Dynamic Leverage and Threshold Effects in Stochastic Volatility Models*

Manabu Asai
Faculty of Economics
Tokyo Metropolitan University

Michael McAleer
School of Economics and Commerce
University of Western Australia

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Abstract

In this paper we examine two methods for modelling asymmetries, namely dynamic leverage and threshold effects, in Stochastic Volatility (SV) models, one based on the threshold effects (TE) indicator function of Glosten, Jagannathan and Runkle (1992), and the other on dynamic leverage (DL), or the negative correlation between the innovations in returns and volatility. A general dynamic leverage threshold effects (DLTE) SV model is also used to enable non-nested tests of the two asymmetric SV models against each other to be calculated. The three SV models are estimated by the Monte Carlo likelihood (MCL) method proposed by Sandmann and Koopman (1998), and the finite sample properties of the estimator are investigated using numerical simulations. As the numerical simulation results show that the MCL estimator is biased, a simple method for correcting the bias is suggested and the performance of the bias-corrected MCL estimators is evaluated. Four financial time series are used to estimate the SV models, with empirical asymmetric effects found to be statistically significant in each case. The empirical results for S&P 500, TOPIX and Yen/USD returns indicate that dynamic leverage dominates the threshold effects model for capturing asymmetric behaviour, while the results for USD/AUD returns show that both the non-nested dynamic leverage and threshold effects models are rejected against each other. For the four data series considered, the dynamic leverage model dominates the threshold effects model in capturing asymmetric effects. In all cases, there is significant evidence of asymmetries in the general DLTE model.

Key words: Stochastic volatility, asymmetric effects, dynamic leverage, threshold effects, indicator function, Monte Carlo likelihood, numerical simulations, bias correction, non-nested models.

1 Introduction

It has long been recognized that the returns of financial assets are negatively correlated with changes in the volatilities of returns (see Black (1976) and Christie (1982)) and, moreover, that such volatilities tend to change over time. In the class of autoregressive conditional heteroskedasticity (ARCH) models pioneered by Engle (1982), several authors have proposed extensions of the ARCH model and found evidence of such negative correlation. For instance, Nelson (1991) proposed the exponential generalized ARCH (EGARCH) model, while Glosten, Jagannathan and Runkle (1992) developed a threshold indicator function GARCH model, which is commonly called the GJR model. The threshold effect is typically called asymmetry when the threshold is set to zero. A common idea used in such asymmetric models is the `leverage' effect, in which negative shocks to returns increase the predictable volatility to a greater extent than do positive shocks.

On the other hand, stochastic volatility (SV) models are based on the direct correlation between the innovations in both returns and volatility. For a theoretical development in continuous time, Hull and White (1987) generalized the Black-Scholes option pricing formula to analyse stochastic volatility and the negative correlation between the innovations. In empirical research, extensions of a simple discrete time model due to Taylor (1986) have been analysed by Wiggins (1987), Chesney and Scott (1989), and Harvey and Shephard (1996) in order to accommodate the direct correlation. Although this extension has been called the asymmetric SV model, we will refer to the asymmetric behaviour based on the direct correlation between the innovations as the "dynamic leverage" SV model to distinguish it from an alternative model of asymmetry.

In addition to the dynamic leverage model, this paper considers an alternative asymmetric SV model using a threshold effects indicator function, as developed by Glosten, Jagannathan and Runkle (1992) in the context of ARCH models. We will refer to the asymmetric behaviour based on a zero threshold indicator function as the "threshold effects" SV model. These two asymmetric SV models, as well as a more general model which incorporates both types of asymmetries, called the "dynamic leverage threshold effects" SV model, will be estimated and tested for an optimal and practical representation of asymmetry. The general model also permits the non-nested dynamic leverage and threshold effects SV models to be tested against each other.

The empirical analysis is concerned with both stock returns and exchange rate returns. Although Gallant, Hsieh and Tauchen (1991) found that the response of conditional volatility to negative and positive shocks was essentially symmetric for the British pound/US dollar exchange rate by using the seminonparametric technique of Gallant and Tauchen (1989), we observed asymmetries in the exchange rate data based on the dynamic leverage and threshold effects SV models, even though such asymmetries may not be captured adequately using the ARCH approach.

For estimation of the SV model, recent developments have been on the likelihood-oriented procedures (see Fridman and Harris (1998), Sandmann and Koopman (1998) and Watanabe (1999)), and on the Bayesian Markov Chain Monte Carlo (MCMC) technique proposed by Jacquier, Polson and Rossi (1994) (see, among others, Chib, Nardari and Shephard (2002) and Shephard and Pitt (1998)). The Monte Carlo results conducted by Fridman and Harris (1998), Sandmann and Koopman (1998) and Watanabe (1999) show that the properties of these methods are very similar to those of Jacquier, Polson and Rossi (1994). While the procedures proposed by Fridman and Harris (1998) and Watanabe (1999) are more computationally demanding than the MCMC technique of Jacquier, Polson and Rossi (1994), the Monte Carlo likelihood method proposed by Sandmann and Koopman (1998) is much easier to implement computationally. With regard to the Bayesian approach, Asai (2003) found some evidence that the method of Chib, Nardari and Shephard (2002) was the best with regard to a numerical efficiency measure that was proposed by Geweke (1992).

The remainder of the paper is organized as follows. Section 2 examines the dynamic leverage (DL), threshold effects (TE), and dynamic leverage threshold effects (DLTE) SV models, and investigates their relationships. A non-nested testing procedure to discriminate between the DL and TE models is also discussed. Section 3 discusses some estimation techniques for SV models, and Section 4 presents the results of some Monte Carlo experiments regarding the finite sample performance of the estimators of the alternative SV models. As the numerical simulation results show that the MCL estimator is biased, a simple method for correcting the bias is suggested and the performance of the bias-corrected MCL estimators is evaluated. In Section 5, the two asymmetric SV models and the DLTE model are estimated using S&P 500 Composite returns, the Tokyo stock price index (TOPIX) returns, and the exchange rates between the USA and Australia and between Japan and the USA. Section 6 gives some concluding remarks.

2 Asymmetric Effects in Stochastic Volatility Models

In this paper we consider two types of asymmetric behaviour in SV models. Dynamic leverage captures asymmetry by the negative correlation between returns and volatility innovations, as follows:

$$y_t = \sigma\varepsilon_t \exp(h_t / 2), \quad \varepsilon_t \sim N(0,1), \quad t = 1,...,T, \tag{1}$$

$$h_{t+1} = \phi h_t + \eta_t, \quad \eta_t \sim N(0,\sigma_\eta^2), \quad E(\varepsilon_t \eta_t) = \rho\sigma_\eta, \tag{2}$$

where $\mathrm{y}_t = R_t - \mu_t$ is the mean-adjusted return on an asset. Since many financial time series exhibit little or no dynamic behaviour in the mean but pronounced serial dependence in the variance (see Bollerslev, Chow and Kroner (1992), Bollerslev, Engle and Nelson (1994), and Li, Ling and McAleer (2002) for useful surveys), the estimation of μ_t is not the subject of interest in this paper. We will refer to this type of asymmetry, namely when $\rho \neq 0$, as the dynamic leverage (DL) SV model. When $\rho = 0$, there is no dynamic leverage, although alternative asymmetries may be present (such as $\gamma \neq 0$ in equation (3) below).

