Properties of Realized Variance for Pure Jump Processes: Calendar Time Sampling versus Business Time Sampling

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determine optimal sampling frequency
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IBM sampling scheme: BTS always outperforms CTS (!)

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IBM sampling frequency: 12 seconds (bias and MSE reduction over 65%!)
IBM sampling scheme: BTS always outperforms CTS

Motivation & Related Literature

- Realized variance (RV) defined as the sum of squared intra-period returns as a "feasible" or "discretized" version of the quadratic variation process

  see e.g. Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Labys (2003), Barndorff-Nielsen and Shephard (2003), French, Schwert, and Stambaugh (1987), Hsieh (1988), Meddahi (2002), Merton (1980)

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A Pure Jump Process For High Frequency Financial Data

- Let the logarithmic price at time t, $P(t)$, follow CPP-MA(s), i.e.

$$P(t) = P(0) + \sum_{j=1}^{M(t)} (\varepsilon_j + \eta_j) \qquad \text{where} \qquad \eta_j = \rho_0 \nu_j + \rho_1 \nu_{j-1} + \ldots + \rho_s \nu_{j-s}$$

where $\varepsilon_j \sim$iid $\mathcal{N}\left(\mu_\varepsilon, \sigma_\varepsilon^2\right)$, $\nu_j \sim$ iid $\mathcal{N}\left(\mu_\nu, \sigma_\nu^2\right)$, and $M(t)$ is a Poisson process with instantaneous intensity $\lambda(t) > 0$

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- Pure jump processes not widely used in finance (that is relative to for example SV and ARCH) . . . notable exceptions include:

Low Frequency:	Press (1967), Maheu and McCurdy (2003, 2004), Piazzesi (2004)
High Frequency:	Bowsher (2002), Oomen (2002b), Rogers and Zane (1998) Rydberg and Shephard (2003)
Option Pricing:	Barndorff-Nielsen and Shephard (2004), Carr and Wu (2003) Geman, Madan, and Yor (2001), Mürmann (2001)

Market Microstructure Noise Interpretation of the Model

- Let P_k denote the logarithmic price after the k^{th} transaction, i.e.

$$P_k = P_0 + \sum_{j=1}^{k} \varepsilon_j + \sum_{j=1}^{k} \eta_j = P_k^e + \sum_{j=1}^{k} \eta_j$$

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- Restrict MA(s) parameters to avoid accumulation of noise. For example, for MA(1) impose $\rho_0 = -\rho_1 = 1$

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✓ Negative serial correlation of returns (OK for transaction data)

✓ Higher order MA(s) and different restrictions can lead to positive serial correlation

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- In calendar time returns are (highly) non-normal
- The joint characteristic function of $\{R(t_1|\tau_1), R(t_2|\tau_2), R(t_3|\tau_3), R(t_4|\tau_4)\}$ *conditional on intensity process* for the restricted CPP-MA(1) can be derived as:

$$
\begin{aligned}
& e^{-\lambda_4-\lambda_3-\lambda_2-\lambda_1} + e^{-\lambda_2-\lambda_3}\Upsilon(1)\,\Lambda_1\left(e^{c_1}\lambda_1\right)\Upsilon(4)\,\Lambda_1\left(e^{c_4}\lambda_4\right)\exp\left\{\xi_1\xi_4\sigma_\nu^2-\lambda_{1,2}-\lambda_{2,3}-\lambda_{3,4}\right\} \\
& + e^{-\lambda_2-\lambda_4}\Upsilon(1)\,\Lambda_1\left(e^{c_1}\lambda_1\right)\Upsilon(3)\,\Lambda_1\left(e^{c_3}\lambda_3\right)\exp\left\{\xi_1\xi_3\sigma_\nu^2-\lambda_{1,2}-\lambda_{2,3}\right\} + e^{-\lambda_4-\lambda_3-\lambda_2}\Upsilon(1)\,\Lambda_1\left(e^{c_1}\lambda_1\right) \\
& + e^{-\lambda_1-\lambda_3}\Upsilon(2)\,\Lambda_1\left(e^{c_2}\lambda_2\right)\Upsilon(4)\,\Lambda_1\left(e^{c_4}\lambda_4\right)\exp\left\{\xi_2\xi_4\sigma_\nu^2-\lambda_{2,3}-\lambda_{3,4}\right\} + e^{-\lambda_4-\lambda_3-\lambda_1}\Upsilon(2)\,\Lambda_1\left(e^{c_2}\lambda_2\right) \\
& + e^{-\lambda_3}\Upsilon(1)\,\Lambda_1\left(e^{c_1}\lambda_1\right)\Upsilon(2)\,\Lambda_1\left(e^{c_2}\lambda_2\right)\Upsilon(4)\,\Lambda_1\left(e^{c_4}\lambda_4\right)\Psi_{12}\Psi_{24} + e^{-\lambda_1-\lambda_2}\Upsilon(3)\,\Lambda_1\left(e^{c_3}\lambda_3\right)\Upsilon(4)\,\Lambda_1\left(e^{c_4}\lambda_4\right)\Psi_{34} \\
& + e^{-\lambda_2}\Upsilon(1)\,\Lambda_1\left(e^{c_1}\lambda_1\right)\Upsilon(3)\,\Lambda_1\left(e^{c_3}\lambda_3\right)\Upsilon(4)\,\Lambda_1\left(e^{c_4}\lambda_4\right)\Psi_{13}\Psi_{34} + e^{-\lambda_3-\lambda_4}\Upsilon(1)\,\Lambda_1\left(e^{c_1}\lambda_1\right)\Upsilon(2)\,\Lambda_1\left(e^{c_2}\lambda_2\right)\Psi_{12} \\
& + e^{-\lambda_1}\Upsilon(2)\,\Lambda_1\left(e^{c_2}\lambda_2\right)\Upsilon(3)\,\Lambda_1\left(e^{c_3}\lambda_3\right)\Upsilon(4)\,\Lambda_1\left(e^{c_4}\lambda_4\right)\Psi_{23}\Psi_{34} + e^{-\lambda_1-\lambda_4}\Upsilon(2)\,\Lambda_1\left(e^{c_2}\lambda_2\right)\Upsilon(3)\,\Lambda_1\left(e^{c_3}\lambda_3\right)\Psi_{23} \\
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\end{aligned}
$$

where $\Psi_{ij} = \left(\exp\left\{\xi_i\xi_j\sigma_\nu^2-\lambda_{i,j}\right\}+\Lambda_1\left(\lambda_{i,j}\right)\right)$, $\Upsilon(i) = \exp\left(\xi_i^2 c_0 + \left(e^{c_i}-1\right)\lambda_i\right)$, and $\Lambda_q(x) = 1-\frac{\Gamma(q,x)}{\Gamma(q)}$.

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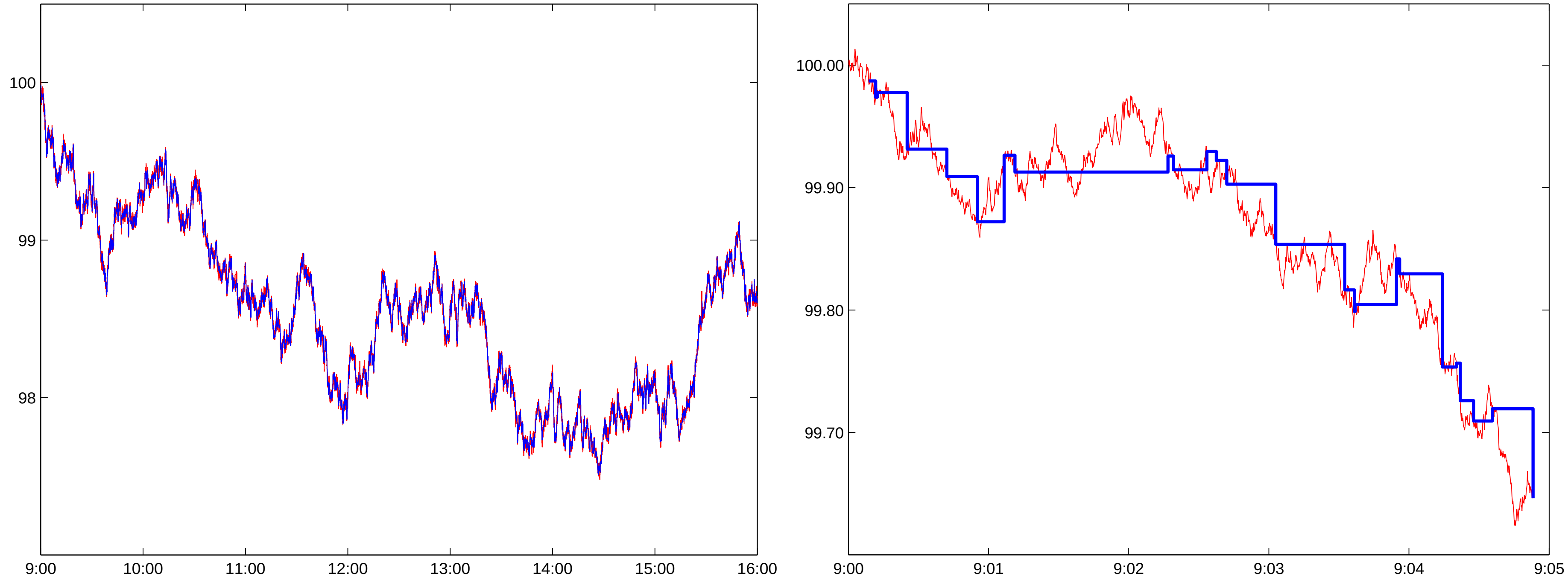
- Stochastic intensity for the CPP-MA(s) ⇔ stochastic volatility for SV
- CPP-MA(s) can capture seasonals, ACD & ARCH effects, serial correlation, fat tails
- A fundamental difference with diffusive process: CPP-MA(s) is of finite variation



Alternative Sampling Schemes

- ⋆ General Time Sampling: Under GTS_N, the price process is sampled at time points $\{t_0^g, \ldots, t_N^g\}$ over the interval $[t_0, t_0 + T]$ such that $t_0^g = t_0$, $t_N^g = t_0 + T$, and $t_i^g < t_{i+1}^g$.

