z)exp(iL_j\varphi).$$

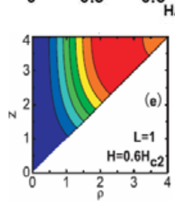
$$\frac{\partial F}{\partial C_{Lj}}=0 \quad \text{and} \quad \frac{\partial F}{\partial C^*_{L_j}}=0$$

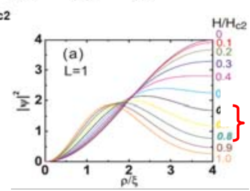


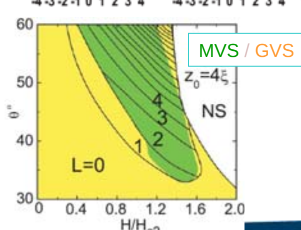
Phase diagram





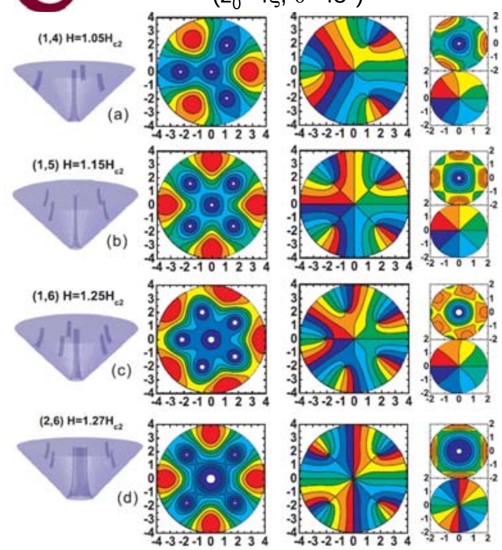




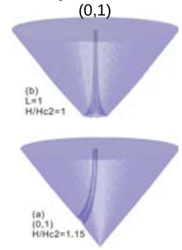


Bend vortices

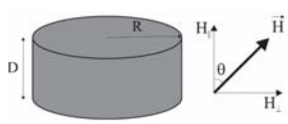
$(z_0=4\xi, \theta=45^0)$



$(z_0=4\xi, \theta=38^0)$
(0,1)

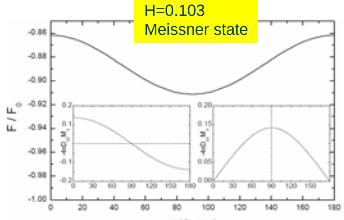


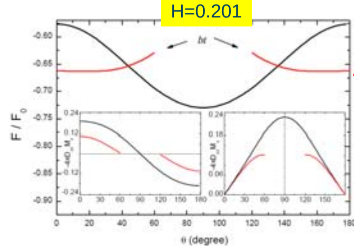
Tilted vortices in cylinder

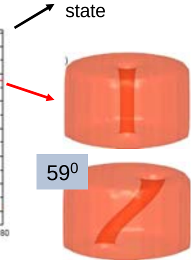


Numerical solution of first GL-equation (i.e. λ>>1)

R=4ξ; D=4ξ
Magnetic field units: $H_{ref} = H_{c3,//}$, max. field of short cylinder for || orientation



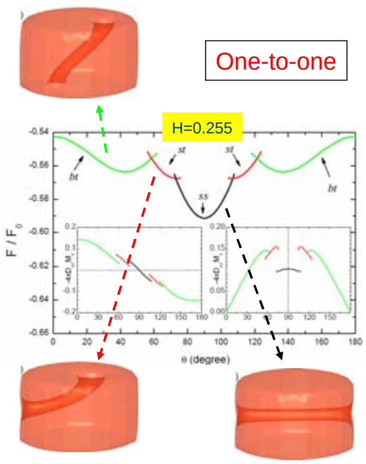




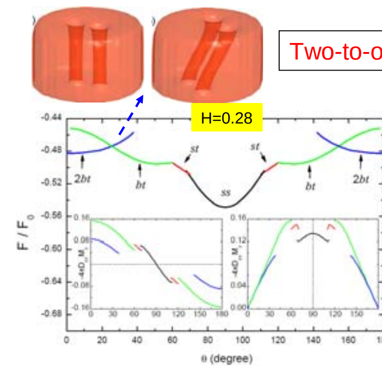
A.R. de C. Romaguera, M.M. Doria, and F.M. Peeters, Phys. Rev. B **75,** 184525 (2007)

Tilted vortices in cylinder

One-to-one



Two-to-one



bt: bottom to top
st: side to top
ss: side to side

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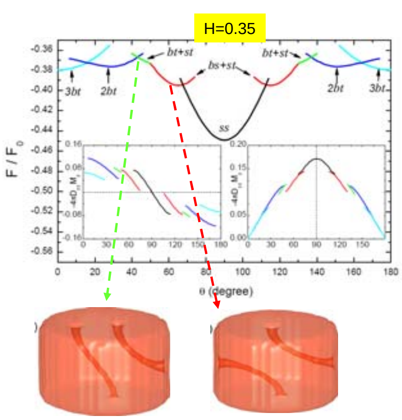
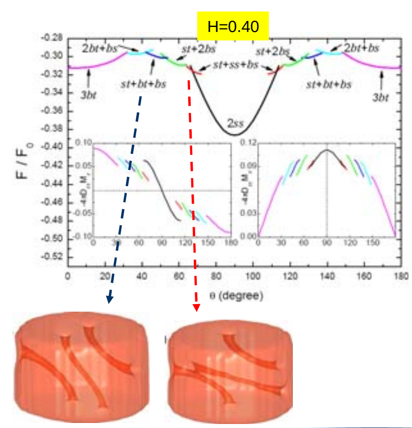
Tilted vortices in cylinder