There are two standard methods of capturing asymmetric behaviour in ARCH-type models, one of which is the exponential generalized ARCH (EGARCH) model of Nelson (1991). Although the EGARCH model has been used quite frequently in empirical applications when asymmetric behaviour is observed, the presence of the absolute value of a standardized shock in the model poses a problem regarding the statistical properties of the model. Shephard (1996) suggested a likely sufficient condition for consistency of the quasi-maximum likelihood estimator (QMLE). McAleer, Chan and Marinova (2002) noted that a similar condition was likely to be sufficient for the existence of moments and for asymptotic normality of the QMLE.

A more frequently used model of asymmetric behaviour in ARCH-type models is the threshold indicator function ARCH (or GJR) model of Glosten, Jagannathan and Runkle (1992). The threshold effect is typically called asymmetry when the threshold is set to zero. Ling and McAleer (2002) established the necessary and sufficient conditions for the existence of moments of the GJR(1,1) model, while McAleer, Chan and Marinova (2002) established the sufficient conditions for consistency and asymptotic normality of the QMLE of GJR(1,1). In view of these recent theoretical results, the development of an alternative to the DL SV model will be based on the threshold model of Glosten, Jagannathan and Runkle (1992).

In the model of asymmetry based on thresholds, volatility is affected by the sign of the previous returns innovation, as follows:

$$h_{t+1} = \phi h_t + \xi_t, \quad \xi_t = \gamma \left\{ I(\varepsilon_t) - E[I(\varepsilon_t)] \right\} + \eta_t, \tag{3}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$, $E(\varepsilon_t \eta_t) = 0$, and $I(\cdot)$ is an indicator function such that $I(x) = 1$ if $x < 0$ and $I(x) = 0$ otherwise. Note that $E[\xi_t] = 0$ and $E[\xi_t^2] = \sigma_\eta^2 + \gamma^2/4$. In the following, we refer to this type of asymmetric behaviour, that is, when $\gamma \neq 0$, as the threshold effects (TE) SV model. When $\gamma = 0$, there is no threshold effect, although alternative asymmetries may be present (such as $\rho \neq 0$ in equation (2) above).

When $\rho \neq 0$ in (2) and $\gamma \neq 0$ in (3), this yields the dynamic leverage threshold effects (DLTE) SV model. The general DLTE model may be interpreted as either: (i) an asymmetric model which exhibits both dynamic leverage and threshold effects; or (ii) an artifact which is used solely for purposes of testing the non-nested DL and TE models against each other. In the latter case, the four possible outcomes of the non-nested tests of the DL and TE models against each other are as follows:
(i) $\rho = 0$ and $\gamma = 0$, which leads to rejection of both DL and TE;
(ii) $\rho \neq 0$ and $\gamma = 0$, which leads to rejection of TE but not DL;
(iii) $\rho = 0$ and $\gamma \neq 0$, which leads to rejection of DL but not TE;
(iv) $\rho \neq 0$ and $\gamma \neq 0$, which leads to rejection of neither DL nor TE.

Tests of non-nested conditional volatility models, specifically GARCH versus EGARCH, and GJR versus EGARCH, have been examined by Ling and McAleer (2000) and McAleer, Chan and Marinova (2002), respectively. For further details regarding non-nested testing procedures in the context of econometric time series and regression models, see McAleer (1995).

An alternative comparison of the DL and TE models can be made as follows. Averaged over the whole sample, we can examine the relationship between the DL and TE models through the correlation of ξ_t and ε_t under alternative models. While $Corr(\xi_t, \varepsilon_t) = \rho$ for the DL model, the correlation coefficient is given by

$Corr(\xi_t, \varepsilon_t) = -\gamma \Big/ \sqrt{2\pi(\sigma_\eta^2 + \gamma^2/4)}$ for the TE model, since $Cov(I(\varepsilon_t), \varepsilon_t) = -E\,|\,\varepsilon_t\,|\,/\,2 = -1\big/\sqrt{2\pi}$. This result indicates that an appropriate choice of γ in the TE model would yield the same $Cov(\xi_t, \varepsilon_t)$ as in the DL model. By appropriate construction of the TE and DL models, if the y_t are generated by the TE model when $\gamma = \gamma_0$ and $\sigma_\eta = \sigma_{\eta 0}$ but the DL model is estimated, the estimate of ρ will be approximately $-\gamma_0 \Big/ \sqrt{2\pi(\sigma_{\eta 0}^2 + \gamma_0^2/4)}$. However, if the data are generated by the true DL model but the TE model is estimated, the estimate of γ will be smaller than the values derived by ρ since the TE model can capture only the threshold effects.

In the DLTE model, the correlation coefficient of ξ_t and ε_t is given by

$$Corr(\xi_t, \varepsilon_t) = \frac{\rho\sigma_\eta\sqrt{\pi} - \gamma\big/\sqrt{2}}{\sqrt{\pi\left(\sigma_\eta^2 + \gamma^2/4\right) - \gamma\rho\sigma_\eta}}. \tag{4}$$

This is also useful for testing the differences between the DL and TE models.

It should be noted that $\left|Corr(\xi_t, \varepsilon_t)\right| = \left|-\gamma \Big/ \sqrt{2\pi(\sigma_\eta^2 + \gamma^2/4)}\right| < \sqrt{2/\pi} = 0.7979$ for the TE model, which implies that this model is not appropriate for describing highly positive or negative correlation.

3 Model Estimation

Before we examine the empirical performance of the two non-nested asymmetric SV models, as well as the general DLTE SV model, it will be useful to discuss estimation of the SV model. There are three categories of estimator: (i) sampling theory based on y_t; (ii) sampling theory based on $\log y_t^2$; (iii) Bayesian Markov Chain Monte Carlo (MCMC) methods. The distinction between the first two categories is important, especially for likelihood-based inference. For instance, Kim, Shephard and Chib (1998) proposed the non-nested likelihood ratio test of the SV model versus the GARCH model, which needs additional estimation of the likelihood of y_t for the second category. The third category is indifferent to the choice of y_t or $\log y_t^2$ because the posterior distributions are invariant to the choice of distribution of y_t or $\log y_t^2$ if the priors are the same.