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- Calendar Time Sampling: Under CTS_N, the price process is sampled at equidistantly spaced points in calendar time over the interval $[t_0, t_0 + T]$, i.e. $t_i^p = t_0 + i\delta$ for $i = \{0, \ldots, NT\}$ where $N = \delta^{-1}$.

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- ⋆ Business Time Sampling: Under BTS_N, the price process is sampled at equidistantly spaced points in business time over the interval $[t_0, t_0 + T]$, i.e. t_i^b for $i = \{0, \ldots, NT\}$ such that $t_0^b = t_0$, $t_N^b = t_0 + T$ and

$$\int_{t_i^b}^{t_{i+1}^b} \lambda(u)du = \frac{1}{NT} \int_{t_0}^{t_0+T} \lambda(u)du \equiv \lambda_N$$

✓ Throughout I translate N to corresponding sampling frequency in minutes

RV in Absence of Market Microstructure Noise

- To set the stage I first consider the CPP-MA(0) (RV is unbiased)

$$MSE\left(GTS_N\right) = \sum_{i=1}^{N}\left(3\sigma_\varepsilon^4\lambda_i^2 + 3\sigma_\varepsilon^4\lambda_i\right) + 2\sigma_\varepsilon^4\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\lambda_i\lambda_j - \lambda_{(0,1)}^2\sigma_\varepsilon^4$$

⋆ Notation: $\lambda_i = \int_{t_i-\tau_i}^{t_i}\lambda(u)du$ and $\lambda_{(0,1)} = \int_0^1\lambda(u)du$

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…which simplifies under BTS to:

$$MSE\left(BTS_N\right) = 2N^{-1} \left(\lambda_{(0,1)}\sigma_\varepsilon^2\right)^2 + 3\sigma_\varepsilon^2 \left(\lambda_{(0,1)}\sigma_\varepsilon^2\right)$$

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- RV is inconsistent under pure jump process
- Consistency in "diffusion" limit where $\lambda \rightarrow \infty$ while $\lambda\sigma_\varepsilon^2$ constant

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Absence of Market Microstructure Noise

- The difference in MSE among different sampling schemes can be derived as:

$$MSE\left(GTS_N\right) - MSE\left(BTS_N\right) = 3\sigma_\varepsilon^4 \sum_{i=1}^{N} \left(\lambda_i^2 - \lambda_N^2\right) + 2\sigma_\varepsilon^4 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\lambda_i \lambda_j - \lambda_N^2\right)$$
$$= 2\sigma_\varepsilon^4 \sum_{i=1}^{N} \vartheta_i^2 > 0$$

where $\vartheta_i = \int_{t_i-\tau_i}^{t_i} \lambda\left(u\right) du - \lambda_N$

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- *In the absence of market microstructure noise, the realized variance measure under BTS is more efficient than under any other conceivable sampling scheme*

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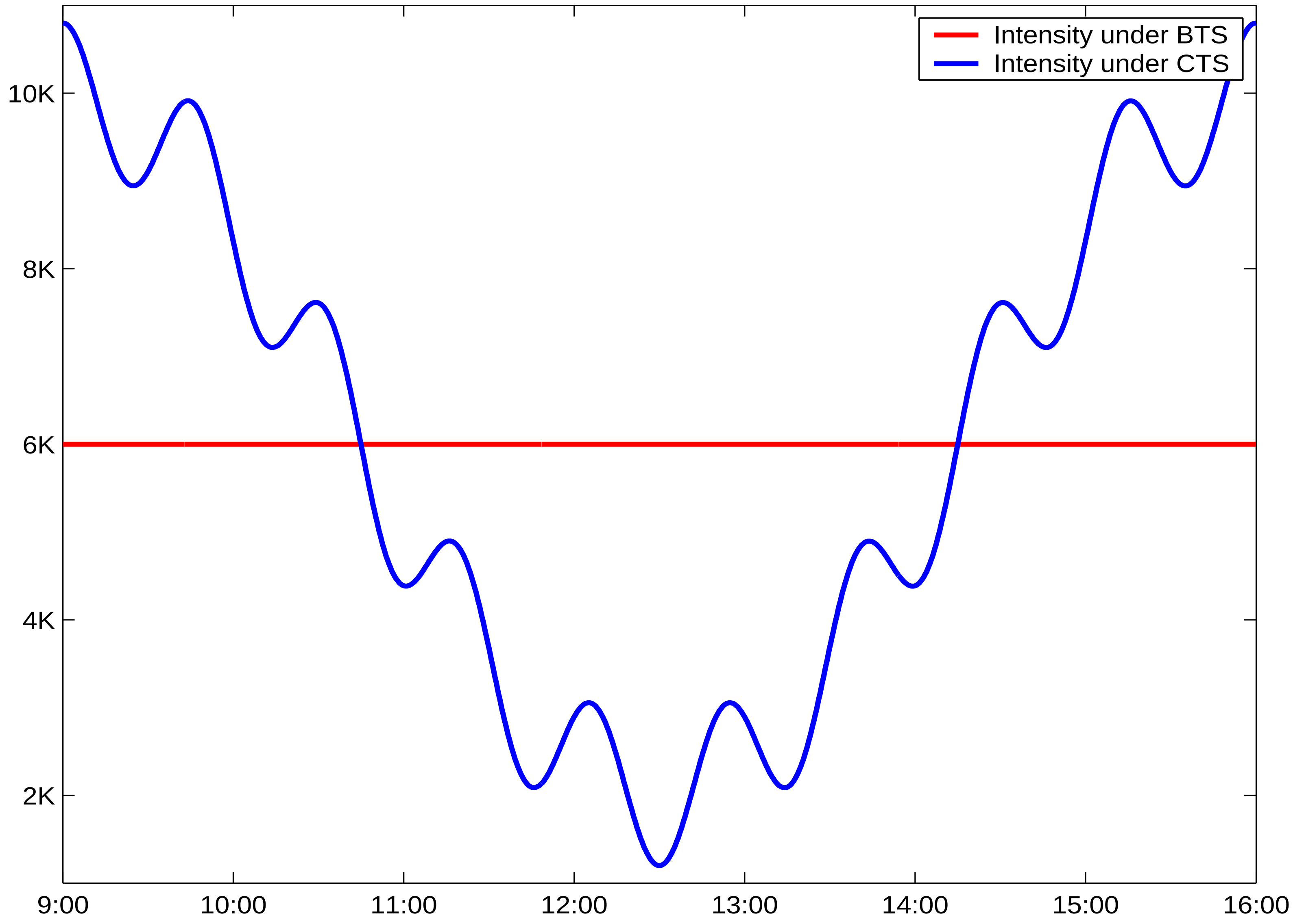
- *In the absence of market microstructure noise, the realized variance measure under BTS is more efficient than under any other conceivable sampling scheme*

- The efficiency gain associated with BTS, relative to CTS, increases with

  (i) an increase in the variability of trade intensity
  (ii) an increase in the variance of the price innovations

