Forecasting jumps in conditional volatility The GARCH-IE model

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Introduction

The Autoregressive Conditional Heteoskedasticity (ARCH) model (Engle, 1982 and Bollerslev, 1986) is often used to describe and forecast conditional volatility in asset returns.

This model is designed to predict the conditional variance of the second observation in a volatile period, but not the first.

We propose the GARCH-IE (IE for innovation effects) model, which adds a component to the GARCH equation such that it becomes possible to forecast a sudden jump in volatility. Hence, the first observation in a sequence of large returns might be predicted.

Main idea: the IE component is a positive-valued outcome of a threshold (censored) regression. This regression contains explanatory variables and an error term.

In a sense, our model is in between the standard GARCH model and the stochastic volatility (Taylor, 1986) model, where in this last model an error term is always included in the conditional volatility equation.

GARCH

Consider a stock return y_t, and

$$y_t = \beta_1 + \beta_2 x_{t-1} + \epsilon_t, \qquad (1)$$

where x_{t-1} concerns explanatory variables for these returns. The GARCH model assumes that

$$\epsilon_t = \eta_t \sqrt{h_t} \qquad (2)$$

and

$$h_t = \omega + \alpha(y_{t-1} - \beta_1 - \beta_2 x_{t-2})^2 + \beta h_{t-1} \qquad (3)$$

Note: no additional error term in (3) and also AR(1) structure.

Innovation effects

Franses and Paap (JAE 2002) assume latent shocks to emerge from a censored (threshold) regression model, where linear combinations of lagged explanatory variables lead to positive shocks, while otherwise shocks have zero effect.

This feature reads as adding v_t with

$$v_t = \left\{ \begin{array}{ll} \theta_0 + \theta_1 x_{t-1} + u_t & \text{if } \theta_0 + \theta_1 x_{t-1} > -u_t \\ 0 & \text{if } \theta_0 + \theta_1 x_{t-1} \leq -u_t \end{array} \right\} \quad (4)$$

to a series y_t, with $u_t \sim N(0, \sigma_u^2)$. As only positive values of v_t are added, the model contains an explicit description of exogenous innovation effects.

Explanatory variables

Franses, van der Leij and Paap (JAE 2002) recommend to consider the moving average of stock prices over k days, that is,

$$\overline{z}_{k,t} = \frac{1}{k} \sum_{i=t-k+1}^{t} z_i.$$

The ratio of the moving average of stock prices of k_1 days over that of k_2 days is defined as

$$x_t = \frac{\overline{z}_{k_1,t} - \overline{z}_{k_2,t}}{\overline{z}_{k_2,t}}, \qquad (5)$$

where usually $0 < k_1 < k_2$. Typically, in practice one takes k_1 to be equal to 1, 2 or 5, and k_2 equal to 50, 100 or 200.

In our empirical work below we use the ratio of 1-day and 50-days moving average.

GARCH-IE

The GARCH-IE model is given by

$$y_t = \beta_1 + \beta_2 x_{t-1} + \epsilon_t, \tag{6}$$

where

$$\epsilon_t = \eta_t \sqrt{h_t}, \tag{7}$$

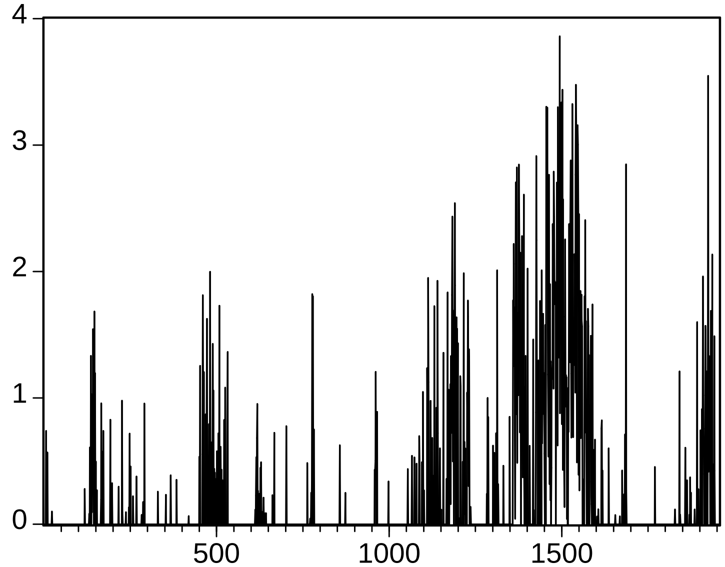
with

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta(h_{t-1} - v_{t-1}) + v_t, \tag{8}$$