For each of the first two categories, there is an optimal method with respect to the finite sample properties, namely the likelihood-oriented procedures of Fridman and Harris (1998), Sandmann and Koopman (1998) and Watanabe (1999). For the first category, Fridman and Harris (1998) and Watanabe (1999) independently applied the non-Gaussian state-space-model filtering and smoothing procedure of Kitagawa (1987) to evaluate the likelihood through recursive numerical integration. It should be noted that another method of estimation is the efficient method of moments (EMM) procedure proposed by Gallant and Tauchen (1996), which matches the score of the auxiliary model through simulation. Although the procedures based on method of moments are known to be suboptimal relative to the likelihood-based methods, Gallant and Tauchen (1996) claim that, if the auxiliary model is an accurate approximation to the distribution of the data, then EMM is as efficient as maximum likelihood. However, there is no method for estimating the instantaneous volatility throughout the sample, $t = 1,\ldots,T$, so that an additional form of estimation, such as the Kalman filter based on $\log\ y_t^2$, is required.

In the second category, Sandmann and Koopman (1998) proposed the Monte Carlo maximum likelihood method, for which the likelihood function can be approximated arbitrarily by decomposing it into a Gaussian part, constructed by the Kalman filter, and a remainder function, for which the expectation is evaluated through simulation. Monte Carlo results conducted by Fridman and Harris (1998), Sandmann and Koopman (1998) and Watanabe (1999) show that their methods have properties which are very close to those of Jacquier, Polson and Rossi (1994), which is an estimation method from the third category. While the procedures proposed by Fridman and Harris (1998) and Watanabe (1999) are more computationally demanding than the MCMC method of Jacquier, Polson and Rossi (1994), the Monte Carlo likelihood method proposed by Sandmann and Koopman (1998) is much easier to implement computationally. Regarding the estimation of the likelihood of y_t, one of several computationally inefficient but accurate methods can be used as this requires no iterations. For example, the auxiliary particle filters proposed by Pitt and Shephard (1999) would be suitable.

A distinguishing feature of the third category is that another measure of efficiency is required to compare the various methods as all the approaches produce a single posterior. Jacquier, Polson and Rossi (1994) proposed a Bayesian approach for estimating SV models using the MCMC technique. Their method is called the single-move sampler since it requires sampling each h_t. Illustrative examples in de Jong and Shephard (1995), Shephard and Pitt (1997) and Kim, Shephard and Chib (1998) regarding the normal SV model suggest that the single-move sampler would produce a highly correlated sample sequence when state variables are highly autocorrelated. The single-move sampler is, therefore, inefficient in the sense that it needs to repeat the sampling a large number of times.

Two methods are more efficient than the single-move sampler, the first of which is the multi-move sampler proposed by Shephard and Pitt (1997). As the original multi-move sampler suffers from serious estimation bias, Watanabe and Omori (2001) suggested a correction. A second method is the so-called "integration sampler", which was proposed by Kim, Shephard and Chib (1998) and extended by Chib, Nardari and Shephard (2002). Based on simulated data, Asai (2003) found that, by using an efficiency factor presented in Geweke (1992), the integration sampler always outperformed the multi-move sampler for sampling (ϕ, σ_η), but that the former was generally less efficient than the latter for sampling σ and the latent volatilities. Using the Yen/Dollar exchange rate data, Asai (2003) also showed that there was an empirical case in which the integration sampler outperformed the multi-move sampler for all the parameters $(\sigma, \phi, \sigma_\eta)$.

Compared with the Monte Carlo Likelihood (MCL) method of Sandmann and Koopman (1998), the Bayesian MCMC methods are computationally demanding. The Monte Carlo results of Sandmann and Koopman (1998), which compare the MCL method with the MCMC method of Jacquier, Polson and Rossi (1994), show that MCL yields a larger bias than MCMC when the unconditional variance of the time-varying log-volatility is relatively small. Since such an outcome suggests that the volatility is not particularly significant, in what follows only the MCL method is used for estimating the three SV models.

Returning to the asymmetric SV models, as the MCL method can incorporate asymmetry and explanatory variables into the volatility equation, it is a straightforward extension to estimate the DL and TE models. For the Bayesian MCMC method, we would use a slight modification of the integration sampler of Chib, Nardari and Shephard (2002). It should be noted that Jacquier, Polson and Rossi (2004) proposed a Bayesian MCMC technique to estimate the DL model. However, this approach is based on Jacquier, Polson and Rossi (1994), which is less efficient than the method of Chib, Nardari and Shephard (2002) with respect to numerical efficiency. Moreover, Yu (2004) showed that it was not clear how to ensure or interpret the leverage effect in the model of Jacquier, Polson and Rossi (2004).

4 Monte Carlo Experiments

Simulation experiments were conducted in order to assess the performance of the MCL estimator. The range of parameter values $\theta = (\sigma, \phi, \sigma_\eta, \gamma)'$ was selected as follows. First, the autoregressive parameter ϕ is set to 0.95, and $(\sigma_\eta, \rho, \gamma)$ is selected so that the coefficient of variation, namely

$$CV = \frac{\mathrm{Var}(h_t)}{\mathrm{E}(h_t^2)} = \exp\left(\frac{\mathrm{Var}(\xi_t)}{1-\phi^2}\right) - 1$$

takes the value of unity in the DLTE model, with a restriction that the correlation coefficient between ξ_t and ε_t is -0.30 or -0.60. Specifically, we set the parameter vector to be

$(\sigma_\eta, \rho, \gamma)$ = {(0.260, -0.30, 0), (0.241, 0, 0.195), (0.253, -0.15, 0.100)},

which represents the DL, TE and DLTE models, respectively. Note that for each parameter set, the value of $\sqrt{\mathrm{Var}(\xi_t)}$ is 0.260, the absolute value of ρ in DL is twice that of ρ in DLTE, and the value of γ in TE is roughly twice that of γ in DLTE. For the case $Corr(\varepsilon_t, \xi_t) = -0.60$ we specify the parameter values to be

$(\sigma_\eta, \rho, \gamma)$ = {(0.260, -0.60, 0), (0.173, 0, 0.388), (0.224, -0.30, 0.225)}.

Again, the value of $\sqrt{\mathrm{Var}(\xi_t)}$ is 0.260 for each parameter set. Second, the values of the location parameter, σ, are chosen such that the expected variance, namely

$$E(y_t^2) = \sigma^2 \exp\left(\frac{\mathrm{Var}(\xi_t)}{2(1-\phi^2)} \right),$$

is set to 0.0009. If the simulated data are regarded as weekly returns, this corresponds to an approximate 22% annualized standard deviation. For convenience in estimation, we mapped σ into α using $\alpha = \log \sigma^2$, which yields $\alpha =$ -7.36. For each θ, we generated a sample of size T = 1000, 2000 and 5000, and estimated the DL, TE and DLTE models using the MCL method.

It should be noted that we exclude the case of highly negative autocorrelation, such as $Corr(\varepsilon_t, \xi_t) = -0.90$, since it may exceed the bound for the TE model, as described in the previous section.