Integrated Incremental Intensity under BTS and CTS: ϑ



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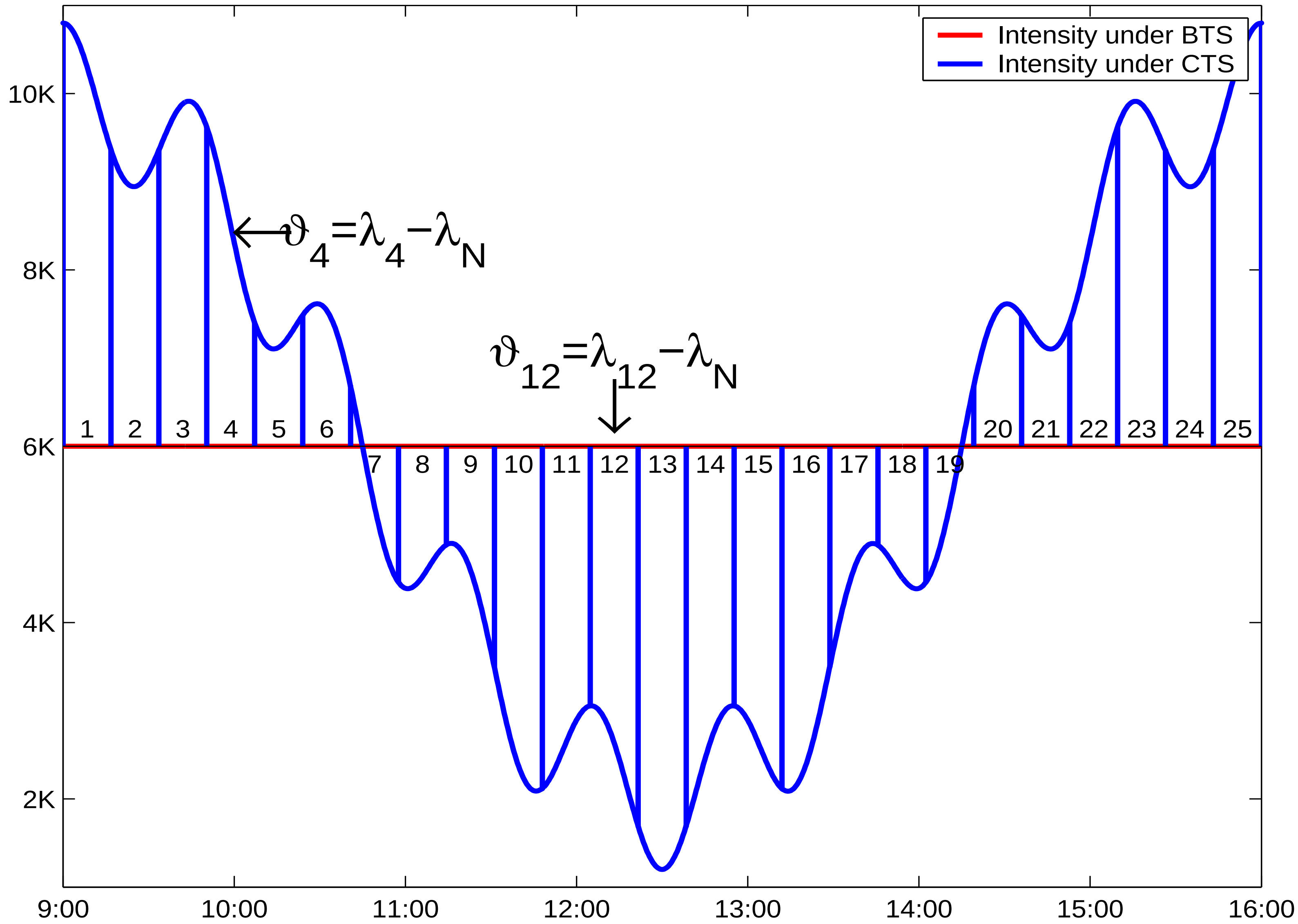

Presence of Market Microstructure Noise – The Bias . . .

- Now move on to the CPP-MA(1), i.e. first order dependence in noise component

$$Bias\left(GTS_N\right) = E_\lambda \left[\sum_{i=1}^{N} R\left(t_i|\tau_i\right)^2\right] - \lambda_{(0,1)}\sigma_\varepsilon^2 = 2\sigma_\nu^2 \sum_{i=1}^{N} \left(1 - e^{-\lambda_i}\right) > 0$$

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- *In the presence of first order market microstructure noise, the bias of the realized variance measure under BTS is larger than under any other sampling scheme*

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- *In the presence of first order market microstructure noise, the bias of the realized variance measure under BTS is larger than under any other sampling scheme*

- When $N = 1$ or $N \to \infty$ all sampling schemes are equivalent.

$$\lim_{N\to\infty} Bias\left(GTS_N\right) = 2\sigma_\nu^2\lambda_{(0,1)}$$

Presence of Market Microstructure Noise – The MSE …

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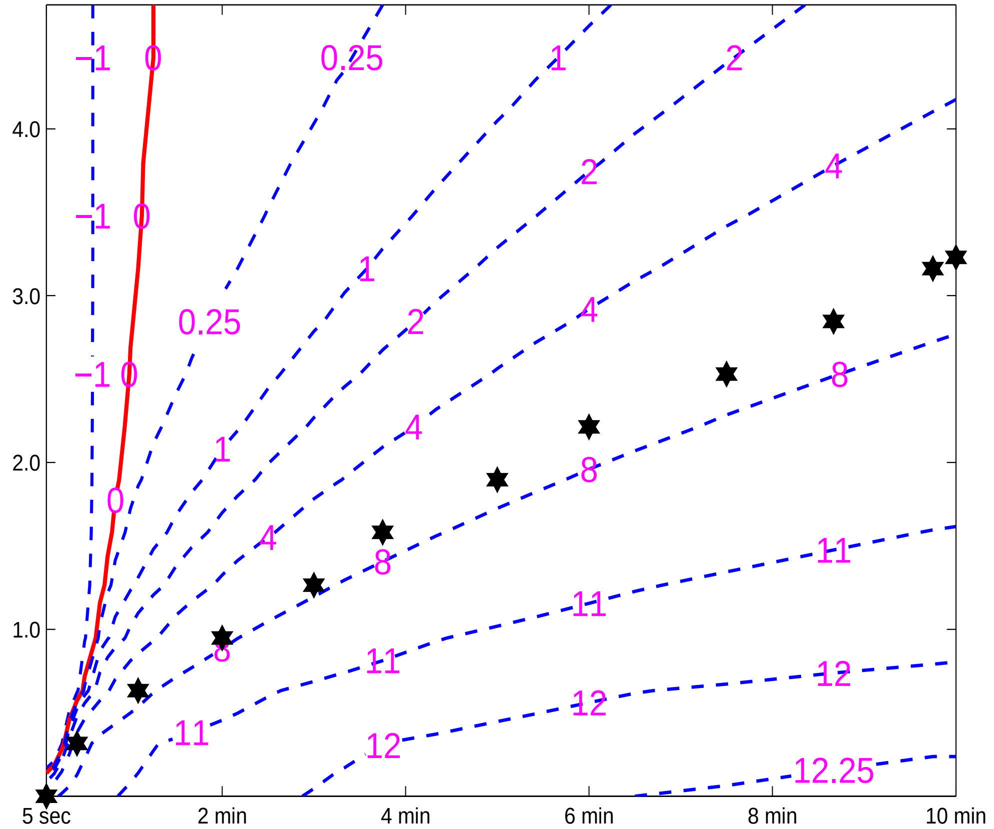

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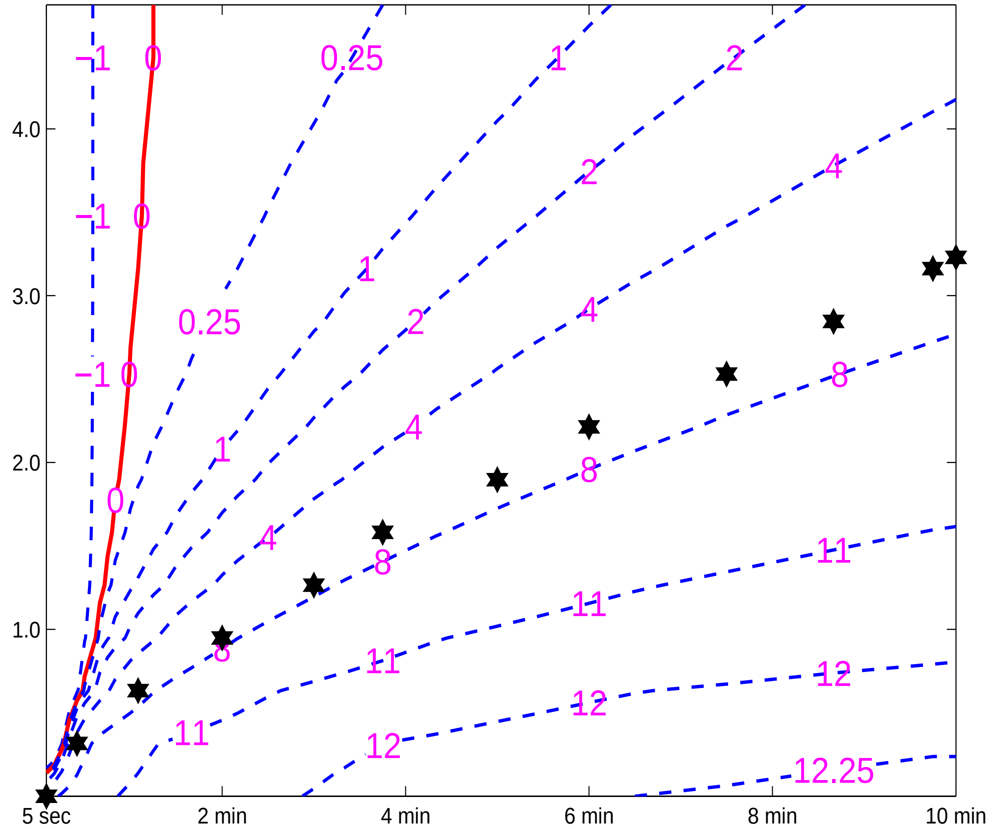