Three-to-one



Three-to-two



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Transverse magnetization and torque in asymmetrical mesoscopic superconductors

Asymmetrical coexistence of vortices with different lengths → transverse magnetization
→ torque

Sphere with $R=6.0\xi$ - lacking one of his quadrants

$\mu=V\mathbf{M}$: magnetic moment of a sample with volume V

$\tau=\mu x\mathbf{H}$: torque

$\tau=\mu H \sin\theta\ \mathbf{e}_\tau$: where θ is the angle between **H** and **M**

For example:

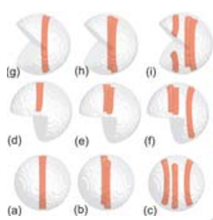
$\xi=100nm$ → $H_{c2} = \Phi_0 /2\pi\xi^2 = 300G$

If $R=0.6\mu m$ then transverse magnetization $M_x = 10^{-3} H_{c2} = 0.3G$,
while the longitudinal Is 10 times larger: $M_z = 10^{-2} H_{c2} = 3G$

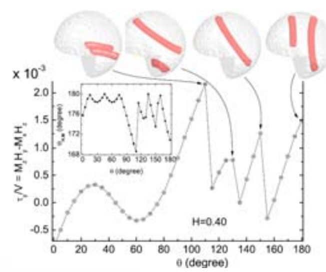
The magnetic moment of this sphere is: $\mu \sim M_z V = 7.0 \cdot 10^{-12}$ emu

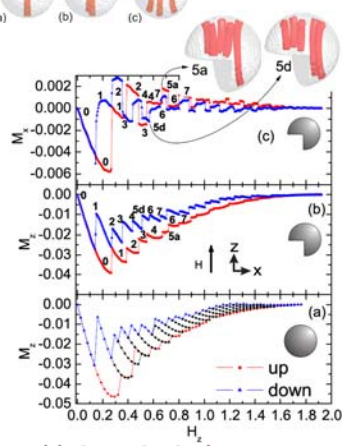
and the torque $\tau_y = 4.0 \cdot 10^{-12}$ erg

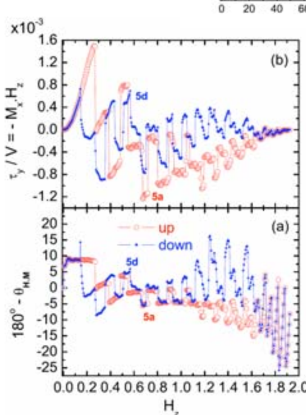
A.R. de C. Romaguera, AmM. Doria, and F.M. Peeters, Phys. Rev. B **76**, 020505(R) (2007)



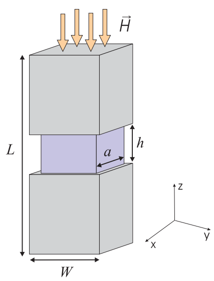
5a=(2,3), 5d=(1,4)

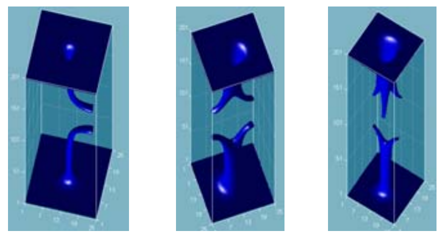






SC wire with a constriction

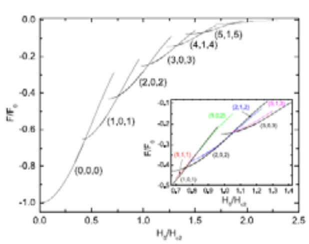


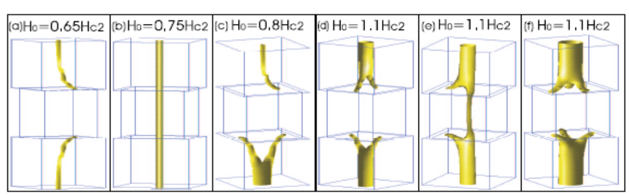


(1,0,1) (2,0,2) (3,0,3)

A. K. Elmurodov, D. Y. Vodolazov, and F. M. Peeters, *Europhys. Lett.*, **74**, 151 (2006)

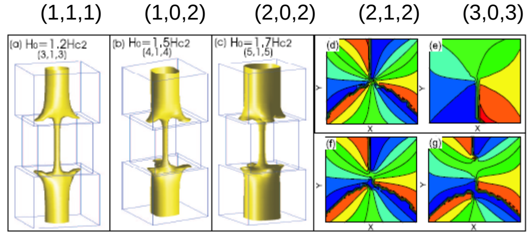
Vortex states





(1,0,1) (1,1,1) (1,0,2) (2,0,2) (2,1,2) (3,0,3)

Spatial break-up of a GVS:
Inside thick wire:
GVS(L=3)
near constriction → GVS(L=2)+1V
Inside constriction
→ 1 vortex



Conclusion

- 3D structures: vortices are 3D objects (tubes instead of disks)
- Added possibility to manipulate the vortices in the third dimension:
  - bend vortice → see e.g. sphere, cone, cylinder in a tilted magnetic field
  - spatial break-up of a GVS (splitting of a vortex) → see wire with a constriction
  - Vortices can exert a torque on an asymmetric sample

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