On Portfolio Optimization: How Do We Benefit from High-Frequency Data?

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ABSTRACT

In this paper, I consider the problem faced by a professional investment manager who wants to track the return of the S&P 500 index with 30 DJIA stocks. The manager constructs many covariance matrix estimators, based on daily returns and high-frequency returns, to form his optimal portfolio. Although prior research has documented that realized volatility based on intraday returns is more precise than daily return constructed volatility, the manager will not switch from daily to intraday returns to estimate the conditional covariance matrix if he rebalances his portfolio monthly and has past 12 months of data to use. He will switch to intraday returns only when his estimation horizon is shorter than 6 months or he rebalances his portfolio daily.

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1. Introduction

Volatility is central for many asset pricing, asset allocation and risk management applications.[1] Thus, following the work of Merton (1980) and Nelson (1992), there is increasing interest among financial economists on the high precision with which volatility can be estimated under the diffusion assumption which is often invoked in theoretical work. The basic insight is that precise estimation of volatility can be obtained from an arbitrarily short span of data provided that returns are sampled sufficiently frequently. In contrast, precise estimation of the drift generally requires long spans of data, regardless of the frequency with which returns are sampled. As a result, there is an apparent tendency toward the use of high-frequency data in the volatility measurement in the literature.

Historically, the choice of the sampling frequency was decided by the data availability. In the early days, monthly returns were used. Later, daily data became available. In recent years, the ever lower costs of data recording and storage have made time-stamped observations on all quotes and transactions, named ultra-high frequency data by Engle (2000), available in many important financial markets. With the advent of high-frequency data, significant progress has been made in the volatility measurement. Among this, a novel model-free approach called realized volatility has been proposed in Andersen and Bollerslev (1998) which uses high-frequency returns. Basically, this approach is to estimate the ex-post realized volatility by summing the squared intraday high-frequency returns. They argue that these realized volatility estimates should be free from measurement error in theory as the sampling frequency of the returns approaches infinity. In practice, to mitigate the contamination by market microstructure frictions which include price discreteness, infrequent trading, and bid-ask bounce, a five-minute return horizon is chosen as the effective "continuous time record" for high liquid assets as suggested in Andersen, Bollerslev, Diebold, and Labys (2000) and Andreou and Ghysels (2001). A number of papers implement this approach and use five-minute returns to estimate the realized volatility and examine its properties. Among them, Andersen, Bollerslev, Diebold, and Labys (2000, 2001) examine foreign exchange rates, Andersen, Bollerslev, Diebold, and Ebens (2001, henceforth, ABDE) examine DJIA stocks, Ebens(2000) examines the DJIA index, and Areal and Taylor (2002) examine FTSE-100 index futures. These findings provide support for the realized volatility approach in the statistical sense.

Despite the availability of intraday data, it is still very common to use daily returns to estimate volatility, especially unconditional variance. A natural question to ask is whether there is benefit of using intraday data to estimate volatility in practice. Bai, Russell, and Tiao (2001, BRT, hereafter) provide a measure of the usefulness of high frequency data in estimating volatility. They examine the precision of unconditional variance estimates that use high-frequency data

[1] Here I use the term "volatility" to refer to any element of covariance matrix of asset returns, thus in this paper, the use of volatility is to denote variances and covariances rather than standard deviations.

and derive an analytical expression for it as a function of the prominent high-frequency data characteristics including leptokurtosis, autocorrelation in the returns, deterministic patterns and volatility clustering in intraday variance. Their simulation and empirical results indicate that once these features are accounted for, the benefit of using high frequency data to estimate daily volatility is much smaller than the one under ideal situation in which the asset prices follow the geometric Brownian motion. The simple sum of squares estimator using intraday returns can be much less efficient than those from daily returns when high-frequency returns have significant negative lag one autocorrelation and strong leptokurtosis.

High-frequency data are more useful to estimate time-varying volatility. Fleming, Kirby, and Ostdiek (2003, henceforth, FKO) evaluate the economic benefits of the realized volatility approach in the context of investment decisions. They consider a risk-averse investor who uses conditional mean-variance analysis to allocate funds across four asset classes: stocks, bonds, gold, and cash, and rebalances his portfolio daily. Their results indicate that the value of switching from daily to intradaily returns to estimate the conditional covariance matrix can be substantial. The investor would be willing to pay 50 to 200 basis points per year to capture the incremental gains generated by the realized-volatility-based estimator. Moreover, volatility timing at the daily levels leads to performance gains over longer horizons. However, in this latter case, FKO (2003) only compare realized-volatility-based volatility-timing strategy with the ex ante efficient static portfolios for a range of longer horizons, thus it is still unclear whether there is benefit of switching from daily to intradaily returns to measure the conditional covariance matrix over longer horizons.

In this paper, I consider a professional investment manager whose objective is to track the S&P 500 index with 30 stocks in the Dow Jones Industrial Average (DJIA).[2] The minimum tracking error volatility portfolio can be found by expressing every stock's return in excess of the return on the benchmark, and solving for the portfolio with the lowest variance of excess returns. Previous research (Jagannathan and Ma, 2003) shows that once the nonnegativity constraint is imposed, portfolios constructed using sample covariance matrices perform as well as portfolios constructed using covariance matrices estimated using factor models and shrinkage methods, whether they use monthly returns or daily returns. Also, the sample covariance matrix of daily returns performs better than the monthly return covariance matrix. To extend the research with higher frequency data, I construct large covariance matrix estimates based on intraday returns and daily returns respectively, and evaluate whether there is benefit of using high-frequency data. The manager first uses past 12 months of data, daily or high-frequency data to estimate the covariance matrix every month and hold the portfolio for one month and redo the estimation once every month. For this purpose, I construct estimates of the conditional covariance matrix based on daily returns and intraday returns, which include the sample

[2] I focus on the 30 DJIA stocks for their importance and computational tractability. It should be easily extended to a limited number of other liquid stocks.

covariance matrix adjusted or unadjusted for microstructure effects, the optimal shrinkage estimator of Ledoit and Wolf (2001), and the rolling estimators suggested by Foster and Nelson (1996) and Andreou and Ghysels (2002). Consistent with FKO (2003) and BRT (2001), I find that adding overnight returns and first-order correlation in the measure of the realized volatility will greatly reduce the tracking errors. But unlike FKO (2003), there is no evidence that the tracking errors from realized-volatility-based estimator are smaller than those from daily returns, even when the simplest sample covariance matrix of daily returns is used. They are almost the same after the former estimator adopts the optimal decay rate and the bias corrections using both daily and intraday data.

If the manager rebalances his portfolio daily, things are different. I find that there are substantial performance gains with switching to the realized-volatility based estimator. Even when the manager still rebalances monthly, if he only utilizes less than 6 months of previous daily or high-frequency data to estimate the covariance matrix, there are significant incremental gains generated by the realized-volatility-based estimator. In this latter case, I find that standard deviation of the tracking errors from daily-return based covariance matrix estimator increases quickly with the decrease of the sample size, while those from realized-volatility-based estimator are very stable, whether 6 months' high frequency data or only one month's high-frequency data are used. The out-of-sample performances from all of them are much better than from previous one year's high-frequency data. This indicates that longer horizon of high-frequency data may not be necessary and there is a tradeoff between efficiency gains from the larger sample size and biases from market microstructure frictions. The results with the minimum variance portfolio from the 30 stocks in DJIA are also consistent with above findings. By measuring high-dimensional covariance matrix with realized volatility approach, this paper contributes to the current literature by answering under which circumstances, there are potential performance gains with high-frequency data and how to utilize them, thus holds promise for the further development of better decision making in practical risk management and portfolio allocation.

The rest of the paper is organized as follows. Section 2 proposes the problem faced by a professional investment manager. Section 3 describes features of daily and high-frequency data and develops methodology for constructing the conditional covariance matrix estimates. Section 4 describes the data and presents the empirical results. Section 5 examines the robustness of the above results by constructing the minimum variance portfolio with the 30 DJIA stocks and doing the same empirical analysis as before. I also demonstrate with a Monte Carlo simulation exercise that high-frequency data indeed help when the asset prices follow the geometric Brownian motion with the same covariance matrix as used in the empirical analysis. Finally, I discuss the implications of my findings for future research and conclude.

2. Portfolio Optimization Problem

There is a lot of evidence that superior returns to investment performance are elusive. Numerous studies indicate that on average professional investment managers do not outperform passive benchmarks. In turn, the methods of optimally tracking a benchmark, especially when full replication of the benchmark is not desired or not practical, have received attention in academics and practitioners. In practice, managers are often evaluated relative to some benchmark, therefore it has become one of their objectives to minimize the portfolio's tracking error volatility, or the volatility of the difference between a portfolio's return and the return on the benchmark.[3]

Following Chan, Karceski and Lakonishok (1999), I assume a professional investment manager who is trying to track the return of the S&P 500 index. Suppose the manager can allocate funds across the 30 Dow Jones stocks and he uses conditional mean-variance analysis with past one year of data to make his allocation decisions and rebalances his portfolio monthly, that is, he constructs his minimum variance portfolio using returns in excess of the benchmark.[4] Every month, he uses past 12 months of data of these DJIA stocks and the S&P 500 index, daily or high-frequency returns, for estimating the covariance matrix, and forms his portfolio accordingly.[5]

Let R_t denote the 1×30 vector of excess returns formed by substracting the S&P 500 index return from the individual stock returns on day t. Suppose there are 21 trading days in one month, then at the beginning date t of every month m (that is, $t = 21 * (m-1) + 1$), the manager will use the return series from $[R'_{t-252}, R'_{t-251}, ..., R'_{t-1}]$ (previous one year's data) to estimate covariance and generate a covariance matrix forecast Σ_m for month m. After that, to construct his minimum tracking error variance portfolio in month m, the manager simply applies the following weights with the 30 DJIA stocks

$$w_m = \frac{\Sigma_m^{-1}\mathbf{1}}{\mathbf{1}'\Sigma_m^{-1}\mathbf{1}} \tag{1}$$

where $\mathbf{1}$ is a 30×1 vector of ones.[6] The portfolio is held for one month and

[3] Estimates of the mean returns are very noisy. In fact, Jagannathan and Ma (2003) find that tangency portfolios even do not perform as well as the minimum variance portfolios in terms of out-of-sample Sharpe Ratio. In practice, professional investment managers are often evaluated relative to some benchmarks, thus it is necessary to use a subset of available high liquid stocks to construct the minimum tracking error portfolios.

[4] In section 4, I also consider another case, i.e, the manager rebalances his portfolio daily. Other things are the same; he uses past 12 months of data and the same estimation methods to construct his minimum tracking error portfolio. Similar problems emerge when he only has less than 6 months of past data to use. Section 5 explores the problem when the manager wants to form the time-varying minimum variance portfolio using the 30 DJIA stocks.

[5] Here the use of a rolling window allows for time variations in variances as in French, Schwert and Stambaugh (1987) and time variations in covariances as in Bollerslev, Engle and Wooldridge (1988). A considerable literature has shown that expected returns are notoriously difficult to predict while return variances and covariances are much easier to estimate from historical data. For this reason, when the manager mimimizes the tracking error volatility, he focuses on forecasting the second moments rather than expected returns.

[6] Note that constructing the minimum tracking error variance portfolio is the same as

its realized return is recorded. This procedure begins from the point where the manager has enough data to estimate the covariance matrices and is repeated monthly at the beginning of every following month. It is easy to see that the optimal portfolio weights vary through time as Σ_m changes. Thus for each different estimation method, the manager has the ex post performance of its minimum tracking error portfolio, which is rebalanced monthly. He then uses the ex post standard deviation of its minimum tracking error volatility portfolio as a measure of how precise a covariance estimator is.