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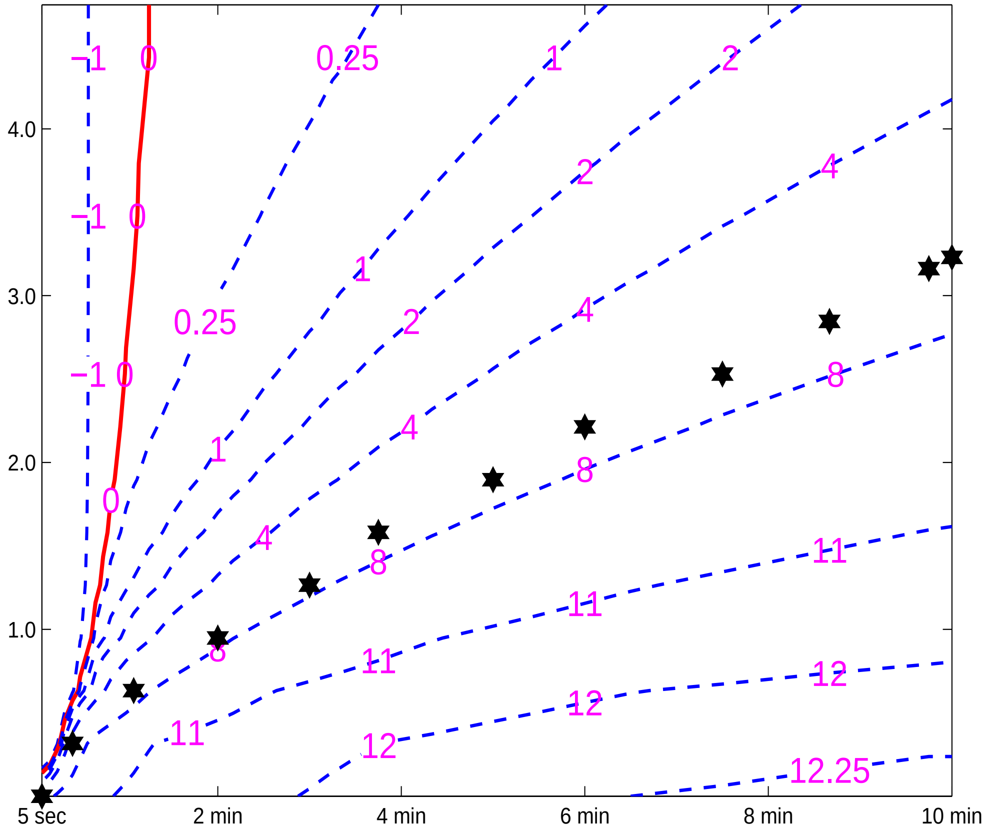
- Similar results for higher order dependence, i.e. CPP-MA(q)



Empirical Analysis

- Estimate the model parameters of the restricted CPP-MA(1) and CPP-MA(2) using IBM transaction data

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  (ii) measure the improvement in MSE resulting from BTS relative to CTS.

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- Data available through Trade and Quote (TAQ) database from NYSE.

  January 1, 2000 until 31 August 2003 (917 days)

  Transactions from all exchanges between 9.45 and 16.00

  Filter for instantaneous price reversals (detect 1358)

  Total number of transactions is 5,522,929

Empirical Analysis – Parameter Estimation

- σ_ε, σ_ν, and ρ estimated in "business time" using all transactions

$$Cov\left(R_k, R_{k-1}\right) = -\left(1-\rho\right)^2 \sigma_\nu^2$$

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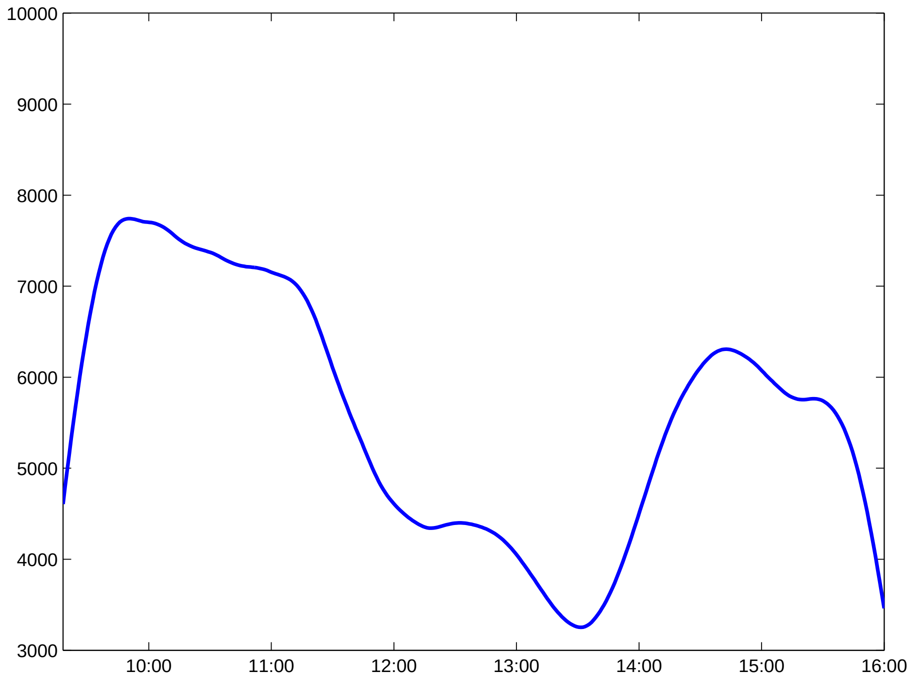

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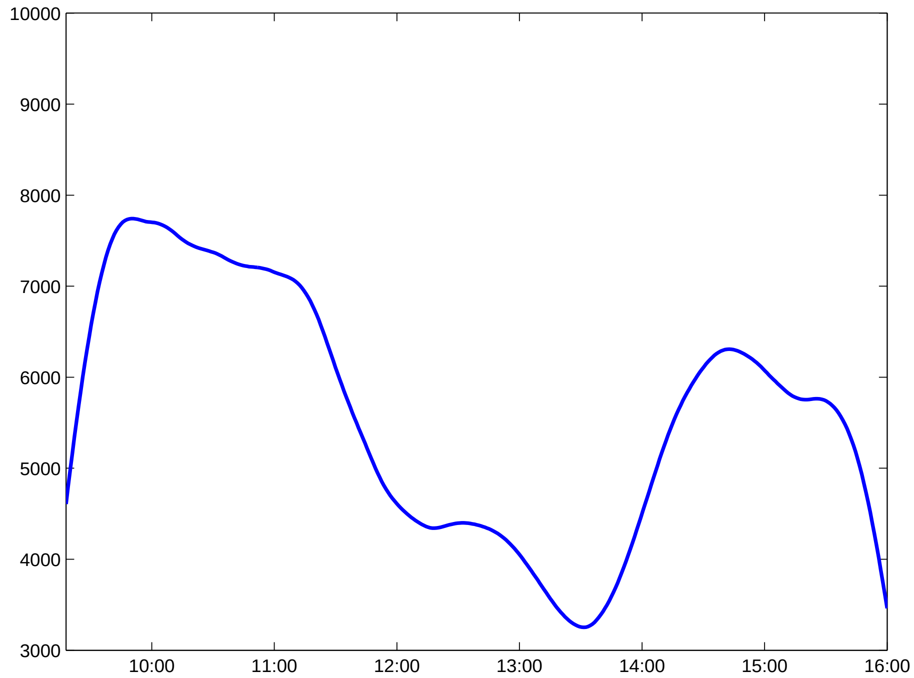

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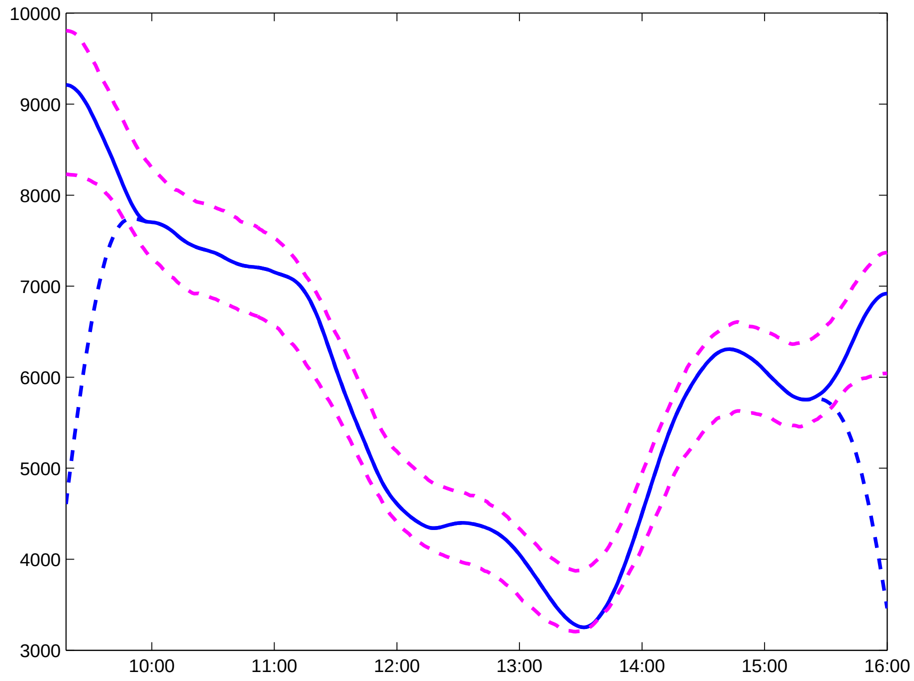

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Impact of Measurement Error on Optimal Sampling Frequency

