# Some nontrivial elements in the stable homotopy groups of sphere

For connected finite type spectra $X, Y$, there exists Adams spectral sequence (ASS) $\{E_r^{s,t}, d_r\}$ such that:
(1) $d_r : E_r^{s,t} \to E_r^{s+r,t+r-1}$ is the differential,
(2) $E_2^{s,t} \cong Ext_A^{s,t}(H^*X, H^*Y)$ and
(3) converges to $[\Sigma^{t-s}Y, X]_p$
i.e. $E_2^{s,t} \cong Ext_A^{s,t}(H^*X, H^*Y) \Longrightarrow [\Sigma^{t-s}Y, X]_p$. When $Y$ is sphere spectrum $S$, it is $E_2^{s,t} \cong Ext_A^{s,t}(H^*X, Z_p) \Longrightarrow \pi_{t-s}(X)_p$. When $X$ is sphere spectrum $S$, Moore spectrum $M$, Toda-Smith spectrum $V(1), V(2)$ respectively, $\pi_{t-s}(X)_p$ is respectively the stable homotopy group of $S, M, V(1), V(2)$. Today, we detected some new nonzero elements of the stable homotopy groups of sphere and Toda-Smith spectrum $V(1)$ by using of the ASS. If a family of homotopy generators $x_i$ in $E_2^{s,*}$ converges nontrivially in the ASS,then we get a family of homotopy elements $f_i$ in $\pi_*S$ and we say that $f_i$ is represented by $x_i \in E_2^{s,*}$ and has filtration $s$ in the ASS. so far, not so many families of homotopy elements in $\pi_*S$ have been detected. For example, a family $\xi_{n-1} \in \pi_{p^nq+q-3}S (n \geq 2)$, which has filtration 3 in the ASS and is represented by $h_0b_{n-1} \in Ext_A^{3,\ p^nq+q}(Z_p, Z_p)$ .

This thesis contains four chapters. In the first chapter,we find the convergence of the products $\widetilde{\gamma}_t\widetilde{l_1}g_0 \in Ext_A^{t+5,(t+1)p^2q+(t+2)pq+tq+t-3}(Z_p, Z_p) (3 \leq t < p-2)$ in Adams spectral sequence,where A is $mod\ p$ Steenrod algebra, $\widetilde{\gamma}_t \in Ext_A^{t,tp^2q+(t-1)pq+(t-2)q+t-3}$ $(Z_p, Z_p)$ converges to $\gamma_t = j_0j_1j_2\gamma^t i_2i_1i_0 \in \pi_*(S)$.

In the second chapter,by the algebraic method,we prove the existence of a new nontrivial family $\widetilde{\gamma}_{s+3}h_nh_m, (m \geq n+2 > 5, s < p-3)$which filtration is $s+5$ in the stable homotopy groups of spheres $\pi_{q(p^m+p^n+(s+3)p^2+(s+2)p+(s+1))-5}S$.

In the third chapter,we use the estimation about $Ext_P^{s,t}(Z_p, Z_p)$ ,which is the subalgebra of $mod\ p$ Steenrod algebra $A$ which is generated by all $P^i (i \geq 0)$, we find $Ext_A^{4+s,p^2q+pq+q+s}(H^*V(1), Z_p) = Z_p\{b_0^p g_0\}$ $(s \geq 1)$.At the same time,Massey product and Toda brackets are very important to determine some new families of homotopy elements in $\pi_*(S)$ ,in this chapter,a nontrivial product $i_1i_0(\xi_1) \centerdot i_1j_1\beta i_1i_0 \neq$ 0was detected by Moss .In the end ,The relation $i_1i_0(\xi_1) \centerdot i_1j_1\beta i_1i_0 = (\beta')^p\alpha''\beta i_1i_0 \neq 0$ (where $p \geq 7$ )$in$ $\pi_*V(1)$ was obtained.

In the forth chapter,we discuss the property of the ring spectrum $V_r(2)$.At last,we get the convergence of $\gamma_{tp^n/r}(p \geq 7, t \geq 1, 1 \leq r \leq 2^n < \frac{p-3}{2})$ in the Adams-Novikov spectral sequence(ANSS).

**Key Words:** stable homotopy groups sphere spectrum Toda-Smith spectrum Adams spectral sequence May spectral sequence