3. Econometric Methodology

In this section, I provide a number of covariance matrix estimators based on daily data or high-frequency data, which will be used in the following empirical analysis. These estimators can be used to generate the covariance matrix forecasts based on the past 12 months of data (the estimation period). I first construct covariance matrix estimators and forecasts with daily return data. These include the daily return sample covariance matrix, its variants that incorporate the corrections for microstructure effects suggested by Scholes and Williams (1977), Dimson (1979), Cohen, Hawawini, Maier, Schartz and Whitcome (1983, henceforth, CHMSW), Jagannathan and Ma (2003), etc. and the optimal shrinkage estimator proposed by Ledoit and Wolf (2003). Following Foster and Nelson (1996) and FKO (2003), I also construct a rolling covariance matrix estimator. Then I introduce notation, and provide different methods to estimate the conditional covariance matrix using high-frequency data taking into account of several features of intradaily returns. As is documented in BRT (2001), high-frequency financial data have deterministic intraday volatility, serial correlation, fat tails and volatility clustering, which do not conform well with the geometric Brownian motion assumption. To deal with these features, besides sample covariance matrix estimators, I also develop several rolling estimators based on lagged returns, which follow the work of Foster and Nelson (1996), Andreou and Ghysels (2001), Areal and Taylor (2002), and FKO (2003). FKO (2003) argue that these estimators avoid complicated parametric assumptions and nest a variety of GARCH and stochastic volatility models as special cases, thus are computationally efficient and provide a natural way to evaluate the performance gains using realized volatility.

3.1 Covariance Matrix Estimators Based on Daily Returns

3.1.1 Sample Covariance Matrix Estimator

The starting point for forecasting return covariances is given by the sample covariance matrix:

constructing the minimum variance portfolio using returns in excess of the benchmark subject to the restriction that the portfolio weights sum to one.

$$S = \frac{1}{252-1}\sum_{k=1}^{252}(R_{t-k} - \bar{R})'(R_{t-k} - \bar{R}) \quad (2)$$

where R_{t-k} is the 1×30 vector of excess returns formed by substracting the S&P 500 index return from the individual stock returns on day $t-k$, $\bar{R}$ is the in sample historical average of these return vectors. The manager uses past one year's data to estimate the covariance matrix, thus the sample size is 252. He then uses the above matrix to predict return variances and covariances on day t.

3.1.2 Sample Covariance Matrix Estimator (CHMSW)

Due to nontrading effect, the observed daily returns may not be equal to the true daily returns. To correct this bias, CHMSW (1983) propose a new estimator. Suppose that the true date n return on stock k is $r^t_{k,n}$, and the corresponding observed return is $r_{k,n}$. The CHMSW estimator is based on the following relation between the covariance of the true returns on any two stocks j and k, $j \neq k$ and the covariance of the observed returns on the same two stocks at different leads and lags:

$$cov(r^t_{j,n}, r^t_{k,n}) = cov(r_{j,n}, r_{k,n}) + \sum_{m=1}^{L}[cov(r_{j,n}, r_{k,n-m}) + cov(r_{j,n-m}, r_{k,n})] \quad (3)$$

This equation remains valid when either or both assets are portfolios. With this equation, we can estimate the true covariances using daily observed returns. In practice, following CHMSW (1983) and Shanken (1987), L is set to 3 for daily returns. To construct the minimum tracking error volatility portfolio, the manager only needs to replace the returns in above equation with excess returns relative to the benchmark.

3.1.3 Sample Covariance Matrix Estimator (SW)

The Scholes-William estimator is the special case of the above equation, with L equal to 1.

3.1.4 Sample Covariance Matrix Estimator (NW)

There is a potential problem in implementing SW and CHMSW estimator: the estimated covariance matrix from equation (3) may not be positive semi-definite. This is problematic for solving the portfolio variance minimization problem since estimated variances can be negative when the estimated covariance matrix is not positive semi-definite. In addition, an estimator of S that is not positive definite may be troublesome because S^{-1} may behave poorly at the same time. In order to address this issue, I propose the following estimator according to Newey and West(1987):

$$cov(r^t_{j,n}, r^t_{k,n}) = cov(r_{j,n}, r_{k,n}) + \sum_{m=1}^{L} w(m, L)[cov(r_{j,n}, r_{k,n-m}) + cov(r_{j,n-m}, r_{k,n})] \tag{4}$$

where

$$w(m, L) = 1 - \frac{m}{L+1} \tag{5}$$

is the Bartlett Kernel.

3.1.5 Optimal Shrinkage Estimator

This estimator is from Ledoit and Wolf (2003). Assume that the return on stock j in trading day t is generated by the market model,

$$r_{j,t} = \alpha_j + \beta_j r_{M,t} + \varepsilon_{j,t} \tag{6}$$

where $r_{M,t}$ is the day t return on the market index, and $\varepsilon_{j,t}$ is a residual term. If $r_{M,t}$ is uncorrelated with the residual return $\varepsilon_{j,t}$ and the residual returns are mutually uncorrelated, the covariance matrix estimator F of the returns on a set of stocks is given by

$$F = s^2_m BB' + D \tag{7}$$

where B is the vector of β's, s^2_m is the sample variance of $r_{M,t}$, and D is a diagonal matrix that has the sample variances of the residuals along the diagonal.

The optimal shrinkage estimator is a weighted average of the sample covariance matrix and the market model based estimator:

$$\Sigma = \frac{\lambda}{T} F + (1 - \frac{\lambda}{T}) S \tag{8}$$

where λ is a parameter that determines the shrinkage intensity which can be estimated from the data and T is the sample size. This estimator is positive semi-definite and Ledoit and Wolf (2003) shows that it outperforms many factor models empirically. To use it to construct the minimum tracking error variance portfolio, all that the manager needs to do is to use excess returns instead of raw returns.

3.1.6 Optimal Rolling Sample Covariance Matrix Estimator

Foster and Nelson (1996) provide an optimal rolling estimator that has the smallest asymptotic mean squared error (MSE). FKO (2003) extend this estimator to multivariate case and gives an excellent exposition about its application in the current portfolio optimization problem.

Following FKO (2003), suppose we have a conditional covariance matrix estimate Σ_t on day t with the general form:

$$\Sigma_t = \sum_{k=1}^{\infty} W_{t-k} * (R_{t-k} - \bar{R})'(R_{t-k} - \bar{R}) \qquad (9)$$

where W_{t-k} is a symmetric 30×30 matrix of weights, and $*$ denotes element-by-element multiplication, or the Hadamard product of two matrices. It includes many estimators such as the sample covariance matrix estimator and GARCH models as its special cases with the suitable choices of the weighting scheme. Foster and Nelson(1996) prove that, under weak assumptions, the optimal weight is given by an exponential function;

$$W_{t-k} = \alpha \exp(-\alpha k)\mathbf{1} \cdot \mathbf{1}' \qquad (10)$$

where $\mathbf{1}$ is a 30×1 vector of ones. Under this weight, the optimal rolling sample covariance matrix estimator will be

$$\Sigma_t = \exp(-\alpha)\Sigma_{t-1} + \alpha \exp(-\alpha)(R_{t-1} - \bar{R})'(R_{t-1} - \bar{R}) \qquad (11)$$

This estimator Σ_t is positive semi-definite and is a special example of a multivariate GARCH model such as Engle and Kroner(1995). The advantage with this estimator is that it has only one parameter α to estimate. The optimal decay rate α controls the rate at which the weights decay with the lag length. Unlike the overparameterized multivariate model, this parsimonious form is computationally easy to implement and this is very important in a real-time portfolio optimization problem.

3.2 Covariance Matrix Estimators Based on Intradaily Returns

The realized volatility approach advocated by ABDE (2001) and Barndorff-Nielsen and Shephard (2002) suggests that intradaily returns can be used to construct daily volatility estimates which in theory should be more accurate than those based on daily returns.

Normalize the unit time interval to represent a trading day. The daily return is simply the sum of all the high-frequency intraday returns in the trading period and the overnight return:

$$r_{j,t} = r_{j,t,0} + r_{j,t,1} + r_{j,t,2} + ... + r_{j,t,N} \qquad (12)$$

where N is the sampling frequency for the trading period in a day and $r_{j,t,k}$ is the return of stock j in interval k ($1 \leq k \leq N$) on day t. $r_{j,t,0}$ is the overnight return for stock j on the same day. The overnight period is defined as the close of the previous trading day to the open of the subsequent trading day, e.g., overnight -Monday is the period from close-Friday to open-Monday.[7] Overnight returns are computed as the natural logarithm of the stock price relative, adjusted for dividends and splits.

[7] Therefore, the daily return is from close of the previous trading day to close of the subsequent trading day, which follows common use in the literature.

Denote $R_{t,n}$ as the 1×30 vector of excess returns for all the 30 DJIA stocks relative to the S&P 500 index in interval n $(0 \leq n \leq N)$ on day t. The manager then uses the previous one year's intradaily data to estimate the covariance matrix.

3.2.1 Sample Covariance Matrix Estimator (excluding overnight returns)

In much research related to realized volatility, overnight returns are ignored either because financial markets are open 24 hours a day such as FX markets, thus there are no overnight returns (see ABDL (2000, 2001)), or because only realized intraday volatility is considered as in ABDE (2001). For simplicity, I exclude overnight returns now. Since there is little evidence that there are changes in expected returns at the high-frequency(such as five-minute) level, I have the following sample covariance matrix estimator:

$$S = \frac{1}{252}\sum_{k=1}^{252}\sum_{n=1}^{N} R'_{t-k,n}R_{t-k,n} \tag{13}$$

where N is the number of intraday intervals in a trading day. Basically, this estimator replaces the cross-product of daily returns with the sum of the cross-product of sufficiently finely sampled high-frequency returns. Although the former is also an unbiased estimate for the realized daily integrated volatility, it is an extremely noisy estimator and may include large measurement errors; in contrast, realized volatility measures using high-frequency returns are asymptotically free of measurement error under the diffusion assumption.

3.2.2 Sample Covariance Matrix Estimator (including overnight returns)

Although the stock markets are closed overnight, the arrival of information during non-trading hours will still lead to non-zero overnight returns. Excluding overnight returns will lead information loss and bias the covariance matrix estimator. To reduce these negative effects, I include the outer product of the vector of overnight returns as another term when constructing the realized volatility. Although it is an imprecise estimator of the integrated covariance matrix over the non-trading period, the information gains may dominate the imprecision. The estimator is given as:[8]

$$S = \frac{1}{252}\sum_{k=1}^{252}\sum_{n=0}^{N} R'_{t-k,n}R_{t-k,n} \tag{14}$$

[8] I tried raw overnight returns and demeaned overnight returns where the sample mean is deducted from the raw returns. Both produced very similar results. To save space, I only include empirical results obtained from demeaned overnight returns in this paper.

3.2.3 Sample Covariance Matrix Estimator (adjusted for intraday volatility pattern)

It is widely documented that there is a distinct U-shaped pattern in stock return volatility over the trading day, i.e. volatility is high at the open and close of trading and low in the middle of the day, see Wood, McInish and Ord (1985) and Harris (1986) for early evidence. Therefore, as the sampling period shrinks, stock returns do not conform well with the normal white noise assumption. Andersen and Bollerslev (1997) and Martens, Chang, and Taylor (2002) suggest using the flexible Fourier form (FFF) to filter out the deterministic seasonals from high-frequency data. In the present portfolio optimization problem, I use a computationally easier to implement method to remove the intraday pattern, which follows Areal and Taylor (2002).