1 Simulate transaction data based on the CPP-MA(2) model ($\rho = 0.6$, $\lambda_{(0,1)} = 5000$, $\sigma_\nu/\sigma_\varepsilon = 1.1$, annualized return volatility of 25%)

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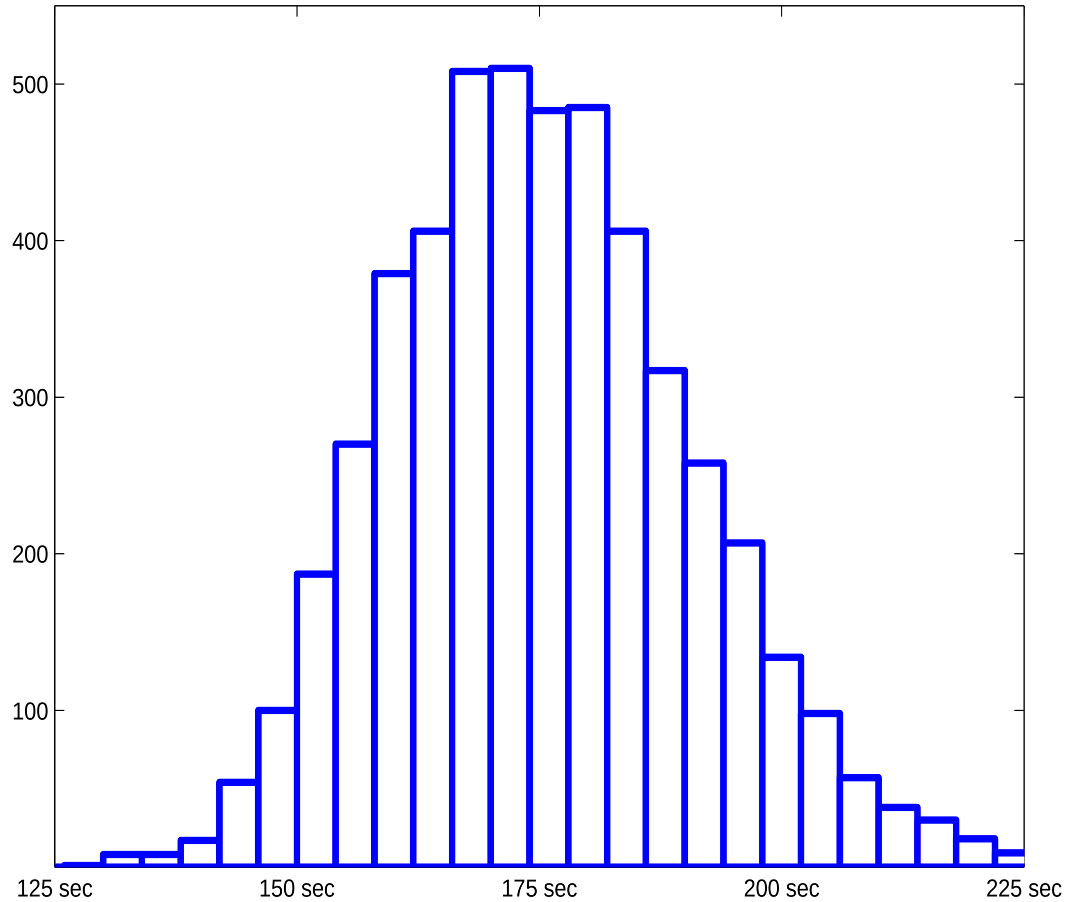
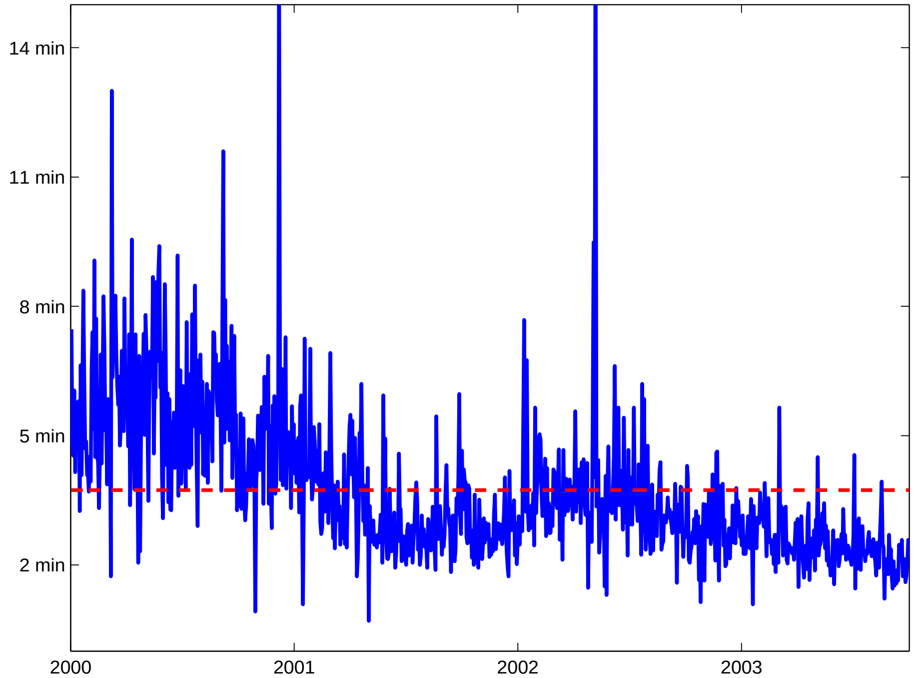


- No bias due to measurement error! (this is particularly important when analyzing illiquid securities)

Optimal Sampling Frequency and Sampling Scheme Efficiency

Optimal Sampling Frequency



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- Downward trend

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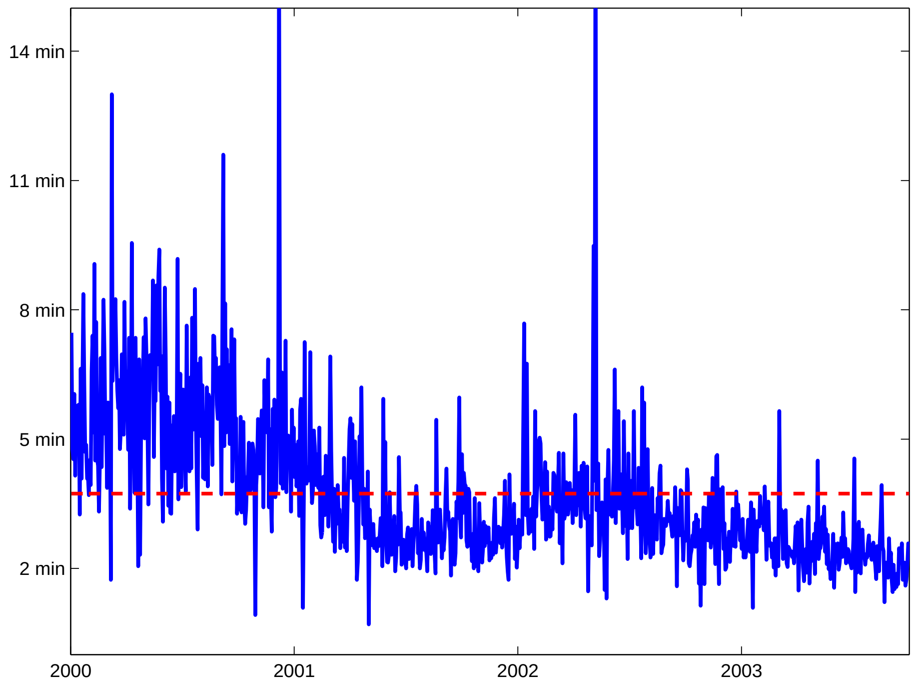

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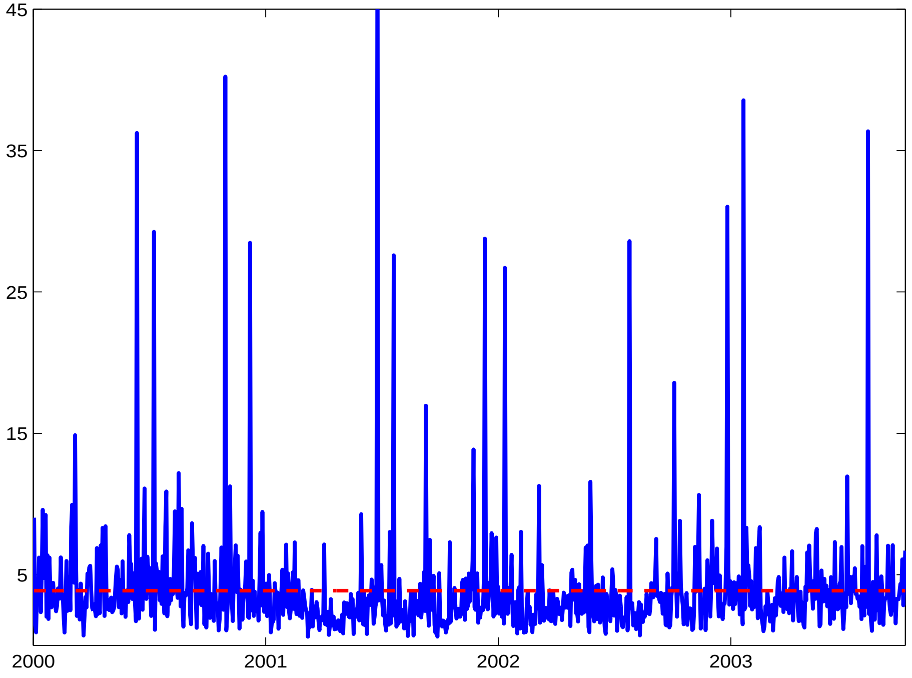