4.1 Estimates based on correct models

Table 1 shows the results for $Corr\left(\varepsilon_t, \xi_t\right) = -0.30$ under various true models and sample sizes. Table 1(a) reports the sample means and standard deviations of the MCL estimates for $K = 500$ replications, while Table 1(b) presents the 95% coverage probabilities of the Monte Carlo simulations. Thus, for each replication i, a confidence interval is computed as $\hat{\theta}_{ij} \pm 1.96\sqrt{\mathrm{Var}(\hat{\theta}_{ij})}$, where $\mathrm{Var}(\hat{\theta}_{ij})$ is the relevant element of the covariance matrix of the estimator $\hat{\theta}_{ij}$. The coverage probability, $\hat{p}$, corresponds to the number of times the true value of θ (which, for the purposes of the experiments, is assumed to be known), θ_0, falls within the confidence interval for each replication, divided by the number of replications, K. Standard errors are computed from the Bernoulli formula $\sqrt{(1/K)\hat{p}(1-\hat{p})}$.

Table 1 indicates that there is a small bias in the estimator of ρ relative to the true value. The bias seems to increase as $|\rho|$ becomes large, being around 0.03 for $\rho = -0.15$ and around 0.06 for $\rho = -0.30$. This may explain the relatively low coverage probability for ρ compared with γ, especially in DLTE. The bias in ρ remains even for sample size $T = 5000$, which may lead to the result that the coverage probability for ρ decreases as the sample size increases for $\rho = -0.30$. Estimated values of $Corr(\varepsilon_t, \xi_t)$ and $\sqrt{\mathrm{Var}(\xi_t)}$, which are ρ and σ_η, respectively, in the DL model, also support the results regarding the bias. The absolute value of the estimate of ρ in DL is roughly twice that of ρ in DLTE, and the estimate of γ in TE is twice that of γ in DLTE, which are similar to the true parameter values. In TE (where $\rho = 0$), the sample mean of the correlation coefficient is close to the true value, but those for $\rho =$ 0.15 and $\rho = -0.30$ are about -0.33 and -0.37, respectively.

The estimates of ρ and γ are significant in DL and TE, respectively, but neither is significant in DLTE for $T = 1000$ and 2000. The sample mean of $\sqrt{\mathrm{Var}(\xi_t)}$ approaches the true value as ρ approaches zero, and the bias seems to disappear as the sample size increases. Although it might be expected, the standard deviations in Table 1 are very close to those in Table 3 of Sandmann and Koopman (1998), which report the finite sample properties of the MCL estimates when $\rho = 0$ and $\gamma = 0$.

Table 2 presents the results for $Corr(\varepsilon_t, \xi_t) = -0.60$ under various true models. The bias for ρ increases as ρ deviates significantly from zero. While the bias is about 0.05 for $\rho = -0.30$, it is about 0.10 for $\rho = -0.60$. Estimates of γ are close to the true values. In addition to these results, the evidence that the sample mean of $\sqrt{\mathrm{Var}(\xi_t)}$ approaches the true value as ρ approaches zero implies that a little bias for σ_η is affected by the bias for ρ. Before we discuss a way of correcting such biases, it will be helpful to present the results for misspecified models, as the main concern in the paper is to examine the relationships among the DL, TE and DLTE models.

4.2 Estimates based on misspecified models and LR tests

Table 3 shows the estimation results for two models when a third model is true. The sample size for Table 3 is $T = 2000$, with the results for $T = 1000$ and 5000 being omitted as they are qualitatively similar. Table 3(a) presents the sample means and standard deviations of the MCL estimates of TE and DLTE when the true model is DL. As expected, the estimated correlation coefficient between ξ_t and ε_t is far from the true value -0.3. While the estimate of γ is significant in the incorrect TE but not in DLTE, the estimate of ρ is significant in DLTE. Table 3(b) indicates that DL can capture the true correlation between ξ_t and ε_t even when TE is the true model. While the estimate of ρ is significant in the incorrect DL but not in DLTE, the estimate of γ is significant in DLTE. Thus, Tables 3(a) and 3(b) show that the non-nested t-test based on the general DLTE model has power to distinguish between the DL and TE models by rejecting TE when DL is true and also rejecting TE when DL is true. Table 3(c) shows that the estimates of ρ and γ are significant in the incorrect DL and TE models, respectively, when DLTE is true.

Table 4 reports the rejection frequencies of the likelihood ratio (LR) tests of the respective null hypotheses under alternative true models. Table 4(a) and (b) correspond to the cases for $Corr(\varepsilon_t, \xi_t)$ to be -0.30 and -0.60, respectively. The LR test is based on $\log y_t^2$ since it does not require any additional computation. Table 4 indicates that the rejection frequency of the LR test is close to the nominal size of 5% when the DL and TE models are true, and that the LR test has sufficient power to reject the respective null hypotheses when the general DLTE model is true. Table 4(a) and (b)b also indicates that the rejection frequency of the LR test when the DLTE is true increase as the sample size increases. Comparing Table 4(a) with (b), the rejection frequency when the DLTE is true for $Corr(\varepsilon_t, \xi_t) = -0.30$ is relatively smaller than that for $Corr(\varepsilon_t, \xi_t) = -0.60$, as expected.

4.3 Bias correction

As stated above, the estimates of ρ have a small bias even for sample size $T = 5000$, with the estimates of σ_η being sensitive to the estimates of ρ. In order to cope with this problem, we propose an effective method for correcting the bias.

Based on the response surface methodology, we consider two regressions as follows:

$$\hat{\rho}-\rho=a_1+a_2\rho+\text{error},$$
$$\hat{\sigma}_\eta-\sigma_\eta=b_1+b_2\rho+b_3/T+b_4\sigma_\eta/T+\text{error},$$

where a_i (i = 1,2) and b_i (i = 1, …, 4) are coefficients. The first regression does not depend on sample sizes as the bias of ρ does not seem to be affected by the sample sizes that are typically used in empirical analysis. The first and second terms of the right-hand side of the second regression follow from the same idea, with the third and fourth terms diminishing as T increases. In order to estimate these models, we used twelve observations listed in Tables 1(a) and 2(a) for the DL and DLTE models. Table 5 shows the estimation results. All parameters except for the constant term for ρ are significant at the five percent level. The p-value of the constant term is 0.053. Several other specifications were considered, including terms such as ρ/T, but these were all insignificant. Based on these results, two natural bias-corrected estimators are given as follows:

$$\tilde{\rho}=\left(\hat{\rho}+0.014\right)/1.147,$$
$$\tilde{\sigma}_\eta=\frac{\hat{\sigma}_\eta-(0.005-0.039\tilde{\rho}-39.138/T)}{1+181.79/T}.$$

This method can be applied not only to the DL and DLTE models, but also to the TE model, with the latter obtained upon setting $\tilde{\rho}=0$.