Suppose for any stock j,

$$\begin{aligned} var(r_{j,t}|\sigma_{j,t}) &= \sigma_{j,t}^2, &(15)\\ var(r_{j,t,n}|\sigma_{j,t}) &= \lambda_{j,n}\sigma_{j,t}^2, &(16)\\ \sum_{n=0}^{N}\lambda_{j,n} &= 1. &(17)\end{aligned}$$

where $\sigma_{j,t}$ is the latent daily volatility for stock j on day t, and $\lambda_{j,n}$ is the proportion of a trading day's total return variance that is attributed to period n, assuming that intraday returns are uncorrelated. The realized volatility for trading day t is estimated by weighting the intraday squared returns,

$$\widetilde{\sigma_{j,t}}^2 = \sum_{n=0}^{N} w_{j,n} r_{j,t,n}^2 \qquad (18)$$

Areal and Taylor (2002) derive the optimal weight as

$$w_{j,m} = \frac{1}{(N+1)\widetilde{\lambda_{j,m}}} = \frac{\sum_t \sum_{n=0}^{N} r_{j,t,n}^2}{(N+1)\sum_t r_{j,t,m}^2} \qquad (19)$$

The estimate $\widetilde{\sigma_{j,t}}^2$ with this weight is a consistent and unbiased estimate of the realized volatility and has the least variance. From this expression, it is easy to see that the optimal weight $w_{j,m}$ is inversely proportional to $\lambda_{j,m}$. Thus the weight for the overnight returns is much less than that for the other returns since the average overnight return is much larger than the average intraday return in any high-frequency interval. This weight can be also used to construct the realized covariance by

$$\widetilde{cov}(r_{i,t}, r_{j,t}) = \sum_{n=0}^{N} \sqrt{w_{i,n}}\sqrt{w_{j,n}} r_{i,t,n} r_{j,t,n} \qquad (20)$$

3.2.4 Sample Covariance Matrix Estimator (adjusted for correlations)

Stock return series are approximately Gaussian and uncorrelated when measured over monthly or longer horizon. As the sampling frequency increases, however, temporal dependence and departures from normality become strikingly apparent. As is well known, non-synchronous trading and bid-ask spread will typically induce negative autocorrelation for individual security returns and positive autocorrelation for the market index returns, particularly at lag one. This correlation tends to become stronger as the sampling frequency increases. To take account of this, French, Schwert and Stambaugh (1987) use the sum of squared daily returns plus twice the sum of the products of adjacent returns to estimate the variance of the monthly returns. Here I focus on estimation of daily volatility using high-frequency data and this suggests the following estimator that allows for serial dependence:

$$S = \frac{1}{252}\sum_{k=1}^{252}\{\sum_{n=0}^{N} R'_{t-k,n}R_{t-k,n}+\sum_{l=1}^{L}[\sum_{n=l+1}^{N} R'_{t-k,n}R_{t-k,n-l}+\sum_{n=l+1}^{N} R'_{t-k,n-l}R_{t-k,n}]\} \tag{21}$$

where L is the maximum number of lead and lag covariances to be included.[9] Since there is no evidence that overnight returns have significant correlation with intraday returns, the lead and lag covariances included in above estimator only apply for high-frequency returns.

3.2.5 Optimal Rolling Sample Covariance Matrix Estimator

To encompass the realized volatility approach within the rolling estimator framework, it is only necessary to replace the outer product of the vector of demeaned returns in equation (11) with the realized covariance matrix:

$$\Sigma_t = \exp(-\alpha)\Sigma_{t-1} + \alpha\exp(-\alpha)\sum_{n=0}^{N} R'_{t-1,n}R_{t-1,n} \tag{22}$$

Intuitively, it should be more efficient than the daily return based estimator from (12) as argued in ABDE (2001), the realized volatility is asymptotically free of measurement error, while the cross-product of daily returns is extremely

[9] An alternative approach to purge the high-frequency returns of the serial correlation induced by microstructure frictions would estimate sample autocorrelation and use a MA model to filter the raw return series, then estimate realized volatilty with filtered returns. This is adopted in ABDE (2001). I also experimented with the use of filtered five-minute returns. This gives similar results to using contempraneous unfiltered returns, and has worse out-of-sample performance compared to the lag correlation included estimator, thus is not included in this paper.

noisy although it is unbiased.

3.3 Monthly Covariance Matrix Forecast

In section 3.1 and 3.2, I provide 11 daily covariance matrix estimates based on daily returns or high-frequency returns. It is easy to produce a new estimator from the combination of above estimators. For example, I can also include some lead and lag covariances in (22) to get another rolling covariance matrix estimator. For the purpose of monthly rebalancing, a monthly covariance matrix has to be constructed to get the optimal portfolio weight from these daily estimates, as can be seen in (1).

There are several methods used in this paper. A simplest monthly forecast would use the daily estimate multiplied by a constant, for instance, $21 * \Sigma_t$, as a choice since it is assumed that there are 21 trading days in a month. From (1), the choice of this constant is irrelevant; it is cancelled out. Therefore, daily covariance matrix estimate can be used directly to get the optimal portfolio.

Jagannathan and Ma (2003) provide a method to estimate the covariance matrix of monthly returns using data on daily returns. Let $\widetilde{R_0}$ the 252×30 matrix of demeaned returns from $[R'_{t-252}, R'_{t-251}, ..., R'_{t-1}]$. For $j = 1, 2, ..., 20$, $\widetilde{R_{-j}}$ is the matrix of lag-j demeaned returns. Denote

$$\Omega_j = \frac{\widetilde{R'_0}\widetilde{R_{-j}}}{252} \tag{23}$$

$$S = 21\Omega_0 + \sum_{j=1}^{20}(21-j)(\Omega_j + \Omega_j^{'}) \tag{24}$$

S is positive semi-definite and consistent. This estimator takes account of all the autocovariances and cross-covariances between all the daily returns at different leads and lags in a month and is in spirit similar to Newey and West (1987) estimator. It is easily extended to high-frequency data case.

Andreou and Ghysels (2002) treat integrated volatility as a continuous time stochastic process, sample it at high frequency and get a rolling sample estimator-Historical Quadratic Variation (HQV). This is especially useful for the high-frequency data. The following monthly covariance matrix estimator is constructed according to this approach. Let $\sum_{t=1}^{21}\sum_{n=0}^{N} R'_{t,n}R_{t,n}$ be the monthly realized volatility from day 1 to 21. It can seen as a continuous time stochastic process. If this process is sampled at daily frequency, then there will be 232(=252-21+1) monthly realized volatilities in a year. A sample mean of these monthly realized volatility series can be used to compute the optimal portfolio weight.

The last method assumes that the daily covariance matrix estimates follows a AR(1) process. After I have used the past one year's returns to get the sample mean of the daily covariance estimate S, at the beginning date of a month $t+1$,

$$\widetilde{\Sigma_{t+1}} = \alpha\Sigma_t + (1-\alpha)S \tag{25}$$

is used to forecast the covariance matrix on day $t+1$. Here α is the autoregressive parameter, Σ_t can be outer-products of lag daily excess return vectors or realized volatility or optimal rolling sample estimator on day t, etc. After this, daily covariance matrix forecasts from day $t+2$ to $t+21$ can be recursively constructed. With these daily forecasts, a monthly covariance matrix estimate is given by $\sum_{j=1}^{21} \widetilde{\Sigma_{t+j}}$.

4. Empirical Results

4.1 Data

My empirical analysis from high-frequency returns is based on data from the Trade and Quotation(TAQ) and Chicago Mercantile Exchange(CME) database. The TAQ data files contain continuously recorded information on the trades and quotations for the securities listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and the National Association of Security Dealers Automated Quotation system (NASDAQ). The database is published monthly, and has been available since January 1993. The sample period is from January 2, 1993 until June 30, 2000, for a total of 1894 trading days. Since a full replication of the S&P 500 index with all the stocks is not practical, I restrict my analysis to the 30 DJIA stocks. The sample consists of all the quotes for the thirty DJIA firms, as of the reconfiguration of the DJIA index on November 1, 2000.[10] A list of the relevant ticker symbols is contained in the tables below.

The DJIA stocks are among the most actively traded U.S. equities and represent about one-fifth of the value of all US stocks. The median duration between trades for all of the stocks across the full sample is less than one minute. Although all of them are high liquid, it is not practically feasible to push the continuous record asymptotics and the length of the observation interval beyond their transaction frequency. In addition, market microstructure features, such as price discreteness, nonsynchronous trading, and bid-ask spread, can seriously distort the distributional properties of high-frequency intraday returns, especially in the current multivariate context, if varying degrees of interpolation are employed in the calculation of the returns for different stocks. Following ABDE (2001), I construct five-minute returns which represent a reasonable compromise between the accuracy of the theoretical approximations and the market microstructure considerations.

The daily transaction record extends from 9:30 EST until 16:00 EST. Each quote consists of a bid and an ask price along with a "time stamp" to the nearest second. To avoid the opening effect of the exchange on the quotes price data and possible reporting errors in the opening of the stock markets, I remove a 30 minute window of data and consider quotes price after 10:00 EST. Hence, the intraday sample covers the records from 10:00 EST to 16:00 EST, with a total of 72 five-minute returns for each trading day. The five-minute return

[10] I also experimented with the use of transcation prices instead of price quotes for the thirty DJIA stocks in part of the sample period, which produced very similar empirical results.

series for the thirty stocks in DJIA are constructed from the difference between the average of the log bid and the log ask that are recorded at or immediately before the corresponding five-minute marks. Overnight returns are computed accordingly as the difference between the mid-points of log quotes at 10:00 EST and the previous 16:00 EST, adjusted for dividends and splits.[11]

Intraday observations on the S&P 500 cash index can be obtained from Chicago Mercantile Exchange (CME). The period covered by the data is the same as those of the thirty DJIA stocks, from 9:00 CST to 15:00 CST each trading day from January 2,1993 until June 30,2000.[12] Similarly, the five-minute return and overnight series of the S&P 500 index are constructed, except that they are the logarithmic difference between the cash index levels at each five-minute and overnight interval.

Daily returns of the 30 DJIA stocks and the S&P 500 index of the same period can be obtained from the Center for Research in Security Prices (CRSP) daily data files.

4.2 Empirical Results

In this section, I compare the out-of-sample performance of minimum tracking error portfolios formed using a number of covariance matrix estimators based on daily data or high-frequency data, with an eye to judging which models improve the manager's ability to optimize portfolio risk. I first provide summary statistics for daily returns and five-minute returns. Then I form the minimum tracking error portfolios according to the covariance matrix estimators given in section 3. These portfolios are rebalanced daily and monthly and evaluated according to their out-of-sample performance. Finally, the t-tests for the difference between the mean returns and mean squared returns on the portfolios reveal in which case and what kind of covariance matrix estimators based on high-frequency data can improve upon those based on daily returns.

Table 1 and Table 2 provide summary statistics for daily returns and five-minute returns for each of the thirty DJIA stocks and the S&P 500 cash index, respectively. From Table 2, it is clear that the means of all the five-minute return series are close to zero. Given their sample standard deviations, a simple t-test shows that the means of high-frequency returns are effectively zero from a statistical perspective. There are some significant differences as the sampling frequency increases. The kurtosis of the S&P 500 index and all the individual stocks, except EK, PG and T, become larger as the data are sampled more frequently. The cross-sectional average kurtosis of the 30 DJIA stocks increases from 8.2539 to 14.0095 as five-minute returns replace daily returns. In the same

[11] During the sample period, there are some mergers and acquisition deals for some of the stocks. I always use PERMNO as the unique standard to decide the return series for a security. A security through its entire trading history can be checked with one PERMNO, regardless of name changes or capital structure changes.