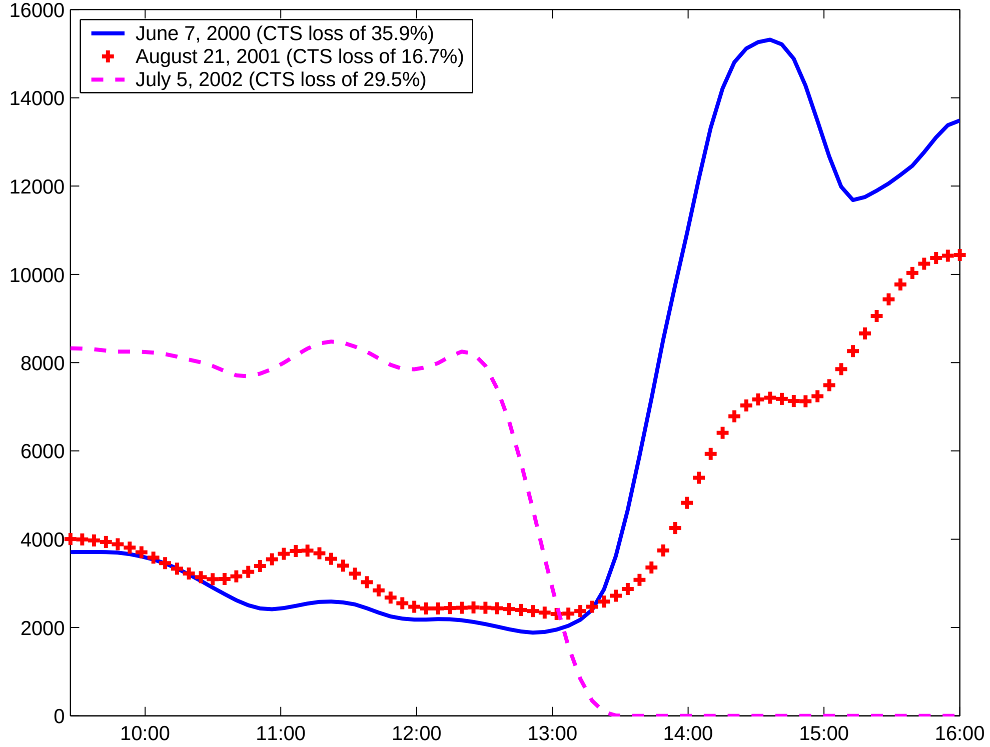
- Largest on days with highly irregular trading patterns, early market closures, or sudden moves in market activity

CTS loss on Irregular Trading Days

- On June 7, 2000 Dow Jones Business News headlined:

"*Wall Street Closes Higher, Paced By IBM Rebound On Goldman Sachs Comments*"

"…A late-day rally in IBM shares helped push stocks higher Wednesday…International Business Machines (IBM) jumped 8 3/8 to 120 3/4 after Goldman Sachs analyst Laura Conigliaro told CNBC that the computer maker should see revenue improvements in the second half of the year"



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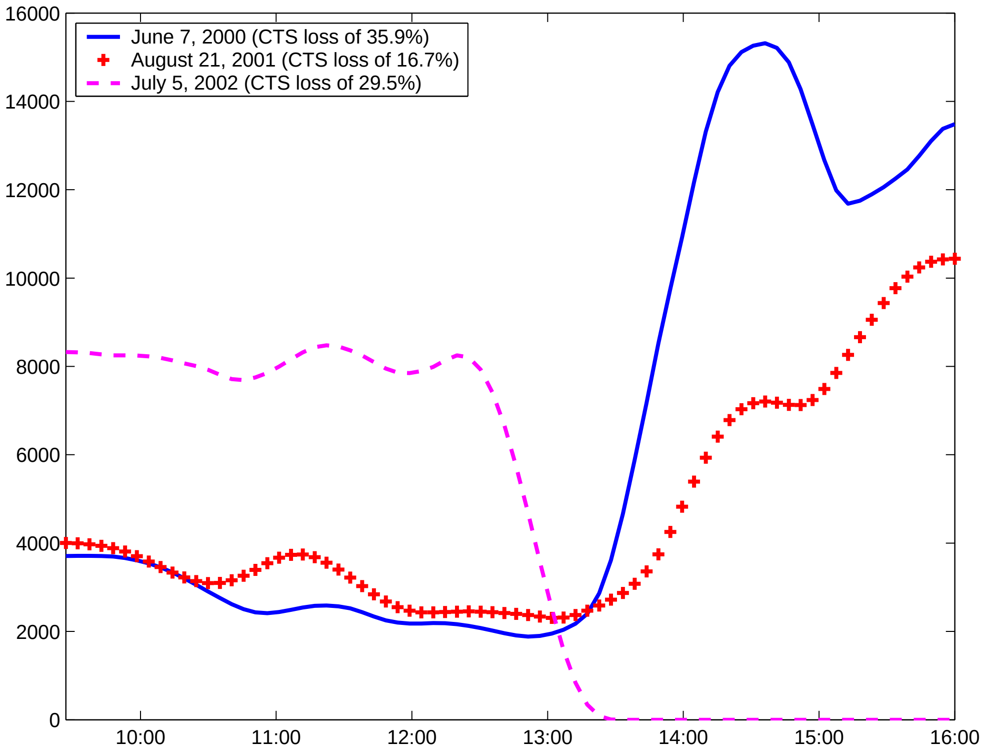