We performed Monte Carlo experiments to investigate the performance of the bias-corrected MCL estimators. Table 6 shows the results for $T=2000$ and $Corr(\xi_t,\varepsilon_t)=-0.3$. Several other cases examined are qualitatively similar to the results in Table 6, and hence they are omitted. Compared with the original estimates in Table 1, Table 6 shows that the biases in σ_η and ρ are reduced dramatically in the DL and DLTE models, and that the coverage probabilities about ρ are much closer to 0.95 than those reported in Table 1. The estimates of the correlation coefficients between ξ_t and ε_t, and the estimates of $\sqrt{\text{Var}(\xi)}$, are all close to the true values of -0.3 and 0.260, respectively. More precise estimates would require further extensive

Monte Carlo experiments using response surface methodology. The results in Table 6, however, indicate that the simple bias-correction method suggested above can be quite effective.

5 Empirical Results

This section examines the MCL estimates of asymmetric behaviour in the three SV models for four sets of empirical data, namely Standard and Poor's 500 Composite Index (S&P), the Tokyo stock price index (TOPIX), the US Dollar/Australian Dollar exchange rate (USD/AUD), and the Japanese Yen/US dollar exchange rate (YEN/USD). The sample period for S&P is 1/6/1986 to 12/4/2000, giving T = 3723 observations, that for TOPIX is 1/4/1990 to 9/30/1999, giving T = 2403 observations, that for USD/AUD is 1/6/1986 to 12/4/2000, giving T = 3723 observations, and that for Yen/USD is 1/4/1990 to 12/28/1999, giving T = 2467 observations. We define returns y_t as 100 × {log P_t - log P_{t-1}} minus the sample mean, where P_t is the closing price on day t. Figure 1 shows the mean subtracted returns of all four series. There seems to be an outlying observation early in the sample for S&P, and there are clusterings of volatility in each series.

For stock returns, a negative correlation would be expected between the innovations in returns and volatility. Table 7 shows the MCL estimates for S&P. Although the estimates of ρ and γ are significant, and have the expected signs, at the five percent level in the DL and TE models, respectively, the non-nested LR test based on the general DLTE model rejects the TE null hypothesis but does not reject the DL null hypothesis. The result that DL is preferred to TE for S&P implies that threshold effects are inadequate for capturing the asymmetric structure of stock returns, whereas dynamic leverage is appropriate. Table 8 for TOPIX returns leads to a similar implication in that the individual estimates of ρ and γ are significant, and have the expected signs, at the five percent level in the DL and TE models, respectively, but the non-nested LR test based on the general DLTE model rejects TE in favour of DL. In both Tables 7 and 8, there is significant evidence of asymmetries in the general DLTE model.

The Monte Carlo results of Sandmann and Koopman (1998) show that their MCL method yields a larger bias than the Bayesian MCMC approach of Jacquier, Polson and Rossi (1994) when the unconditional variance of the time-varying log-volatility is relatively small, especially when the value of CV defined in the previous section is close to 0.01. When CV is equal to or greater than one, the MCL estimates are very close to the true value, as are the Bayesian MCMC estimates. For S&P the value of CV is 1.060, which will guarantee robustness of the results with respect to the coefficient of variation.

Tables 9 and 10 present the MCL estimates for USD/AUD and Yen/USD returns. In Table 9, the value of CV is 1.171, and the estimate of γ, though having the expected sign, is insignificant in the TE model, which corresponds to the result of Gallant, Hsieh and Tauchen (1991) for the British Pound/US Dollar rate. However, the estimate of ρ is significant in the DL model, indicating a negative correlation between the USD/AUD returns and volatility innovations. It is interesting to note that the estimates of both ρ and γ are significant in the general DLTE model, which leads to rejection of both the non-nested DL and TE models against each other. Thus, the non-nested LR test does not reject either model in favour of the other. The results for Yen/USD returns in Table 10 are similar to those for USD/AUD returns in Table 9, except that the estimate of γ is not significant in the general DLTE model, thereby leading to the rejection of the TE model in favour of the DL model. In both Tables 9 and 10, there is significant evidence of asymmetries in the general DLTE model.

The method of Sandmann and Koopman (1998) is used to obtain the smoothed volatility estimates of the DL and TE models for each of the four series, which are given in Figures 2 and 3, respectively. Apart from a spike early in the sample for S&P, which corresponds to an outlying observation, there do not appear to be extreme volatility estimates elsewhere in the series. Sample correlations between the pairs of volatility estimates for the four series in Figures 2(a)-2(d) and 3(a)-3(d) are very similar at 0.99223, 0.99053, 0.99464 and 0.99725, respectively. The differences in the smoothed volatility estimates in Figures 2 and 3 may be interpreted as follows. Let $Corr(\varepsilon_t, \xi_t)$ be denoted ρ1 and ρ2 for DL and TE, respectively, and the corresponding volatilities be denoted h1 and h2, respectively. In the empirical analysis, it is always the case that ρ1 < ρ2 < 0. There is a tendency for h1 > h2 after a large negative price shock. For example, there is a large negative shock around T = 500 in Figure 1(a), with Figures 2(a) and 3(a) showing that the corresponding volatilities satisfy h1 > h2. Similarly, there is a tendency for h1 < h2 immediately after a large positive shock. For example, there is a positive shock in Figure 1(c) around T = 3300, with Figures 2(c) and 3(c) showing that the corresponding volatilities satisfy h1 < h2.

6 Conclusion

In this paper we considered two methods for modelling asymmetries in Stochastic Volatility (SV) models, namely the threshold effects (TE) model based on the indicator function of Glosten, Jagannathan and Runkle (1992), and the dynamic leverage (DL) model based on the negative correlation between the innovations in returns and volatility. A general dynamic leverage threshold effects (DLTE) SV model, which could be interpreted as either an asymmetric model which exhibits both dynamic leverage and threshold effects; or as an artifact which is used solely for purposes of testing the non-nested DL and TE models against each other, was also analysed.

The three SV models were estimated by the Monte Carlo likelihood method proposed by Sandmann and Koopman (1998), and the finite sample properties of the estimator were investigated using numerical simulations. As the numerical simulation results show that the MCL estimator is biased, a simple method for correcting the bias was suggested and the performance of the bias-corrected MCL estimators was evaluated. Four financial time series were used to estimate the SV models, with asymmetric effects found to be statistically significant in each case. The empirical results for S&P 500, TOPIX and Yen/USD returns indicated that the dynamic leverage model dominated the threshold effects model for capturing asymmetric behaviour, while the results for USD/AUD returns showed that both the non-nested dynamic leverage and threshold effects models could be rejected against each other. For the four data series considered, the dynamic leverage SV model dominated the threshold effects SV model in capturing asymmetric behaviour. In all cases, there was significant evidence of asymmetries in the general DLTE model.