[12] June 27,1995 S&P 500 cash index and futures high-frequency data are missing in the CME data files. In addition, CME data files don't include December 18,1998 S&P 500 futures contract data. These two days are excluded from the empirical analysis.

time, the kurtosis of the S&P 500 index increases from 8.5479 to 15.4402. The average of the first order autocorrelation of daily returns for each of the thirty stocks is -0.0071. Almost half of stocks (14 out of 30) have positive first order autocorrelations, and another 16 stocks negatively correlated in the first order; the minimum is -0.0671, and the maximum is 0.0576. This indicates most of the daily return series can be seen as uncorrelated. In contrast, the five-minute returns of all the individual stocks, except INTC and MSFT, have significant negative lag one autocorrelations. The cross-sectional average is -0.1163. The first order autocorrelation of the daily S&P returns is only -0.0008, while that of five-minute S&P returns is 0.1259. These highly significant autocorrelations of the index and the individual stocks may be explained by a non-synchronous trading effect as suggested in Lo and MacKinlay (1990).

Table 3 gives the out-of-sample means, standard deviations, and other characteristics of the minimum tracking error portfolios if the manager rebalances his portfolio monthly based on past 12 months of data, daily returns or intraday returns. In all the covariance matrix estimators based on daily returns (panel A), the sample covariance matrix estimator provides the minimum standard deviation, thus has the best forecast performance.[13] In the rest of the estimators, the sample covariance matrix (NW, L=1), the optimal shrinkage estimator (LW), and the rolling sample estimator (FKO) with the decay rate 0.01 have similar standard deviations.[14] The sample covariance matrix (CHMSW) and the sample covariance matrix (SW) deviate a lot from all the other estimators and perform very poorly according to their standard deviations. This may be explained by the fact that these two estimates are not necessarily positive semi-definite, therefore variance estimates can be negative. They may not be invertible, which are problematic in computing the portfolio weight from equation (1). In fact, both the sample covariance matrix (SW) and the sample covariance matrix (NW, L=1) include the lag 1 autocovariances and cross-covariances, but has different weight with this adjustment. The sample covariance matrix (NW, L=1) is positive semi-definite, while the former is not. The out-of-sample performance of the sample covariance matrix (JM) is also very poor. Although

[13] Monthly returns are not real monthly returns, but constructed from the sum of daily returns in 21 consecutive trading days so that there is no overlap in the estimation period and the forecast period. I also tried monthly returns from CRSP monthly data files. In this case, the sample covariance matrix using daily returns has the best forecast performance among all the estimators, based on daily returns or intraday returns. Here and below, when the manager rebalances monthly, the monthly covariance matrix forecast is contructed from the daily covariance matrix multiplied by a constant, except the sample covariance matrix estimator (JM), the rolling sample estimator (AG), and several other rolling sample estimators which use the second parameter as the autoregressive coefficient.

[14] Unlike FKO (2003), the optimal decay rate with the rolling sample estimator (FKO) does not come from maximum likelihood estimate (MLE), but from the best result among a lot of parameter choice experiments from the interval [0, 1] so that the minimum tracking error with this estimator is only achieved conditional on the identification of the optimal decay rate. It is less efficient but easier to implement in the real-time portfolio optimization problem. The difference between this estimator and the rolling sample estimator (FN) when they both use five-minute returns is that the latter is not adjusted using daily return information, but includes the first-order autocorrelation between the intraday returns to measure the covariance matrix.

including all the autocovariances and cross-covariances between the daily returns in a month is in theory less biased when constructing the monthly covariance matrix estimate, it also introduces greater noise potentially so that the statistical efficiency of the estimator declines.[15] The optimal shrinkage estimator (LW) does not perform better out-of-sample than the sample covariance matrix. It is more useful if a large covariance matrix need to be estimated when there are no long enough time-series data available, while it is no severe under current situation.

The sample covariance matrix is constructed when portfolio weights are unrestricted. Short sale restrictions has little effect on the composition of the minimum tracking error portfolios. This is different from Jagannathan and Ma (2003) where they find that short sale restrictions improve the performance of the sample covariance matrix by a significant amount when they use monthly returns so that it can be comparable to that of the single factor model.[16] On the average, all the thirty stocks have positive weights in the minimum tracking error portfolios. XON and GE have the maximum average weight; about 8% and 7% respectively while the weights of all the other stocks don't exceed 5%. There are only 6 stocks which have negative weights in the full sample period. DD has the minimum weight -3.53%. The second minimum weight is on stock CAT(-1.29%). The short positions with the rest four stocks are all less than 0.6%. The maximum long positions are put on stock GE (12.86%) and XON (11.11%), but there are still significant time variations with the weights.

Panel B of Table 3 provides the out-of-sample performance of the minimum tracking error portfolios whose covariance matrix estimates use five-minute returns and overnight returns. The following patterns emerge. First, including overnight returns in the realized volatility substantially improves the out-of-sample performance. The high-frequency rolling sample estimator (FKO) conditional on its optimal decay rate 0.03 performs as well as the daily sample covariance matrix estimator when the realized volatility includes overnight returns. Second, the sample covariance matrix estimator (AT) which takes account of intraday volatility and the rolling sample covariance estimator (AG) which samples the monthly realized volatility more frequently do not perform better than the sample covariance matrix using realized volatility approach. All of them perform worse than the sample covariance matrix using daily returns. Third, when lag one autocovariances and cross-covariances are included in the

[15] Jagannathan and Ma (2003) get similar out of sample performances when they use this estimator, the sample covariance matrix (CHMSW), the sample covariance matrix (SW) and the sample covariance matrix. There are two differences from this paper: 1. they use past 60 months or 120 months of daily returns while I use past 12 months of daily returns in order to be consistent with the high-frequency data; 2. they use randomly chosen 500 stocks to track the S&P 500 index while I use 30 DJIA stocks.

[16] Jagannathan and Ma (2003) don't get significant improvement in performance when constucting sample covariance matrix from daily returns wtih shortsale restrictions. They also find short sale constraints are not important for the single factor model which verifys the Green and Hollifield (1992) hypothesis that the presence of a single dominant factor is critical for the minimum variance portfolio to have a lot of short positions. The dominant factor gets cancelled out when the single factor model is used to generate the minimum tracking error portfolio.

realized daily volatility, the sample covariance matrix estimator improves a lot - its performance becomes comparable to that of daily return based sample covariance matrix.[17] Fourth, when the lag one correlation and overnight returns are incorporated into the realized volatility, the last six rolling sample estimators perform marginally better than the daily return based sample covariance matrix with appropriate choices of the decay rate and the autoregressive coefficient. Another interesting fact to note is that there is only one stock that has negative weight -0.6% in the full sample for the sample covariance matrix estimator with overnight returns and lag 1 covariances. On the average, all the stocks have long positions and XON has the maximum average weight 7%.

Table 4 compares the out-of-sample performance of each covariance estimator if the manager balances his portfolio daily using the past 12 months of data. Consistent with the monthly rebalancing case, the sample covariance matrix (CHMSW) and the sample covariance matrix (SW) perform very poorly and are very different from other estimation methods. In all the other daily return based covariance matrix estimates, the sample covariance matrix has the out-of-sample tracking error as small as those from the optimal shrinkage estimator (LW) and the rolling sample estimator (FKO) with the optimal decay rate 0.01. Including the overnight returns, the lag one autocovariances and cross-covariances between intraday five-minute returns in the realized daily volatility will lead the sample covariance matrix to have comparable performance to its daily return based counterpart when only contemporaneous daily returns are used. In addition, there is an apparent difference when the manager rebalances daily rather than monthly. In the former case, the high-frequency rolling sample estimator with the optimal decay rate has a standard deviation 0.53% lower than the daily return based sample covariance matrix. In contrast, when the manager rebalances his portfolio monthly, the sample covariance matrix with daily returns performs at least as well as all kinds of sample covariance matrices with high-frequency returns.

Table 5 confirms the above findings with the t-tests for the difference between the mean returns and mean squared returns on the portfolios constructed from different kinds of covariance matrix estimators and the benchmark - the sample covariance matrix using daily returns. Here I exclude the sample covariance matrix (SW), the sample covariance matrix (CHMSW), and the sample covariance matrix (JM) since they have very different performances from other estimators. The differences in mean returns for all the other estimators are insignificant, therefore the t-test for the difference in mean squared returns can be used as a test for the difference in return variances, which is exactly what the manager wants to minimize. Panel A indicates that all the covariance estimators perform as well as the benchmark, whether daily returns or high-frequency data are used if the minimum tracking error portfolio is rebalance monthly. If it is daily rebalanced, panel B shows that rolling sample estimators, when including overnight returns and lag 1 covariances between intraday returns, perform better than

[17] Higher order (from 2 to 9) autocovariances and cross-covariances are also experimented to be included in the realized volatility. They give roughly close performances as only lag one period is included.

the daily return based sample covariance matrix estimator if appropriate decay rates are selected.

Table 6 further illustrates the relationship between the performance of a sample covariance estimator and its sample size. Here I only include the daily sample covariance matrix and the high-frequency sample covariance matrix - with or without adjustment for microstructure effects. As the estimation period shrinks, the standard deviation of the minimum tracking error portfolio constructed from the daily sample covariance matrix increases. In the extreme case, when only one month of daily returns are used to construct a 30×30 covariance matrix, there is huge estimation error. On the contrary, the standard deviation from the sample covariance matrix using one month of high-frequency data is significantly smaller than that using one year of high-frequency data whether I incorporate the corrections for microstructure effects. In addition, as the estimation period is less than 6 months, the microstructure-adjusted high-frequency sample covariance matrix estimator always performs significantly better than its daily counterpart. With the shortening of the estimation horizon, the performance gains from using high-frequency data rather than daily data increase sharply.

5. Robustness Checks and Simulation Results

In this section, I compare the out-of-sample performance of minimum variance portfolios formed using different covariance matrix estimators to assess the robustness of my results. By constructing the minimum variance portfolios, I only use the thirty DJIA stocks, not the S&P 500 cash index. The construction method is the same, except excess returns relative to the index are replaced with raw returns. Since the five-minute S&P 500 index returns have high positive first order autocorrelation while most of the individual stocks are significantly negative correlated at lag one, by forming the minimum variance portfolios, it may help to alleviate the microstructure contamination from the S&P 500 cash index.[18]

From Table 7, if the manager rebalances his minimum variance portfolio monthly with the previous 12 months of data, the daily sample covariance matrix still performs the best among all the estimators from daily returns or high-frequency returns. The minimum out-of-sample standard deviation among all the high-frequency based estimators is 14.18% per year, while that of the daily sample covariance matrix estimator is 13.51% annualized. If the manager rebalances daily, then the standard deviation of the sample covariance matrix using

[18] When I construct the minimum tracking error portfolios, I also experimented with using the S&P 500 futures contracts instead of the S&P 500 cash index. Unlike the cash index, the first order autocorrelation of the five-minute S&P 500 futures returns is small. The results indicate that when I use the sample covariance matrix estimator adjusted by overnight returns, the tracking errors from using cash index and futures are very close. But if I take account of the autocorrelation between the five-minute returns, the standard deviation from using cash index is always smaller in all the cases. Therefore, I only report the results from using the S&P 500 cash index.

five-minute returns and overnight returns is smaller than its daily counterpart; 14.36% vs 14.65%. If the optimal decay rate is chosen, the rolling sample estimator with high-frequency data performs much better than that based on daily returns; 13.12% vs 14.90%.