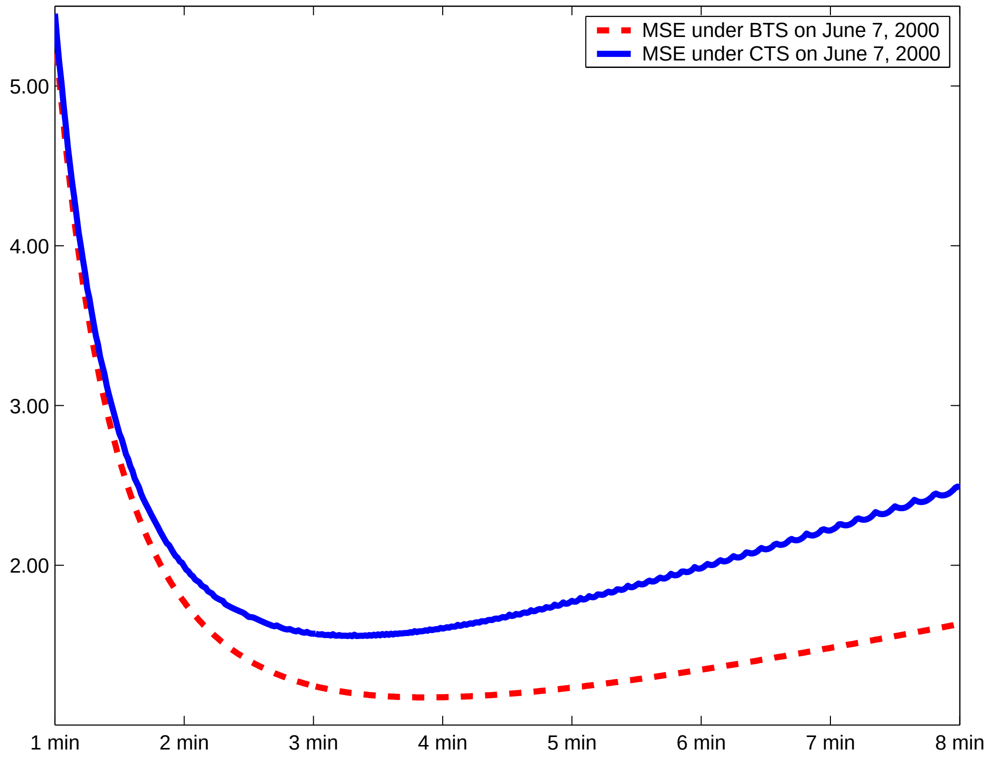

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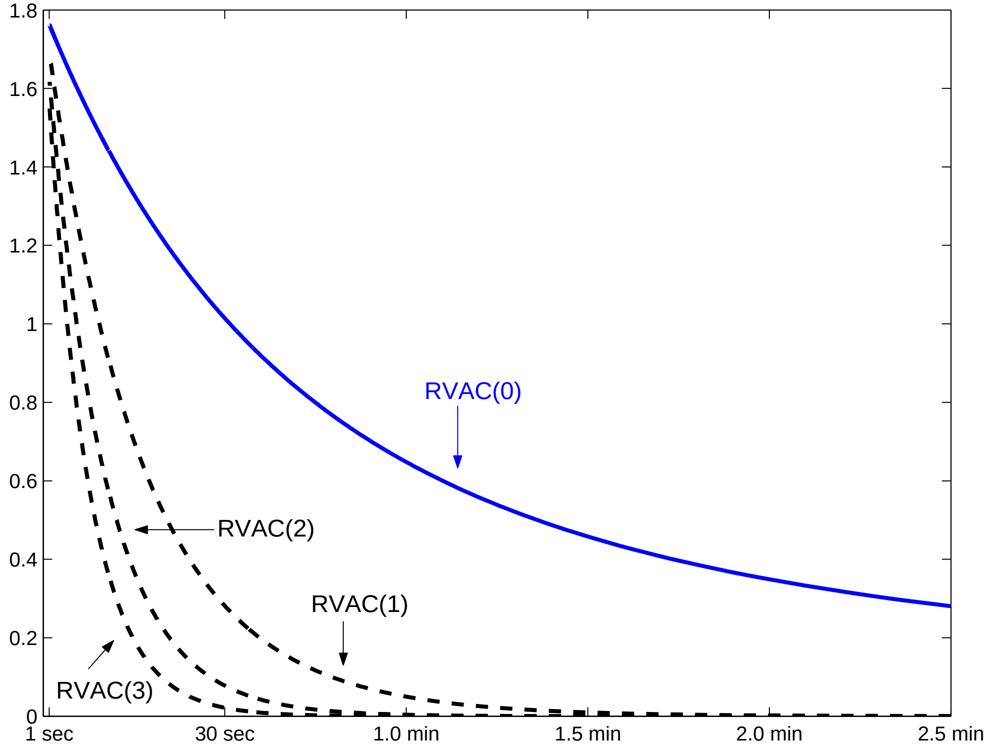
  Close relation of optimal sampling frequency to noise ratio

- Now turn to Newey-West type bias corrected realized variance (Hansen and Lunde 2004):

$$RVAC_{N,q} = \sum_{i=1}^{N} R(t_i|\tau_i)^2 + \sum_{i=1}^{N} R(t_i|\tau_i) \sum_{k=1}^{q} \left(R(t_{i-k}|\tau_{i-k}) + R(t_{i+k}|\tau_{i+k})\right)$$

Bias and MSE of RVAC(q) for CPP-MA(1)

Bias of RVAC(q)



$$Bias = 2N\sigma_\nu^2 \left(1 - e^{-\lambda N}\right) e^{-q\lambda N}$$

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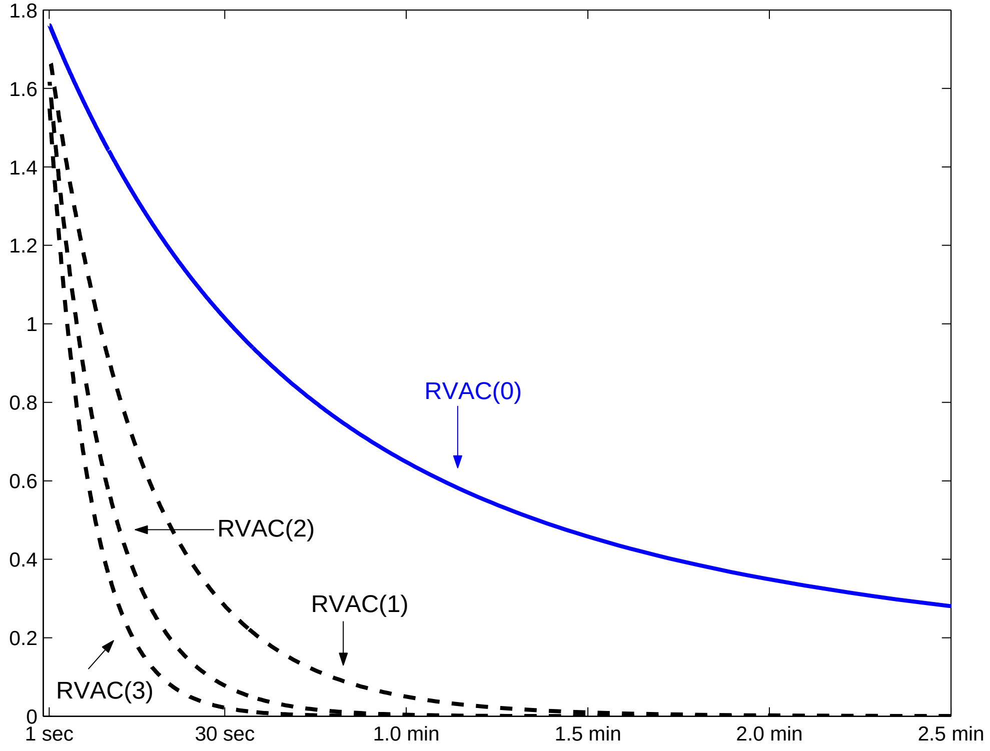

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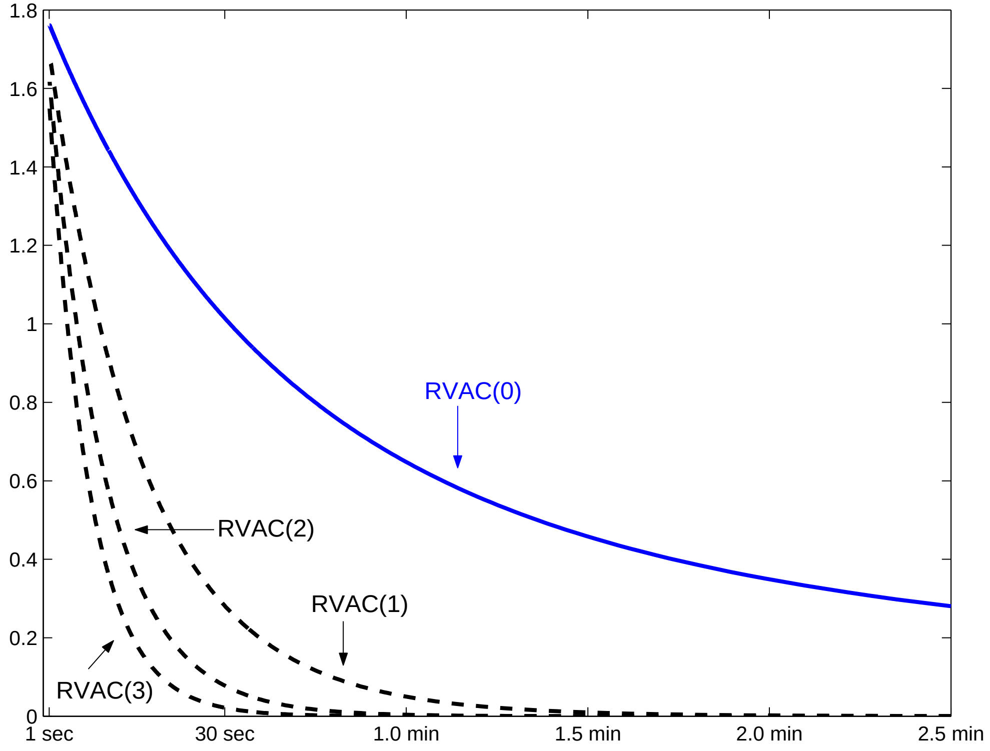
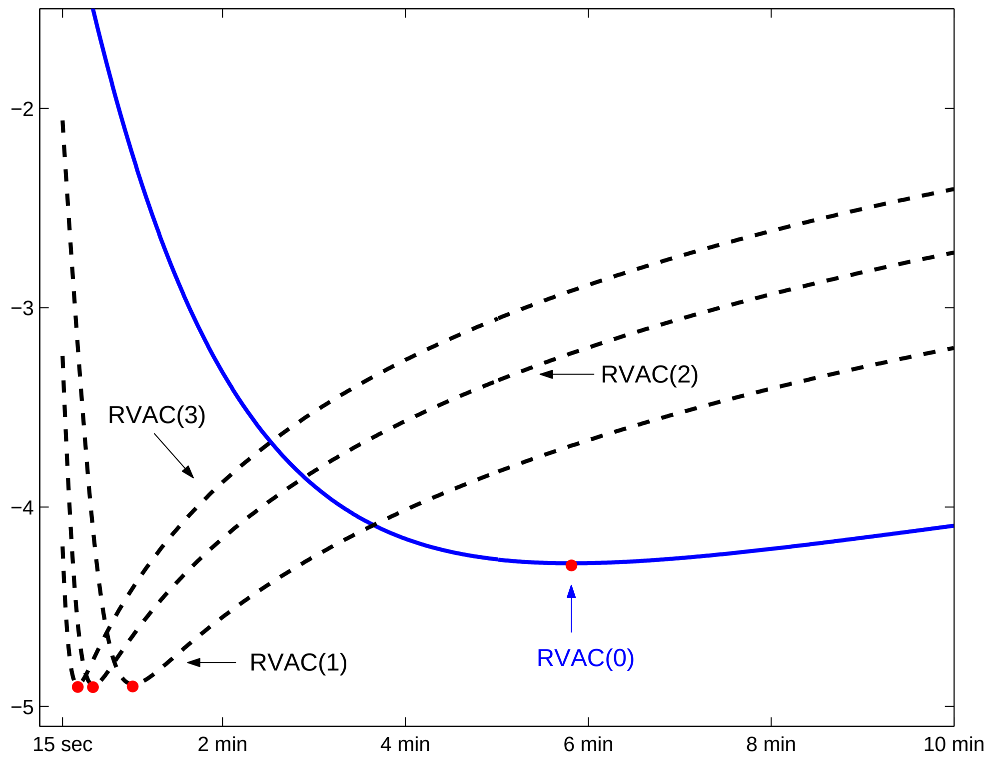