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Table 1: Simulations for the MCL Estimator Based on True Models for

$Corr(\varepsilon_t, \xi_t) = -0.30$

(a) Sample means and standard deviations of the MCL estimates

Model	T	ϕ	σ_η	α	ρ	γ	$\sqrt{\mathrm{Var}(\xi)}$	Corr.
DL	1000	0.941	0.282	-7.414	-0.377			
		(0.018)	(0.038)	(0.156)	(0.119)			
	2000	0.944	0.279	-7.412	-0.369			
		(0.012)	(0.028)	(0.123)	(0.092)			
	5000	0.946	0.276	-7.407	-0.367			
		(0.007)	(0.017)	(0.076)	(0.052)			
TE	1000	0.942	0.253	-7.410		0.209	0.275	-0.303
		(0.018)	(0.037)	(0.152)		(0.057)	(0.037)	(0.075)
	2000	0.944	0.252	-7.411		0.200	0.272	-0.293
		(0.012)	(0.027)	(0.121)		(0.042)	(0.027)	(0.057)
	5000	0.946	0.252	-7.407		0.198	0.271	-0.292
		(0.007)	(0.017)	(0.074)		(0.025)	(0.017)	(0.033)
DLTE	1000	0.941	0.272	-7.412	-0.172	0.112	0.282	-0.332
		(0.018)	(0.040)	(0.157)	(0.236)	(0.105)	(0.038)	(0.133)
	2000	0.944	0.269	-7.422	-0.187	0.100	0.277	-0.330
		(0.012)	(0.028)	(0.125)	(0.164)	(0.070)	(0.027)	(0.101)
	5000	0.946	0.266	-7.408	-0.182	0.103	0.274	-0.328
		(0.007)	(0.017)	(0.076)	(0.103)	(0.044)	(0.017)	(0.059)

Note: `Corr.' denotes the correlation coefficient between ξ_t and ε_t given in equation (4).

(b) 95% Coverage Probabilities

Model	T	ϕ	σ_η	α	ρ	γ
DL	1000	0.968	0.952	0.928	0.874	
		(0.008)	(0.010)	(0.012)	(0.015)	
	2000	0.956	0.918	0.900	0.842	
		(0.009)	(0.012)	(0.013)	(0.016)	
	5000	0.958	0.892	0.878	0.792	
		(0.009)	(0.014)	(0.015)	(0.018)	
TE	1000	0.970	0.974	0.926		0.956
		(0.008)	(0.007)	(0.012)		(0.009)
	2000	0.962	0.952	0.902		0.942
		(0.009)	(0.010)	(0.013)		(0.010)
	5000	0.950	0.942	0.888		0.960
		(0.010)	(0.010)	(0.014)		(0.009)
DLTE	1000	0.968	0.964	0.930	0.888	0.936
		(0.008)	(0.008)	(0.011)	(0.014)	(0.011)
	2000	0.972	0.958	0.906	0.904	0.948
		(0.007)	(0.009)	(0.013)	(0.013)	(0.010)
	5000	0.956	0.932	0.882	0.922	0.950
		(0.009)	(0.011)	(0.014)	(0.012)	(0.010)

Note: The coverage probability is the fraction of times that the true parameter values falls within the confidence interval. Standard errors are given in parentheses and are computed from the Bernoulli formula given on page 11.

Table 2: Simulations for the MCL Estimator Based on True Models for

$$Corr\left(\varepsilon_t, \xi_t\right) = -0.60$$

(a) Sample means and standard deviations of the MCL estimates

Model	T	ϕ	σ_η	α	ρ	γ	$\sqrt{\text{Var}(\xi)}$	Corr.
DL	1000	0.943	0.296	-7.405	-0.704			
		(0.015)	(0.037)	(0.137)	(0.081)			
	2000	0.945	0.293	-7.400	-0.699			
		(0.010)	(0.027)	(0.099)	(0.059)			
	5000	0.947	0.291	-7.399	-0.696			
		(0.006)	(0.016)	(0.066)	(0.036)			
TE	1000	0.945	0.181	-7.398		0.403	0.272	-0.593
		(0.013)	(0.031)	(0.114)		(0.052)	(0.032)	(0.050)
	2000	0.947	0.182	-7.395		0.395	0.269	-0.586
		(0.008)	(0.022)	(0.083)		(0.034)	(0.022)	(0.036)
	5000	0.947	0.183	-7.395		0.393	0.269	-0.583
		(0.005)	(0.014)	(0.055)		(0.023)	(0.014)	(0.022)
DLTE	1000	0.944	0.242	-7.404	-0.346	0.238	0.284	-0.643
		(0.014)	(0.039)	(0.137)	(0.231)	(0.098)	(0.035)	(0.114)
	2000	0.946	0.240	-7.402	-0.359	0.230	0.279	-0.644
		(0.009)	(0.027)	(0.101)	(0.159)	(0.065)	(0.024)	(0.084)
	5000	0.947	0.239	-7.401	-0.360	0.229	0.277	-0.642
		(0.006)	(0.017)	(0.067)	(0.098)	(0.041)	(0.015)	(0.051)

Note: `Corr.' denotes the correlation coefficient between ξ_t and ε_t given in equation (4).

(b) 95% Coverage Probabilities

Model	T	ϕ	σ_η	α	ρ	γ
DL	1000	0.968	0.952	0.928	0.874	
		(0.008)	(0.010)	(0.012)	(0.015)	
	2000	0.956	0.918	0.900	0.842	
		(0.009)	(0.012)	(0.013)	(0.016)	
	5000	0.958	0.892	0.878	0.792	
		(0.009)	(0.014)	(0.015)	(0.018)	
TE	1000	0.964	0.964	0.942		0.962
		(0.008)	(0.008)	(0.010)		(0.009)
	2000	0.954	0.956	0.916		0.958
		(0.009)	(0.009)	(0.012)		(0.009)
	5000	0.952	0.926	0.888		0.950
		(0.010)	(0.012)	(0.014)		(0.010)
DLTE	1000	0.962	0.962	0.932	0.874	0.922
		(0.009)	(0.009)	(0.011)	(0.015)	(0.012)
	2000	0.956	0.942	0.910	0.884	0.948
		(0.009)	(0.010)	(0.013)	(0.014)	(0.010)
	5000	0.954	0.892	0.886	0.876	0.950
		(0.009)	(0.014)	(0.014)	(0.015)	(0.010)

Note: The coverage probability is the fraction of times that the true parameter values falls within the confidence interval. Standard errors are given in parentheses and are computed from the Bernoulli formula given on page 11.

Table 3: Simulations for the MCL Estimator Based on True Models

(a) True model: DL

Model	ϕ	σ_η	α	ρ	γ	$\sqrt{\mathrm{Var}(\xi)}$	Corr.
TE	0.944	0.265	-7.415		0.129	0.273	-0.189
	(0.013)	(0.028)	(0.124)		(0.042)	(0.028)	(0.060)
DLTE	0.944	0.280	-7.412	-0.366	-0.002	0.281	-0.366
	(0.013)	(0.031)	(0.124)	(0.145)	(0.069)	(0.029)	(0.094)

Note: `Corr.' denotes the correlation coefficient between ξ_t and ε_t given in the text.