If the manager only has past 3 months of data to estimate, then the standard deviation of high-frequency sample covariance matrix using five-minute returns and overnight returns is substantially lower than its daily based estimator, 13.73% vs 16.45%, even he rebalances monthly. This is strikingly different when the manager has one year data to use.

The only difference between the minimum variance portfolio and the minimum tracking error portfolio lies in the weights on the component stocks. Among all the stocks, only MMM and XON have positive weights in the full sample period in the minimum variance portfolio constructed from the sample covariance matrix using daily returns. Besides, across all the stocks, the maximum long position is 27.16% while the maximum short position is 23.53%. The average short interest is 28.50% per month, which is in significant contrast to the minimum tracking error portfolio and consistent with the Green and Hollifield (1992) explanation for the presence of a large number of negative weights in mean-variance efficient portfolios.

How precisely can we estimate a large covariance matrix using high-frequency financial data if the asset prices follow the geometric Brownian motion? The following simulation exercise in Table 8 sheds a light on this problem. I set the population covariance matrix to be the sample covariance matrix of the daily excess returns of the 30 DJIA stocks relative to the S&P 500 index from January 2, 1993 to June 30, 2000. Then a random sample of intraday returns is drawn assuming that the daily returns which are the sum of 72 intraday returns have a joint Normal distribution with this covariance matrix.[19] The intraday return series for an individual asset is independent and normally distributed. I then use the simulated return data to estimate the covariance matrix and form the minimum tracking error portfolios. Two minimum tracking error portfolios are constructed: one is based on covariance matrix estimate from intraday returns, another is from daily returns which are obtained from the sum of the intraday returns in the same day. As what has been done before, the portfolio is rebalanced monthly based on past 12 months of data, 6 months of data and 3 months of data, respectively in Panel A, B and C. Panel D, E and F report similar results for the daily rebalancing case. The out-of-sample means and standard deviations of these two portfolios are calculated. I repeat the exercise 1000 times and compute the average of these means and standard deviations. The results of this simulation exercise indicate that if the portfolios are rebalanced monthly and the covariance matrix is estimated with 12 months of data, the average annualized standard deviation of the tracking errors from the daily sample covariance matrix is 4.82%, and is 4.51% if I use the sample covariance matrix estimate based on intraday returns. There is not much dif-

[19] With this assumption, I ignore the difference between the overnight returns and the intraday returns in the simulated data set.

ference in the mean of these two portfolios: the former is -0.04% and the latter is 0.16%. Simple t-tests for the difference between the mean returns and mean squared returns of the two portfolios show that the difference in mean returns is insignificant, while the minimum tracking error from high-frequency returns is significantly smaller than that from daily returns. With the shortening of the estimation horizon, the performance of the portfolios constructed from intraday returns is rather stable, with the standard deviation increases from 4.51% to 4.58%, if I use three months as the estimation window. The standard deviation from daily returns increases 30%, from 4.82% to 6.28% if only three months of daily returns are used. Another significant difference is that when intraday returns are used, almost all the stocks have positive weights every month in the full sample. In contrast, if three months of daily returns are used to estimate the covariance matrix, on the average, the sum of the negative portfolio weights will be about 10% every month.

The daily rebalancing experiment reports very similar results. Consistent with the monthly rebalancing case, as the estimation window changes from 12 months to 3 months, the standard deviation of the portfolio from high-frequency returns is roughly the same, changing from 4.56% to 4.57%. It increases about 29%, from 4.84% to 6.24%, if daily returns are used. At the same time, the average short interest will change from 0% to about -10%.

Compared with the empirical results in Table 4, I find that under the ideal condition that the asset prices follows a geometric Brownian motion, the standard deviation of the minimum tracking error portfolio is reduced by 6%, if 12 months of intraday returns instead of daily returns are used to estimate the covariance matrix. If only three months of data are used, about 30% standard deviation reduction can be obtained from using high-frequency data. These results clearly demonstrate the substantial benefit obtained from using high-frequency data to estimate the covariance matrix under the situation as suggested by Merton (1980). Unlike the empirical results, the larger sample size always leads performance gains in the simulation exercise. This is confirmed by the smaller tracking error from using a longer time span of data to estimate the covariance matrix. The difference lies in the assumptions in generating the data: the simulated return series are drawn i.i.d from a multivariate Normal distribution. There is a tradeoff from using the larger sample size in the empirical analysis. It can also bring more noise induced by market microstructure frictions that are difficult to clean from the data set. Therefore, it is not strange that the out-of-sample performance from using three months of data is better than that from 12 months of data.

6. Conclusion and Future Research

In this paper, I consider the problem faced by a professional investment manager who wants to track the return of the S&P 500 index with 30 DJIA stocks. The manager constructs many covariance matrix estimators, based on daily returns and high-frequency returns, to form his optimal portfolio, and tries to find a best estimator to minimize tracking errors. Although prior research has documented that realized volatility based on intraday returns is more precise than daily return constructed volatility, my experiment indicates that when the manager rebalances his portfolio monthly, he will not switch from daily to intraday returns to estimate the conditional covariance matrix if he has past 12 months of data to use. He will switch to intraday returns only when his estimation horizon is shorter than 6 months or he rebalances his portfolio daily.

Following the work of Merton (1980) and Nelson (1992), it is known that precise estimation of volatility can be obtained from an arbitrarily short span of data in theory provided that returns are sampled sufficiently frequently. With this, one may wonder why my empirical results cannot substantiate this view. In fact, BRT (2001) find that the prominent high-frequency data characteristics such as leptokurtosis, autocorrelation in the returns, deterministic patterns and volatility clustering in intraday variance will significantly reduce the precision of volatility estimates that use high-frequency data. Moreover, market microstructure features such as price discreteness, nonsynchronous trading, and bid-ask spread will further contaminate the data used in empirical research. Under the ideal situation in which the asset prices follow the geometric Brownian motion, my Monte Carlo simulation exercise indicates that there is substantial benefit in using intraday returns to estimate a large covariance matrix compared with using daily returns. Even with the real data, there will be benefit from using high-frequency data with the appropriate selection of covariance matrix estimator and adjustment for microstructure effects as can be seen in the daily rebalance case. Moreover, it is not always better in using high-frequency data with larger size. My experiment illustrates that one month to six months high-frequency data have better performances in estimating conditional covariance matrix than one year data. But with one year data to use, my empirical evidence indicates that it is difficult to beat the sample covariance matrix using daily returns by using high-frequency data after having experimented many estimators suggested in the literature if the performance measurement horizon is as long as one month.

Several issues will be considered in the future research. First, variance is used as a measure of risk in this paper. But it is correct only under some specific conditions, such as the normality of the distributions of returns or when the investor has the quadratic utility. For occasionally occurring extreme events, the Value at Risk (VaR) is a better measure of risk, especially for the thick-tailed distributions commonly used in the empirical research. It is very useful to construct Value at Risk efficient portfolios and explore whether there is incremental benefit from using high-frequency data under this new measure. Second, there may exist a frequency beyond which the manager would like to use intraday returns rather than daily returns. I will use different rebalancing and sampling frequencies to further test the critical frequency. Note that the 30 DJIA stocks are only the representative stocks that can be used in tracking the S&P 500 cash index, it is also useful to do the analysis with a larger or smaller set of stocks. The problem can be more interesting by including transaction costs and having an objective function that minimizes the tracking errors as well as the transactions costs that arise due to rebalancing. And it is also important to include the price impacts in the asset allocation problem.

References

Andersen, T.G. and T. Bollerslev, 1998, Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts, *International Economic Review* 39, 885-905.

Andersen, T.G., T. Bollerslev, F.X. Diebold and H. Ebens, 2001, The Distribution of Stock Return Volatility, *Journal of Financial Economics* 61, 43-76.

Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys, 2001, The Distribution of Exchange Rate Volatility, *Journal of the American Statistical Association* 96, 42-55.

Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys, 2002, Modeling and Forecasting Realized Volatility, *Econometrica*, forthcoming.

Andersen, T.G., T. Bollerslev, and S. Lange, 1999, Forecasting Financial Market Volatility: Sample Frequency vis-a-vis Forecast Horizon, *Journal of Empirical Finance* 6, 457-477.

Andreou, E and E. Ghysels, 2001, Why Does Microstructure Noise Affect the Estimation of Asset Pricing Models? CIRANO Working Paper.

Andreou, E and E. Ghysels, 2002, Rolling-Sample Volatility Estimators: Some New Theoretical, Simulation and Empirical Results, *Journal of Business and Economic Statistics* 20, 363-376.

Areal, N. and S.J. Taylor, 2002, The Realized Volatility of FTSE-100 Futures Prices, *Journal of Futures Markets* 22, 627-648.

Bai, X., J.R. Russell and G.C. Tiao, 2001, Beyond Merton's Utopia (I): Effects of Non-normality and Dependence on the Precision of Variance Estimates Using High-Frequency Financial Data, Working Paper, University of Chicago.

Barndorff-Nielsen, O.E. and N. Shephard, 2002a, Econometric Analysis of Realized Covariation: High Frequency Covariance, Regression and Correlation in Financial Economics, Working Paper, Nuffield College, Oxford.

Barndorff-Nielsen, O.E. and N. Shephard, 2002b, Estimating Quadratic Variation Using Realized Variance, *Journal of Applied Econometrics*, forthcoming.

Blair, B.J., S. Poon, and S. J. Taylor, 2001, Forecasting S&P 100 Volatility: the Incremental Information Content of Implied Volatilities and High Frequency Index Returns, *Journal of Econometrics* 105, 5-26.

Bollerslev, T., R.F.Engle, and J.M.Wooldridge, 1988, A Capital Asset Pricing Model with Time Varying Covariances, *Journal of Political Economy* 96, 116-131.

Campbell, J., A. Lo and C. MacKinlay, 1997, *Econometrics of Financial Markets,* Princeton University Press.

Chan, L.K.C., J. Karceski and J. Lakonishok, 1999, On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model, *Review of Financial Studies* 12, 937-974.

Cohen, K.J., G.A. Hawawini, S.F. Maier, R.A. Schwartz and D.K. Whitcomb, 1983a, Estimating and Adjusting for the Intervalling-Effect Bias in Beta, *Management Science* 29, 135-148.

Cohen, K.J., G.A.Hawawini, S.F.Maier, R.A. Schwartz and D.K. Whitcomb, 1983b, Friction in the Trading Process and the Estimation of Systematic Risk, *Journal of Financial Economics* 12, 263-278.

Dimson, E., 1979, Risk Measurement when Shares are Subject to infrequent Trading, *Journal of Financial Economics* 7, 197-226.

Ebens, H., 2000, On Modeling and Forecasting Stock Volatility, Unpublished Ph.D. Dissertation, Johns Hopkins University.

Engle, R.F, 2000, The Econometrics of Ultra-High Frequency Data, *Econometrica* 68, 1-22.

Engle, R.F and K.F. Kroner, 1995, Multivariate Simultaneous Generalized ARCH, *Econometric Theory* 11, 122-150

Fleming, J., C. Kirby and B. Ostdiek, 2003, The Economic Value of Volatility Timing Using "Realized" Volatility, *Journal of Financial Economics* 67*,* 473-509.