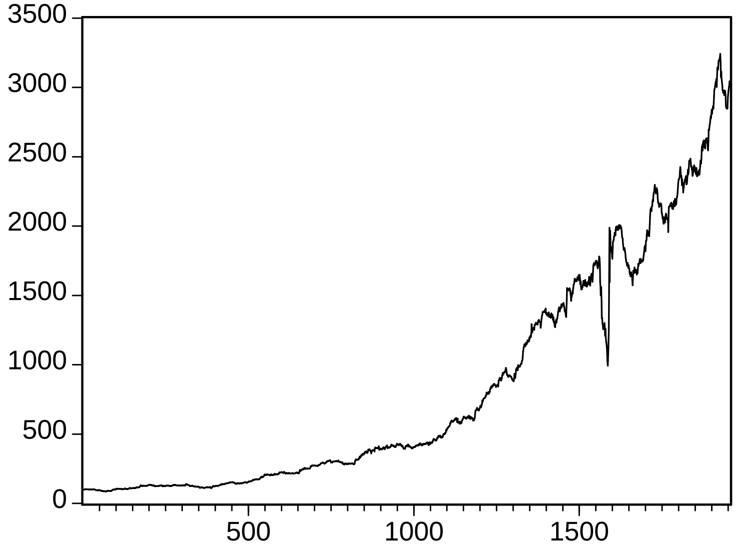
and

$$v_t = \begin{cases} \theta_1 + \theta_2 x_{t-1} + u_t & \text{if } \theta_1 + \theta_2 x_{t-1} > -u_t \\ 0 & \text{if } \theta_1 + \theta_2 x_{t-1} \leq -u_t, \end{cases} \tag{9}$$

where $\eta_t \sim N(0,1)$ and $u_t \sim N(0, \sigma_u^2)$. We assume that η_t is mutually uncorrelated with u_t.



Simulated additional error term v_t from an ARCH-IE model



Simulated stock market index of an ARCH-IE model

Simulation results for an ARCH-IE and an ARCH model, when averaged over 1000 replications for a sample size of 1000.

Statistic	ARCH-IE	ARCH
Mean	0.1019	0.1021
Variance	2.4712	1.8800
Skewness	-0.0308	-0.0112
Kurtosis	6.9041	5.8349

Inference

The log-likelihood is

$$\ell(Y_T|X_{T-1};\beta,\gamma,\theta) = \sum_{t=1}^{T} \ln(f(y_t|Y_{t-1},x_{t-1};\beta,\gamma,\theta)), \tag{10}$$

where β comprises β_1, β_2, γ reflects ω, α and β and θ summarizes θ_1, θ_2.

As v_t has a censored normal distribution, the density function of y_t given its past and x_{t-1} can be written as

$$\begin{aligned} & f(y_t|Y_{t-1},x_{t-1};y_t) = \\ & \Pr[v_t = 0|x_{t-1};\beta,\gamma,\theta] f(y_t|Y_{t-1},x_{t-1},v_t;\beta,\gamma,\theta)|_{v_t=0} \\ & \quad + \int_{-x_t'\theta}^{\infty} \frac{1}{\sigma_u}\phi\left(\frac{u_t}{\sigma_u}\right) f(y_t|Y_{t-1},x_{t-1},v_t)|_{v_t>0} du_t \end{aligned} \tag{11}$$

Nine stock markets

Daily data for years 1990-1999. Forecasting sample is 2000.

We use AIC for within-sample evaluation and the log-likelihood for out-of-sample comparison.

Next, we consider those large absolute returns which were preceded by 20 small returns.

Expected sign of θ_2 is negative. When the recent index is below a longer-term average, we might expect sudden volatility (leverage effect).

GARCH(1,1) and GARCH-IE(1,1) model for daily returns on the Dow Jones from 1/1/1990 to 12/31/1999

	α	β	θ_1	θ_2	σ_u^2
GARCH	**0.049**	**0.942**			
	(0.009)	**(0.011)**			
GARCH-IE	**0.013**	**0.979**	**-1.772**	**-0.387**	**4.267**
	(0.004)	**(0.006)**	**(0.482)**	**(0.078)**	**(1.280)**

Within-sample performance in forecasting absolute returns (AIC)

	GARCH-IE	GARCH
DOWJONES	*2.433*	2.497
NASDAQ	*2.812*	2.856
SP500	*2.389*	2.451
NIKKEI	*3.410*	3.486
FTSE	*2.540*	2.565
DAX	*3.031*	3.121
CAC	*3.132*	3.166
AEX	*2.715*	2.753
HANGSENG	*3.542*	3.601

Out-of-sample performance in forecasting absolute returns for 2000 (LL)

	GARCH-IE	GARCH
DOWJONES	*-416.950*	-418.316
NASDAQ	*-630.853*	-633.571
SP500	*-432.559*	-438.385
NIKKEI	*-437.319*	-443.926
FTSE	-399.702	*-397.876*
DAX	-462.203	*-460.682*
CAC	-460.340	*-459.394*
AEX	*-394.572*	-396.470
HANGSENG	*-517.670*	-523.109

Forecasting performance of GARCH-IE compared with the GARCH model for observations with 20 days of low returns before: Fraction of times that GARCH-IE model gives more accurate forecast

	Within-sample	Out-of-sample
DOWJONES	9 (11)	3 (6)
NASDAQ	1 (5)	0 (0)
SP500	8 (12)	4 (5)
NIKKEI	4 (7)	1 (5)
FTSE	5 (10)	2 (8)
DAX	2 (6)	1 (1)
CAC	4 (5)	2 (4)
AEX	2 (4)	3 (5)
HANGSENG	0 (3)	1 (1)

Conclusion

First attempt to sensibly include additional error term in GARCH model

Inference is not difficult.

Illustrations show an improved within-sample fit, but not always good out-of-sample forecasts.

Perhaps we should look for better x_{t-1} variables.