(b) True model: TE

Model	ϕ	σ_η	α	ρ	γ	$\sqrt{\mathrm{Var}(\xi)}$	Corr.
DL	0.943	0.280	-7.409	-0.375			
	(0.012)	(0.028)	(0.124)	(0.090)			
DLTE	0.944	0.255	-7.411	-0.006	0.198	0.274	-0.295
	(0.012)	(0.027)	(0.121)	(0.173)	(0.069)	(0.027)	(0.105)

Note: `Corr.' denotes the correlation coefficient between ξ_t and ε_t given in the text.

(c) True model: DLTE

Model	ϕ	σ_η	α	ρ	γ	$\sqrt{\mathrm{Var}(\xi)}$	Corr.
DL	0.944	0.283	-7.411	-0.367			
	(0.012)	(0.028)	(0.126)	(0.091)			
TE	0.944	0.263	-7.414		0.165	0.276	-0.239
	(0.012)	(0.027)	(0.124)		(0.042)	(0.027)	(0.058)

Note: `Corr.' denotes the correlation coefficient between ξ_t and ε_t given in the text.

Table 4: Rejection Frequencies (RF) of Likelihood Ratio Tests

(a) Rejection Frequencies for $Corr(\varepsilon_t, \xi_t) = -0.30$

Null hypothesis	True model	RF $T = 1000$	$T = 2000$	$T = 5000$
H_0: γ=0	DL	0.06	0.05	0.05
	DLTE	0.21	0.31	0.65
H_0: ρ=0	TE	0.06	0.06	0.04
	DLTE	0.13	0.19	0.40

(b) Rejection Frequencies for $Corr(\varepsilon_t, \xi_t) = -0.60$

Null hypothesis	True model	RF $T = 1000$	$T = 2000$	$T = 5000$
H_0: γ=0	DL	0.07	0.05	0.04
	DLTE	0.71	0.94	1.00
H_0: ρ=0	TE	0.06	0.05	0.04
	DLTE	0.30	0.56	0.91

Note: The likelihood is based on the distribution of $\log y_t^2$. The nominal significance level is 5% and the corresponding value of the cumulative distribution function of χ^2 (1) is 3.84.

Table 5: Response Surface Regressions for Biases

Dependent variable	Const.	ρ	$1/T$	σ_η/T	S.E.	R^2
$\hat{\rho}-\rho$	-0.014	0.147			0.0099	0.878
	(-2.19)	(8.47)				
	[0.053]	[0.000]				
$\hat{\sigma}_\eta-\sigma_\eta$	0.005	-0.039	-39.138	181.79	0.0018	0.960
	(2.97)	(-11.69)	(-2.80)	(3.26)		
	[0.018]	[0.000]	[0.023]	[0.011]		

Note: t-values are given in parentheses and p-values based on the t-distribution are given in brackets.

Table 6: Simulations for the Bias Corrected MCL Estimators Based on True Models for

$Corr\left(\varepsilon_t,\xi_t\right)=-0.30$ and $T=2000$

	Sample means and standard deviations of the bias corrected MCL estimates				95% Coverage Probabilities	
Model	σ_η	ρ	$\sqrt{\mathrm{Var}(\xi)}$	Corr.	σ_η	ρ
DL	0.258	-0.310			0.964	0.968
	(0.025)	(0.073)				
TE	0.245		0.266	-0.302	0.936	
	(0.024)		(0.024)	(0.053)		
DLTE	0.254	-0.147	0.263	-0.301	0.968	0.964
	(0.025)	(0.144)	(0.026)	(0.081)		

Note: `Corr.' denotes the correlation coefficient between ξ_t and ε_t given in equation (4). The coverage probability is the fraction of times that the true parameter values falls within the confidence interval. Standard errors are given in parentheses and are computed from the Bernoulli formula given on page 11.

Table 7: MCL Estimates for S&P 500 Returns

Model	ϕ	σ_η	α	ρ	γ	LogLike	Corr.
SV	0.9634	0.2297	-0.5103			-8306.5	0
	(0.0083)	(0.0248)	(0.1060)				
DL	0.9527	0.2753	-0.5050	-0.4911		-8284.0	-0.49
	(0.0096)	(0.0279)	(0.0893)	(0.0599)			(0.06)
TE	0.9587	0.2415	-0.5527		0.1565	-8294.0	-0.25
	(0.0090)	(0.0258)	(0.0993)		(0.0329)		(0.04)
DLTE	0.9530	0.2726	-0.5085	-0.4728	0.0136	-8284.0	-0.49
	(0.0096)	(0.0291)	(0.0903)	(0.0871)	(0.0452)		(0.06)

Note: Standard errors are given in parentheses.

Table 8: MCL Estimates for TOPIX Returns

Model	ϕ	σ_η	α	ρ	γ	LogLike	Corr.
SV	0.9492	0.2741	0.0334			-5181.2	0
	(0.0111)	(0.0281)	(0.1143)				
DL	0.9608	0.2643	0.0065	-0.5976		-5151.8	-0.60
	(0.0092)	(0.0260)	(0.1186)	(0.0623)			(0.06)
TE	0.9610	0.2204	-0.0280		0.2444	-5156.0	-0.39
	(0.0087)	(0.0245)	(0.1190)		(0.0333)		(0.05)
DLTE	0.9618	0.2409	-0.0093	-0.4800	0.0829	-5150.8	-0.60
	(0.0087)	(0.0266)	(0.6516)	(0.1061)	(0.0486)		(0.07)

Note: Standard errors are given in parentheses.

Table 9: MCL Estimates for USD/AUD Returns

Model	ϕ	σ_η	α	ρ	γ	LogLike	Corr.
SV	0.8651	0.4417	-1.3579			-10318	0
	(0.0232)	(0.0421)	(0.0579)				
DL	0.8522	0.4623	-1.3637	-0.2290		-10310	-0.23
	(0.0238)	(0.0423)	(0.0596)	(0.0533)			(0.05)
TE	0.8607	0.4452	-1.3332		0.0635	-10317	-0.06
	(0.0238)	(0.0423)	(0.0596)		(0.0387)		(0.04)
DLTE	0.8515	0.4813	-1.4123	-0.3433	-0.1204	-10307	-0.24
	(0.0241)	(0.0449)	(0.0587)	(0.0715)	(0.0584)		(0.05)

Note: Standard errors are given in parentheses.

Table 10: MCL Estimates for YEN/USD Returns

Model	ϕ	σ_η	α	ρ	γ	LogLike	Corr.
SV	0.9412	0.2619	-0.9823			-5300.1	0
	(0.0153)	(0.0351)	(0.0953)				
DL	0.9320	0.2797	-0.9705	-0.2351		-5296.2	-0.24
	(0.0173)	(0.0379)	(0.0875)	(0.0800)			(0.08)
TE	0.9370	0.2681	-0.9705		0.0510	-5299.1	-0.08
	(0.0166)	(0.0367)	(0.0923)		(0.0373)		(0.05)
DLTE	0.9327	0.2899	-0.9801	-0.3904	-0.0928	-5295.1	-0.26
	(0.0168)	(0.0395)	(0.0874)	(0.1203)	(0.0624)		(0.07)

Note: Standard errors are given in parentheses.