French, K.R., G.W. Schwert and R.F. Stambaugh,1987, Expected Stock Returns and Volatility, *Journal of Financial Economics* 19, 3-29.

French, K.R. and R. Roll, 1986, Stock Return Variances: The Arrival of Information and the Reaction of Traders, *Journal of Financial Economics* 17, 5-26.

Foster, D. P. and D.B. Nelson, 1996, Continuous Record Asymptotics for Rolling Sample Variance Estimators, *Econometrica* 64, 139-174.

Frost, P.A. and J. E. Savarino, 1986, An Empirical Bayes Approach to Efficient Portfolio Selection, *Journal of Financial and Quantitative Analysis* 21, 293-305.

Green R.C. and B. Hollifield, 1992, When Will Mean-Variance Efficient Portfolios be Well Diversified? *Journal of Finance* 47, 1785-1809.

Harris, L., 1986, A Transaction data Study of Weekly and Intradaily Patterns in Stock Return, *Journal of Financial Economics* 16*,* 99-117.

Jagannathan,R. and T. Ma, 2003, Risk Reduction in Large Portfolios: A Role for Portfolio Weight Constraints, *Journal of Finance* 58, 1651-1683.

Ledoit, O. and M. Wolf, 2003, Improved Estimation of the Covariance Matrix of Stock Returns with an Application to Portfolio Selection, *Journal of Empirical Finance* 10, December 2003.

Ledoit, O., P. Santa-Clara and M.Wolf, 2002, Flexible Multivariate GARCH Modeling with an Application to International Stock Markets, *Review of Economics and Statistics*, August 2003.

Lo, A. and A.C. MacKinlay, 1990, An Econometric Analysis of Nonsynchronous-Trading, *Journal of Econometrics* 45, 181-212.

Martens, M., 2002, Measuring and Forecasting S&P 500 Index-Futures Volatility Using High-Frequency Data, *Journal of Futures markets* 22, 497-518..

Martins, M., Y.C. Chang and S. J. Taylor, 2002, Intraday Volatility Forecasts Using Different Seasonality Adjustment Methods, *Journal of Financial Research* 25, 283-299.

Merton, R.C, 1980, On Estimating the Expected Return on the Market: An Exploratory Investigation, *Journal of Financial Economics* 8, 323-361.

Nelson, D.B, 1992, Filtering and Forecasting with Misspecified ARCH Models I: Getting the Right Variance with the Wrong Model, *Journal of Econometrics* 52, 61-90.

Newey, W.K. and K.D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703-708.

Scholes, M. and J. Williams, 1977, Estimating Betas from Nonsynchronous Data, *Journal of Financial Economics* 5, 309-327.

Schwert, G.M., 1989, Why Does Stock Volatility Change Over Time? *Journal of Finance* 44, 1115-1153.

Wood, R.A., T.H. McInish and J.K. Ord, 1985, An Investigation of Transactions Data for NYSE Stocks, *Journal of Finance* 40, 723-739.

Table 1

Summary Statistics of Daily Returns

Stock	mean	std	skewness	kurtosis	1st autocorrelation
AA	0.0009	0.0199	0.5549	6.3938	0.0040
AXP	0.0013	0.0206	0.2147	5.4276	-0.0134
BA	0.0006	0.0193	-0.0560	10.5091	0.0001
CAT	0.0008	0.0208	0.0494	6.1556	-0.0059
CCI	0.0016	0.0231	0.4190	6.8107	0.0347
DD	0.0006	0.0184	0.1155	5.2288	0.0229
DIS	0.0007	0.0193	0.5177	8.9438	-0.0300
EK	0.0006	0.0180	-0.2987	11.1385	0.0576
GE	0.0013	0.0153	0.1671	4.8579	0.0166
GM	0.0007	0.0190	0.1094	4.2659	-0.0278
HD	0.0010	0.0207	0.1308	5.0781	0.0297
HON	0.0007	0.0195	-0.5064	10.0622	0.0050
HWP	0.0015	0.0250	0.3268	7.7697	-0.0417
IBM	0.0014	0.0211	0.4350	7.9558	-0.0253
INTC	0.0020	0.0255	0.0687	5.0046	-0.0393
IP	0.0002	0.0194	0.2609	5.4030	-0.0327
JNJ	0.0009	0.0169	0.1120	5.1286	0.0405
JPM	0.0006	0.0183	0.1816	5.6875	-0.0324
KO	0.0007	0.0169	0.1661	6.2156	0.0250
MCD	0.0007	0.0170	0.2781	7.1575	0.0001
MMM	0.0005	0.0158	0.0137	6.1164	-0.0224
MO	0.0004	0.0212	-0.4424	16.7330	-0.0536
MRK	0.0009	0.0182	0.1394	4.9344	0.0102
MSFT	0.0017	0.0223	-0.1639	5.8424	-0.0270
PG	0.0007	0.0186	-2.6223	47.7644	-0.0307
SBC	0.0007	0.0179	0.2508	5.3705	-0.0527
T	0.0004	0.0194	0.3705	9.7474	-0.0031
UTX	0.0011	0.0170	0.1156	4.9028	0.0377
WMT	0.0009	0.0210	0.1889	5.0038	0.0071
XON	0.0007	0.0143	0.3517	6.0071	-0.0671
SP	0.0007	0.0096	-0.3248	8.5479	-0.0008
Median	0.0007	0.0193	0.1528	6.0617	-0.0045
Mean	0.0009	0.0193	0.0483	8.2539	-0.0071
Min.	0.0002	0.0143	-2.6223	4.2659	-0.0671
Max.	0.0020	0.0255	0.5549	47.7644	0.0576

Note: The summary statistics are based on the daily returns for each of the thirty DJIA stocks and the S&P 500 cash index obtained from the CRSP daily data files. The sample covers the period from January 2, 1993 through June 30, 2000, for a total of 1894 daily observations. The mean and median are computed using only 30 DJIA stocks.

Table 2
Summary Statistics of Five-minute Returns

Stock	mean*100	std*100	skewness	kurtosis	1st autocorrelation
AA	0.0002	0.1922	0.1139	9.7813	-0.0806
AXP	0.0000	0.2277	-0.0167	9.3334	-0.1375
BA	-0.0003	0.2013	-0.0263	13.9602	-0.1611
CAT	-0.0001	0.2062	0.1134	10.7671	-0.0828
CCI	-0.0004	0.2439	0.1034	13.3525	-0.1319
DD	0.0000	0.2129	0.0366	10.5078	-0.1589
DIS	-0.0006	0.2093	-0.1108	15.8356	-0.1673
EK	-0.0001	0.1745	-0.0234	10.3928	-0.1266
GE	0.0003	0.1718	0.5323	27.6815	-0.0979
GM	-0.0017	0.1912	0.0313	11.4834	-0.1281
HD	0.0001	0.2454	0.0442	9.9248	-0.2078
HON	0.0003	0.2348	0.1174	20.7058	-0.1431
HWP	-0.0002	0.2157	0.1005	10.8100	-0.0304
IBM	-0.0006	0.1907	0.2505	17.7498	-0.0828
INTC	-0.0001	0.2038	-0.0010	16.4542	0.0658
IP	-0.0007	0.2196	0.0311	10.4674	-0.1471
JNJ	0.0005	0.1809	0.0392	7.9636	-0.1222
JPM	0.0000	0.1755	0.2518	13.7114	-0.0950
KO	0.0009	0.1900	0.0985	8.4408	-0.1798
MCD	0.0004	0.2022	0.0609	9.7339	-0.1765
MMM	-0.0003	0.1725	0.0379	9.7560	-0.0894
MO	-0.0012	0.2203	0.8891	45.9282	-0.1780
MRK	0.0006	0.1930	-0.0155	9.2887	-0.1546
MSFT	0.0007	0.1796	-0.0304	9.8343	0.0775
PG	0.0017	0.1878	0.5999	29.6588	-0.1022
SBC	0.0003	0.1916	0.0361	9.9103	-0.1340
T	-0.0011	0.1837	0.0843	9.2763	-0.1509
UTX	0.0006	0.1887	-0.1201	18.7408	-0.0535
WMT	-0.0001	0.2474	0.0512	9.9494	-0.1798
XON	0.0006	0.1600	0.0220	8.8862	-0.1335
SP	0.0001	0.0796	0.1053	15.4402	0.1259
Median	0.0000	0.1926	0.0417	10.4876	-0.1327
Mean	0.0000	0.2005	0.1100	14.0095	-0.1163
Min.	-0.0017	0.1600	-0.1201	7.9636	-0.2078
Max.	0.0017	0.2474	0.8891	45.9282	0.0775

Note: The summary statistics are based on the five-minute returns within the day for each of the thirty DJIA stocks and the S&P 500 cash index as detailed in the main text. The sample covers the period from January 2, 1993 through June 30, 2000.The mean and median are computed using only 30 DJIA stocks.

Table 3

Out of Sample Performance of Minimum Tracking Error Portfolios (Monthly Rebalance)

Covariance Matrix Estimator	Mean	Std	Max. Weight	Min. Weight	Short Interest
Panel A: Daily Returns					
Sample Covariance Matrix	6.20	4.51	12.86	-3.53	-0.23
Sample Covariance Matrix (CHMSW)	16.47	69.96	981.74	-713.50	-262.23
Sample Covariance Matrix (SW)	53.81	83.08	636.20	-543.59	-91.79
Sample Covariance Matrix (NW, L=1)	5.98	4.74	14.64	-3.35	-0.45
Sample Covariance Matrix (NW, L=3)	5.96	5.04	13.84	-6.38	-1.34
Sample Covariance Matrix (JM)	12.97	51.72	1264.70	-729.99	-93.23
Optimal Shrinkage Estimator (Ledoit)	6.10	4.63	13.18	-2.81	-0.17
Rolling Sample Estimator (FKO, 0.01)	6.45	4.58	12.63	-2.72	-0.12
Panel B: Five-minute Returns					
Sample Covariance Matrix (ex. Overnight)	5.44	5.09	8.81	0.71	0.00
Sample Covariance Matrix	5.65	4.84	8.88	0.74	0.00
Sample Covariance Matrix (AT)	5.24	5.22	8.56	0.72	0.00
Sample Covariance Matrix (Lag 1)	5.63	4.60	10.52	-0.60	-0.06
Rolling Sample Estimator (FKO, 0.03)	5.82	4.48	13.36	-2.35	-0.11
Rolling Sample Estimator (AG)	5.81	4.91	8.68	0.70	0.00
Rolling Sample Estimator (FN, 0.01)	5.68	4.45	10.55	-1.46	-0.06
Rolling Sample Estimator (FN, 0.04)	5.71	4.34	12.68	-3.63	-0.24
Rolling Sample Estimator (FN, 0.1)	5.47	4.60	14.48	-4.28	-0.77
Rolling Sample Estimator (FN, 0.04, 0.95)	5.59	4.33	11.24	-1.73	-0.09
Rolling Sample Estimator (FN, 0.04, 0.5)	5.56	4.40	10.69	-1.35	-0.05
Rolling Sample Estimator (FN, 0.04, 0.1)	5.58	4.54	10.62	-0.74	-0.06

Mean and standard deviation are those of the tracking errors when tracking the S&P 500 index. Both are in percentage per year. Maximum and minimum portfolio weights and short interest are in percentages. Short interest is the sum of negative portfolio weights. The portfolio is monthly rebalanced based on past 12 months of data. In the last three estimators, the first parameter is the decay rate, and the second is the autoregressive coefficient.