MSE of RVAC(q)



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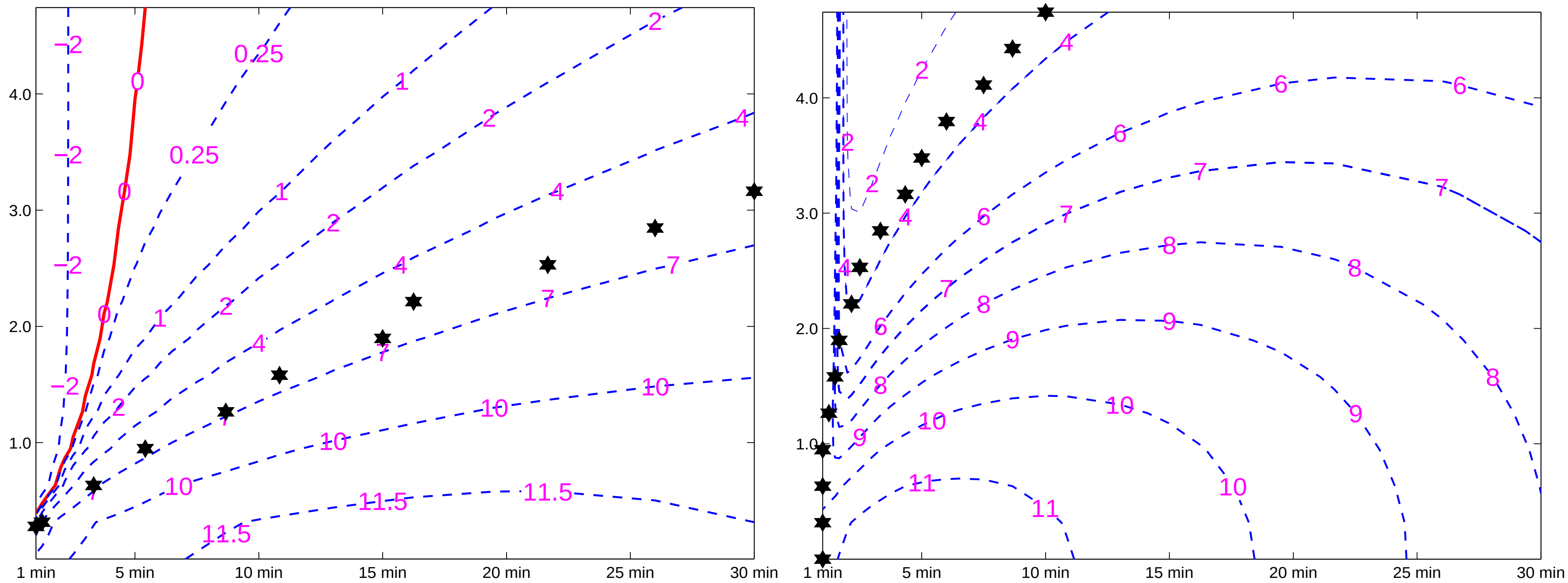
At given frequency, bias decreases but MSE increases with increase in q

CTS loss of RVAC(1) for CPP-MA(1)

- Compare MSE under BTS and CTS for RV (left graph) and RVAC(1) (right graph)



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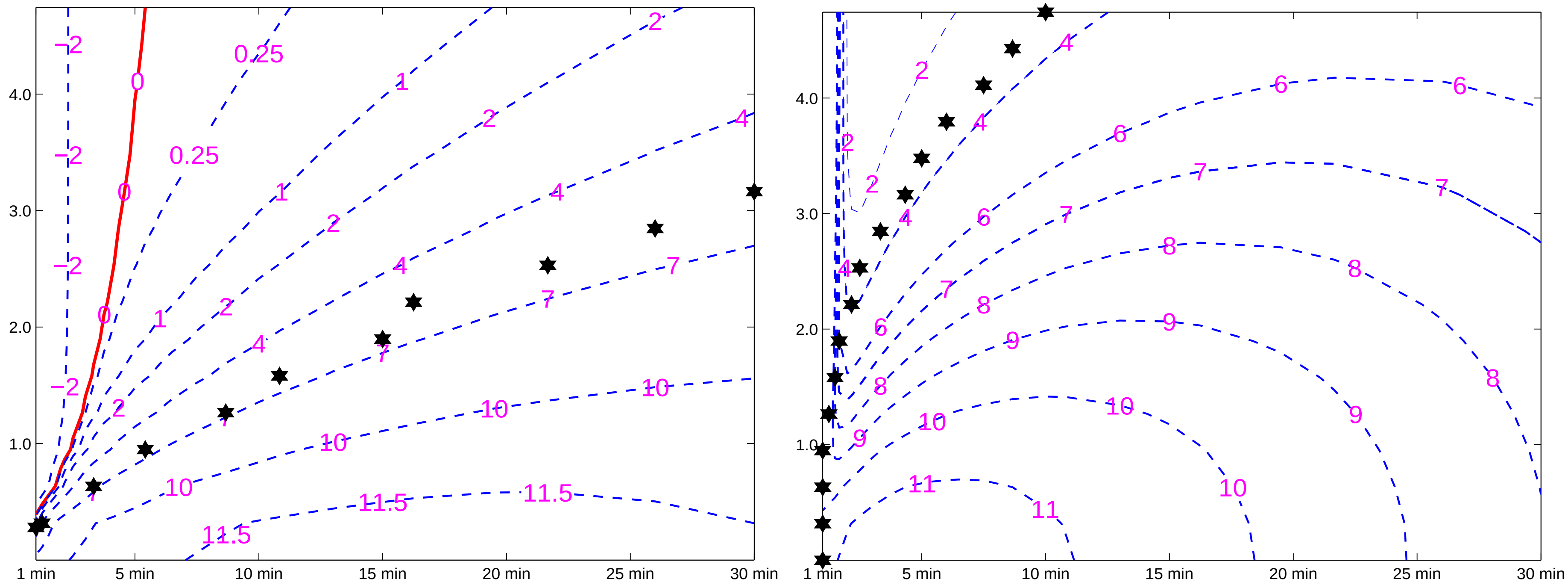

✓ BTS superior to CTS along optimal sampling frequency

✓ Optimal sampling frequency much higher for RVAC(1) than for RV

Empirical Results for IBM

	CPP-MA(1)			RVAC(0)				RVAC(1)			
IBM	$\sigma_\nu/\sigma_\varepsilon$	σ_ε	$\lambda_{(0,1)}$	Freq.	Bias	MSE	CTSloss	Freq.	Bias	MSE	CTSloss
Jan 03	1.66	1.27	8583	170	8.61	5.14	2.51	12	1.23	1.59	2.63
Feb 03											
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- 166 days with 1,224,127 transactions. Relative bias and MSE in percentage points.

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Jul 03	1.22	1.34	6203	140	7.78	2.67	2.19	14	1.79	0.96	3.55
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Jul 03	1.22	1.34	6203	140	7.78	2.67	2.19	14	1.79	0.96	3.55
Aug 03	1.05	1.15	5907	119	7.20	1.25	2.66	14	1.93	0.44	4.71
Jan 03 - Aug 03	**1.39**	**1.28**	**7377**	**148**	**8.01**	**3.83**	**2.52**	**13**	**1.55**	**1.22**	**3.50**

- 166 days with 1,224,127 transactions. Relative bias and MSE in percentage points.
- First order correction $\Rightarrow$ bias $\downarrow$, MSE $\downarrow$, optimal sampling frequency $\uparrow$
- BTS superior to CTS for each month in sample

Empirical Results for S&P500 Spiders

SPY	CPP-MA(1)			RVAC(0)				RVAC(1)			
	$\sigma_\nu/\sigma_\varepsilon$	σ_ε	$\lambda_{(0,1)}$	Freq.	Bias	MSE	CTSloss	Freq.	Bias	MSE	CTSloss
Jan 03	2.24	0.74	19666	147	8.01	2.94	5.72	7	0.41	0.76	3.64
Feb 03	2.23	0.78	23454	130	7.51	4.47	5.64	6	0.38	1.08	3.67
Mar 03	2.15	0.83	27747	112	6.98	9.13	5.24	5	0.41	2.00	3.56
Apr 03	2.13	0.70	24087	120	7.24	2.51	6.95	6	0.47	0.61	4.12
May 03	2.18	0.60	22819	128	7.48	1.37	6.56	6	0.43	0.34	4.37
Jun 03	1.99	0.59	24467	108	6.87	1.21	6.59	5	0.57	0.29	5.20
Jul 03	1.72	0.75	28043	82	5.95	8.29	4.52	4	0.69	2.76	3.76
Aug 03	1.78	0.55	24736	95	6.42	1.03	4.77	5	0.72	0.23	3.99
Jan 03 - Aug 03	**2.05**	**0.69**	**24389**	**115**	**7.05**	**3.86**	**5.75**	**6**	**0.51**	**1.01**	**4.04**

- 166 days with 4,048,665 transactions. 25K transaction per day!
- Higher noise ratio than for IBM but downward trend (market efficiency improved?)
- Bias correction leads to 60%-80% reduction in MSE!

The End…

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- Substantial gains associated with bias correction

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