Figure 1: Mean Subtracted Returns

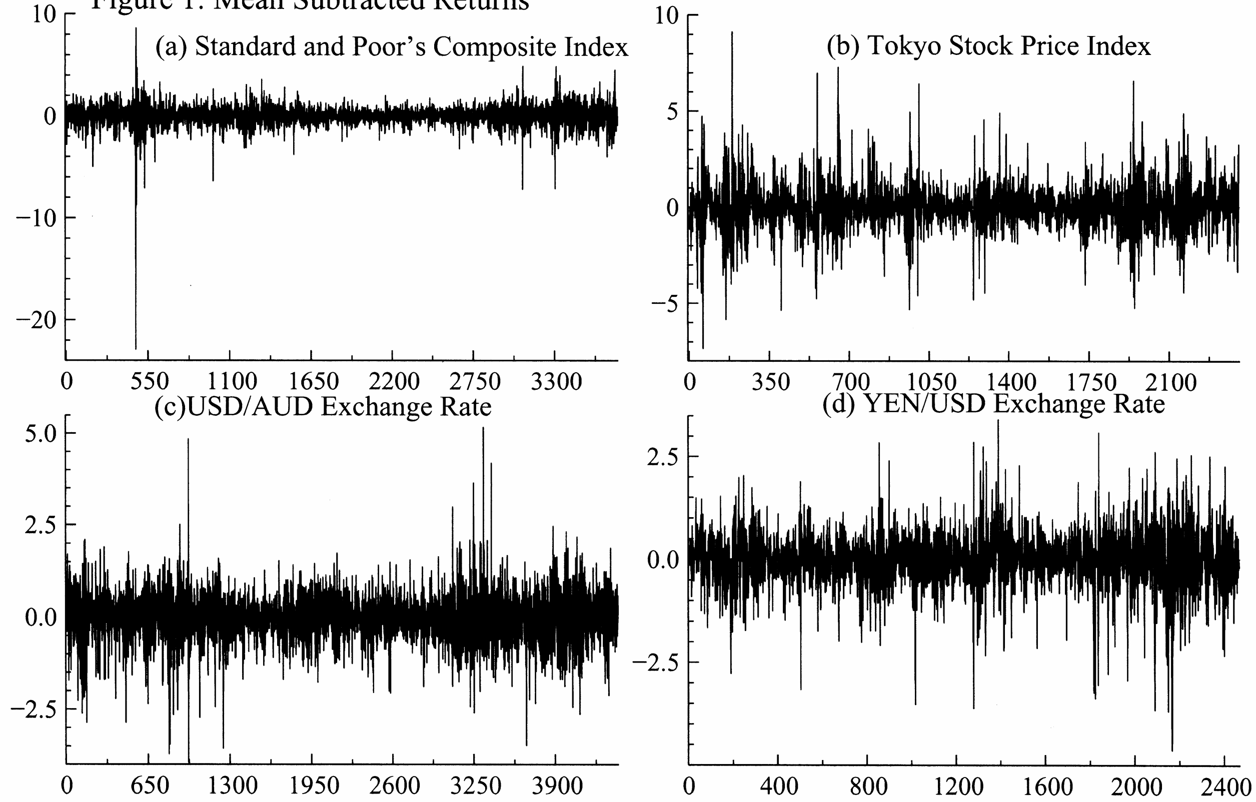

Figure 2: Volatility Estimates of DL

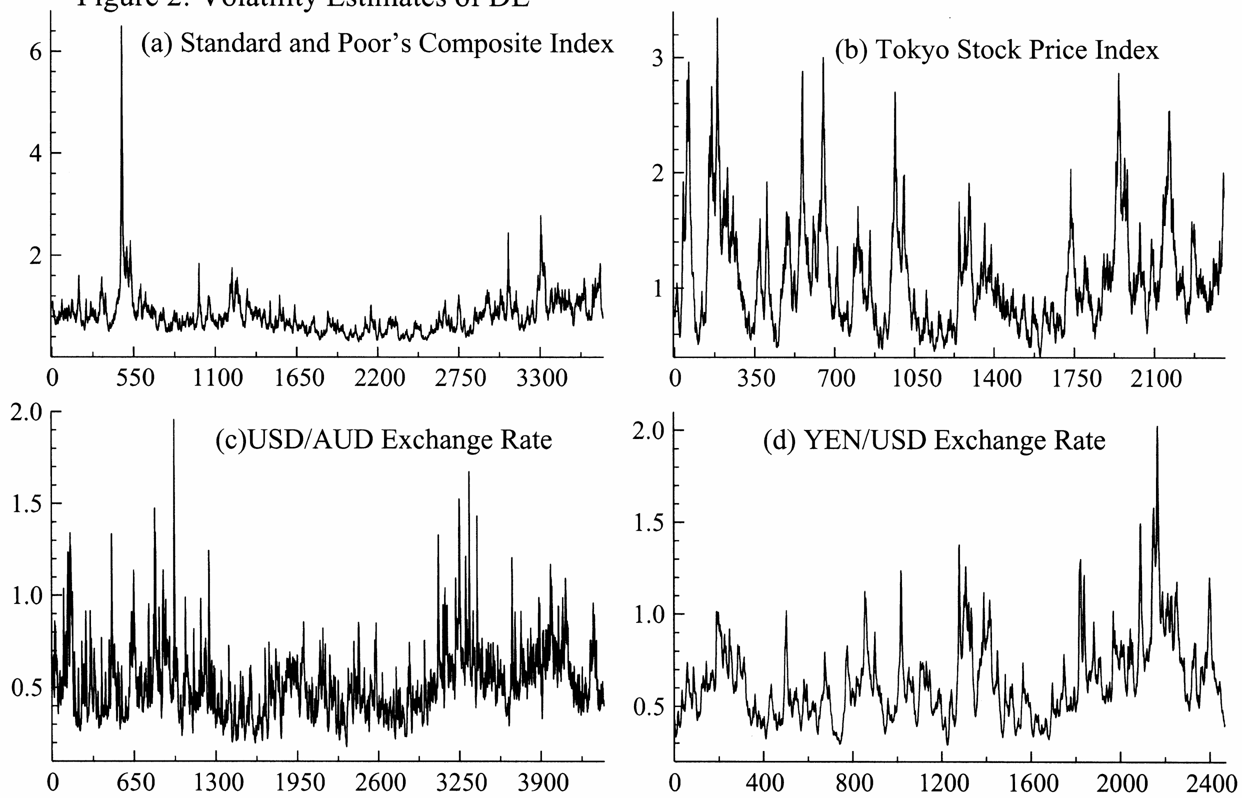

Figure 3: Volatility Estimates of TE