Table 4

Out of Sample Performance of Minimum Tracking Error Portfolios (Daily Rebalance)

Covariance Matrix Estimator	Mean	Std	Max. Weight	Min. Weight	Short Interest
Panel A: Daily Returns					
Sample Covariance Matrix	6.37	4.44	13.78	-3.72	-0.25
Sample Covariance Matrix (CHMSW)	-31.77	218.93	26321.13	-27500.02	-530.01
Sample Covariance Matrix (SW)	6.09	17.47	636.20	-543.59	-21.30
Sample Covariance Matrix (NW, L=1)	6.27	4.56	14.65	-3.89	-0.48
Sample Covariance Matrix (NW, L=3)	6.19	4.88	13.84	-13.97	-1.36
Optimal Shrinkage Estimator (Ledoit)	6.37	4.43	13.31	-2.97	-0.18
Rolling Sample Estimator (FKO, 0.01)	6.57	4.42	14.36	-4.52	-0.31
Panel B: Five-minute Returns					
Sample Covariance Matrix (ex. Overnight)	5.52	5.09	9.13	0.66	0.00
Sample Covariance Matrix	5.79	4.70	8.82	0.68	0.00
Sample Covariance Matrix (Lag 1)	5.54	4.42	10.53	-0.68	-0.06
Rolling Sample Estimator (FKO, 0.03)	6.69	4.26	14.04	-2.97	-0.16
Rolling Sample Estimator (FN, 0.01)	5.69	4.20	10.69	-2.36	-0.08
Rolling Sample Estimator (FN, 0.04)	5.65	3.91	11.93	-4.66	-0.53
Rolling Sample Estimator (FN, 0.1)	6.61	4.06	12.74	-8.28	-3.44

Mean and standard deviation are those of the tracking errors when tracking the S&P 500 index. Both are in percentage per year. Maximum and minimum portfolio weights and short interest are in percentages. Short interest is the sum of negative portfolio weights. The portfolio is daily rebalanced based on past 12 months of data.

Table 5

Test for Equality of Means of Minimum Tracking Error Portfolios

Covariance Matrix Estimator	Equality in Mean Return	Equality in Mean Squared Return
Panel A: Monthly Rebalance		
Daily Returns		
Sample Covariance Matrix (NW, L=1)	-0.7087	1.2705
Sample Covariance Matrix (NW, L=3)	-0.4303	1.8586
Optimal Shrinkage Estimator (Ledoit)	-0.4801	1.1456
Rolling Sample Estimator (FKO, 0.01)	0.8872	0.8123
Five-Minute Returns		
Sample Covariance Matrix (ex. Overnight)	-0.6631	1.3430
Sample Covariance Matrix	-0.6107	0.9204
Sample Covariance Matrix (AT)	-0.8672	1.7428
Sample Covariance Matrix (Lag 1)	-0.8005	0.1050
Rolling Sample Estimator (FKO, 0.03)	-1.1042	-0.4680
Rolling Sample Estimator (AG)	-0.4376	1.2065
Rolling Sample Estimator (FN, 0.01)	-0.7730	-0.4425
Rolling Sample Estimator (FN, 0.04)	-0.6052	-0.6215
Rolling Sample Estimator (FN, 0.1)	-0.7368	0.0135
Rolling Sample Estimator (FN, 0.04, 0.95)	-0.8573	-0.7857
Rolling Sample Estimator (FN, 0.04, 0.5)	-0.9487	-0.6854
Rolling Sample Estimator (FN, 0.04, 0.1)	-0.8863	-0.1655
Panel B: Daily Rebalance		
Daily Returns		
Sample Covariance Matrix (NW, L=1)	-0.2570	3.7712
Sample Covariance Matrix (NW, L=3)	-0.2348	7.4103
Optimal Shrinkage Estimator (Ledoit)	-0.0038	-0.6595
Rolling Sample Estimator (FKO, 0.01)	1.6463	-2.0711
Five-Minute Returns		
Sample Covariance Matrix (ex. Overnight)	-0.7445	6.8456
Sample Covariance Matrix	-0.6458	3.5229
Sample Covariance Matrix (Lag 1)	-1.1475	-0.5138
Rolling Sample Estimator (FKO, 0.03)	0.6164	-4.0195
Rolling Sample Estimator (FN, 0.01)	-0.9148	-4.2934
Rolling Sample Estimator (FN, 0.04)	-0.7065	-6.0286
Rolling Sample Estimator (FN, 0.1)	0.1983	-1.7805

This table reports the t-tests of equal mean returns and equal mean squared returns of the minimum tracking error portfolios. For each such portfolio, I test whether its mean return and mean squared return are statistically different from those of the portfolio constructed from the sample covariance matrix of daily returns. The t-tests are calculated as mean(r1-r2)/std(r1-r2)*sqrt(No of Obs), where r2 is either the return (in testing the equality of mean return) or squared return (in testing the equality of mean squared return) of the portfolio constructed the sample covariance matrix of daily returns, r1 is either the return or squared return of the portfolio constructed from another covariance matrix estimator.

Table 6

Out of Sample Performance of Minimum Tracking Error Portfolios (Monthly Rebalance)

Covariance Matrix Estimator	Mean	Std	Max. Weight	Min. Weight	Short Interest
Panel A: Past 12 months of Data					
Daily Sample Covariance Matrix	6.20	4.51	12.86	-3.53	-0.23
HF Sample Covariance Matrix (ex. Overnight)	5.44	5.09	8.81	0.71	0.00
HF Sample Covariance Matrix	5.65	4.84	8.88	0.74	0.00
HF Sample Covariance Matrix (Lag 1)	5.63	4.60	10.52	-0.60	-0.06
Panel B: Past 6 months of Data					
Daily Sample Covariance Matrix	7.14	4.63	16.55	-8.56	-1.42
HF Sample Covariance Matrix (ex. Overnight)	5.50	4.96	9.68	0.17	0.00
HF Sample Covariance Matrix	5.58	4.66	9.43	0.36	0.00
HF Sample Covariance Matrix (Lag 1)	5.71	4.12	10.74	-1.71	-0.10
Panel C: Past 3 months of Data					
Daily Sample Covariance Matrix	7.65	5.52	21.36	-20.10	-6.16
HF Sample Covariance Matrix (ex. Overnight)	5.49	4.61	10.24	-0.80	-0.06
HF Sample Covariance Matrix	5.58	4.29	10.02	-0.50	-0.01
HF Sample Covariance Matrix (Lag 1)	5.75	3.95	12.19	-3.60	-0.28
Panel D: Past 2 months of Data					
Daily Sample Covariance Matrix	7.57	7.39	29.94	-27.11	-22.70
HF Sample Covariance Matrix (ex. Overnight)	5.64	4.55	10.55	-1.76	-0.12
HF Sample Covariance Matrix	5.84	4.30	10.53	-1.47	-0.06
HF Sample Covariance Matrix (Lag 1)	5.92	4.07	13.89	-5.66	-0.56
Panel E: Past 1 month of Data					
Daily Sample Covariance Matrix	-90.02	204.59	2589.01	-2206.50	-733.70
HF Sample Covariance Matrix (ex. Overnight)	5.14	4.62	12.70	-3.92	-0.21
HF Sample Covariance Matrix	5.19	4.45	12.82	-3.80	-0.23
HF Sample Covariance Matrix (Lag 1)	5.93	3.99	36.72	-48.83	-2.34

Mean and standard deviation are those of the tracking errors when tracking the S&P 500 index. Both are in percentage per year. Maximum and minimum portfolio weights and short interest are in percentages. Short interest is the sum of negative portfolio weights. The portfolio is monthly rebalanced. Daily estimator uses daily returns, while HF estimator uses five-minute and/or overnight returns.

Table 7

Out of Sample Performance of Minimum Variance Portfolios

Covariance Matrix Estimator	Mean	Std	Max. Weight	Min. Weight	Short Interest
Panel A: Monthly Rebalance (Past 12 months of Data)					
Daily Returns					
Sample Covariance Matrix	15.27	13.51	27.16	-23.53	-28.50
Rolling Sample Estimator (FKO, 0.01)	6.74	33.04	29.48	-24.90	-30.46
Five-minute Returns					
Sample Covariance Matrix (ex. Overnight)	18.08	15.19	15.93	-7.45	-11.00
Sample Covariance Matrix	17.73	14.36	17.28	-9.86	-11.23
Sample Covariance Matrix (Lag 1)	16.44	14.18	23.89	-11.81	-20.30
Rolling Sample Estimator (FKO, 0.03)	14.27	14.27	28.60	-27.00	-31.42
Rolling Sample Estimator (FN, 0.01)	16.92	14.26	25.48	-14.68	-21.31
Rolling Sample Estimator (FN, 0.04)	17.37	14.70	27.35	-16.17	-23.48
Rolling Sample Estimator (FN, 0.1)	17.78	16.38	37.07	-22.73	-29.90
Rolling Sample Estimator (FN, 0.04, 0.95)	17.19	14.95	28.81	-15.68	-22.75
Rolling Sample Estimator (FN, 0.04, 0.5)	16.77	14.52	25.97	-14.43	-20.76
Rolling Sample Estimator (FN, 0.04, 0.1)	16.49	14.23	24.15	-12.56	-20.24
Panel B: Daily Rebalance (Past 12 months of Data)					
Daily Returns					
Sample Covariance Matrix	16.58	14.65	27.86	-23.87	-28.70
Rolling Sample Estimator (FKO, 0.01)	17.24	14.90	32.64	-27.63	-33.31
Five-minute Returns					
Sample Covariance Matrix (ex. Overnight)	18.88	15.12	16.17	-8.08	-11.01
Sample Covariance Matrix	18.74	14.36	17.43	-10.73	-11.25
Sample Covariance Matrix (Lag 1)	17.29	14.29	24.05	-11.87	-20.35
Rolling Sample Estimator (FKO, 0.03)	17.55	14.73	32.01	-32.96	-31.89
Rolling Sample Estimator (FN, 0.01)	17.65	13.82	25.78	-14.97	-21.64
Rolling Sample Estimator (FN, 0.04)	17.68	13.28	27.96	-16.70	-24.09
Rolling Sample Estimator (FN, 0.1)	17.71	13.12	32.45	-20.60	-28.06
Panel C: Monthly Rebalance (Past 3 months of Data)					
Daily Sample Covariance Matrix	12.56	16.45	42.78	-43.73	-77.19
HF Sample Covariance Matrix (ex. Overnight)	17.60	14.34	19.20	-10.72	-11.43
HF Sample Covariance Matrix	17.17	13.73	22.06	-12.48	-13.46
HF Sample Covariance Matrix (Lag 1)	17.05	14.17	36.07	-16.90	-25.44

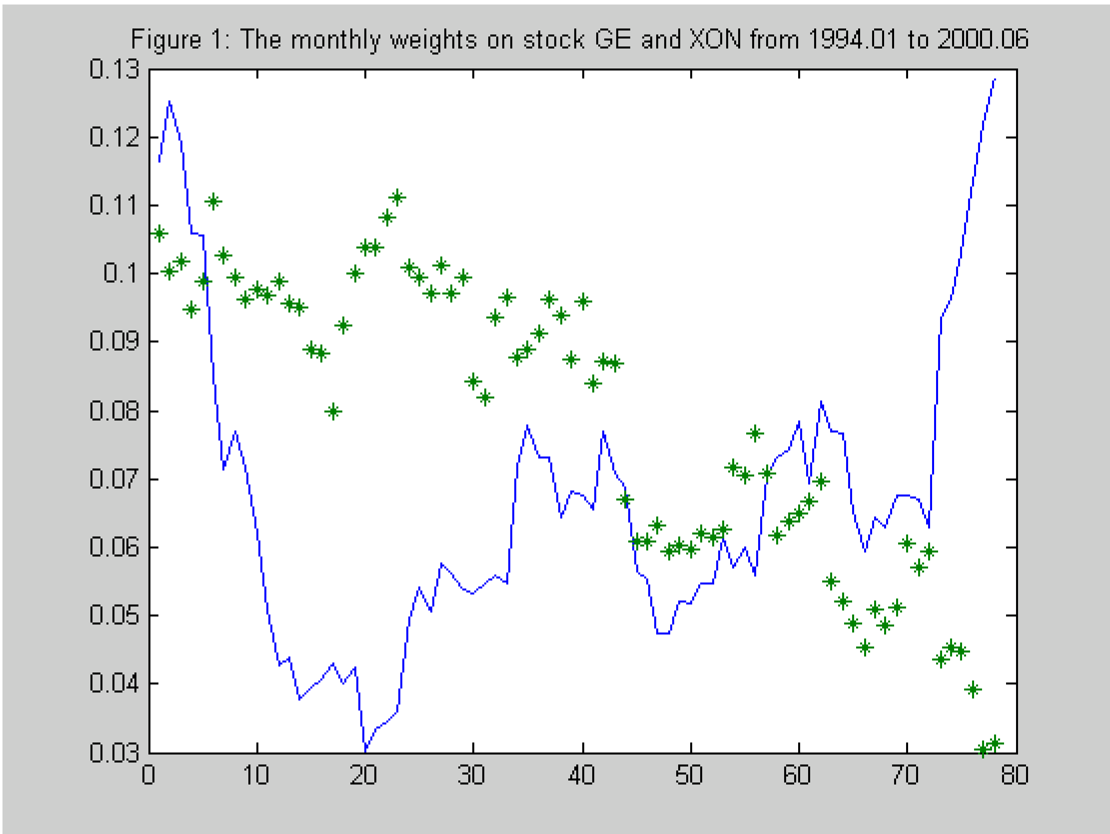
Mean and standard deviation are those of the minimum variance portfolios. Both are in percentage per year. Maximum and minimum portfolio weights and short interest are in percentages. Short interest is the sum of negative portfolio weights. In the last three estimators of Panel A, the first parameter is the decay rate, and the second is the autoregressive coefficient. In Panel C, HF estimator uses five-minute and overnight returns.

Table 8

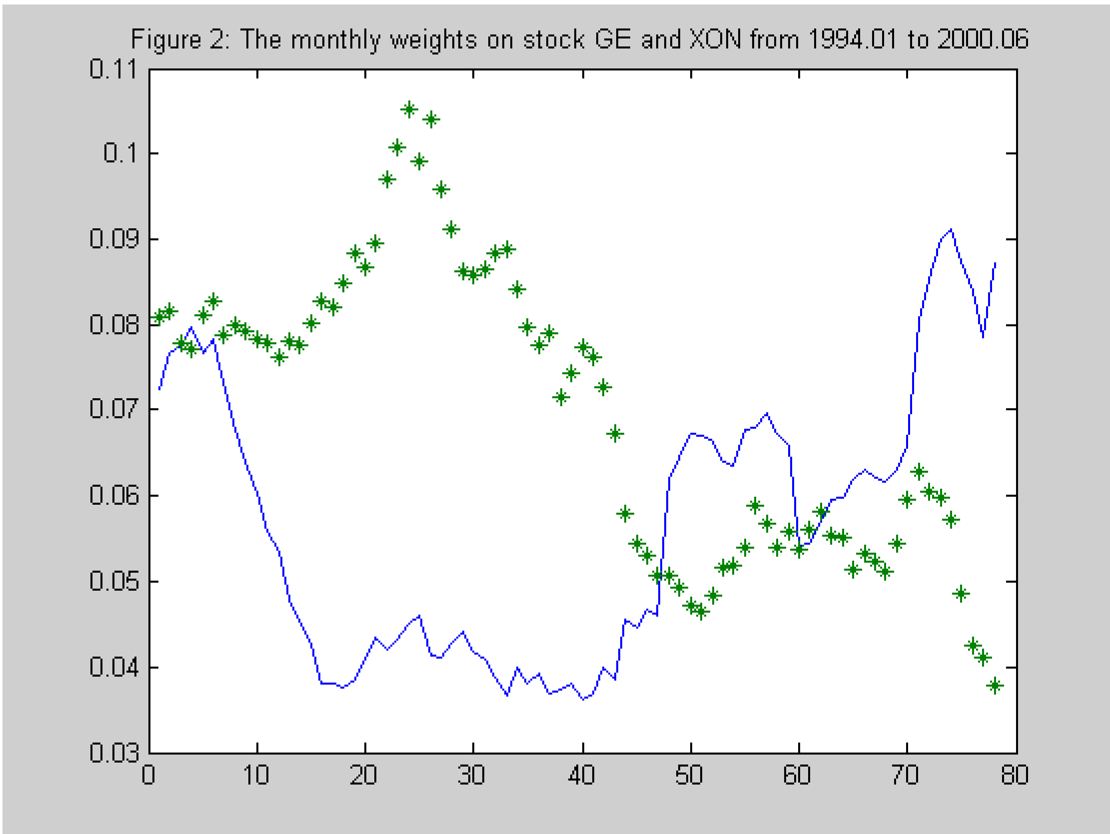
Out of Sample Performance of Minimum Tracking Error Portfolios (Simulation Exercise)

Covariance Matrix Estimator	Mean	Std	Max. Weight	Min. Weight	Short Interest
Monthly Rebalance					
Panel A: Past 12 months of Data					
Daily Sample Covariance Matrix	-0.04	4.82	13.35	-2.35	-0.88
HF Sample Covariance Matrix	0.16	4.51	10.24	0.33	0.00
Panel B: Past 6 months of Data					
Daily Sample Covariance Matrix	0.16	5.25	15.98	-4.67	-2.90
HF Sample Covariance Matrix	0.08	4.54	10.48	0.17	0.00
Panel C: Past 3 months of Data					
Daily Sample Covariance Matrix	-0.12	6.28	21.58	-10.08	-10.40
HF Sample Covariance Matrix	-0.01	4.58	10.76	-0.02	-0.01
Daily Rebalance					
Panel D: Past 12 months of Data					
Daily Sample Covariance Matrix	-0.20	4.84	13.64	-2.60	-0.85
HF Sample Covariance Matrix	-0.20	4.56	10.29	0.31	0.00
Panel E: Past 6 month of Data					
Daily Sample Covariance Matrix	-0.44	5.19	16.68	-5.18	-2.78
HF Sample Covariance Matrix	-0.32	4.57	10.54	0.13	0.00
Panel F: Past 3 months of Data					
Daily Sample Covariance Matrix	-0.29	6.24	23.86	-11.67	-10.26
HF Sample Covariance Matrix	-0.29	4.57	10.90	-0.12	0.00

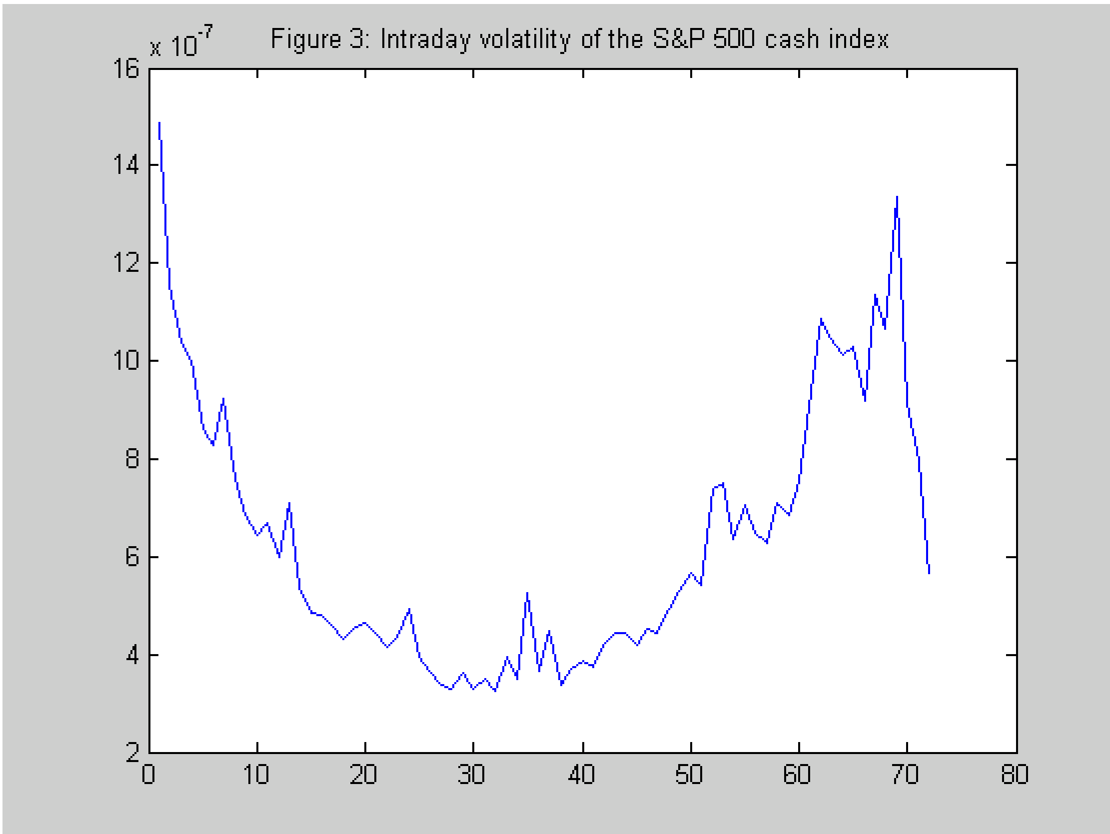
The sample covariance matrix of the daily excess returns of the 30 DJIA stocks relative to the S&P 500 index from January 2, 1993 to June 30, 2000 is used as the true covariance matrix. Intraday returns are simulated using the true covariance matrix. Daily returns are constructed with the sum of 72 intraday returns. Mean and standard deviation are those of the tracking errors when tracking the S&P 500 index. Both are in percentage per year. Maximum and minimum portfolio weights and short interest are in percentages. Short interest is the sum of negative portfolio weights. The portfolio is monthly rebalanced in panel A, B, C and daily rebalanced in panel D, E, F. Daily estimator uses daily returns, while HF estimator uses intraday returns. The results are averages over 1000 independent replications.



Note: This figure shows the monthly weights on GE and XON from January 1994 to June 2000, when the minimum tracking error portfolios are formed using 30 DJIA stocks to tracking the S&P 500 index. The monthly covariance matrix estimator is constructed using the sample covariance matrix based on daily returns. "-" stands for GE, and "*" stands for XON.



Note: This figure shows the monthly weights on GE and XON from January 1994 to June 2000, when the minimum tracking error portfolios are formed using 30 DJIA stocks to tracking the S&P 500 index. The monthly covariance matrix estimator is constructed using the sample covariance matrix based on five-minute returns and overnight returns, adjusted by their first order autocorrelations and cross-correlations. "-" stands for GE, and "*" stands for XON.



Note: The figure shows the return volatility of the S&P 500 cash index over the trading day. Intraday volatility is computed based on 72 five-minute returns from 10:00 EST to 16:00 EST each trading day.