Temporal Aggregation, Causality Distortions, and a Sign Rule

By

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Literature on temporal aggregation and Granger-causality distortions

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} \varphi_{1,11} & \varphi_{1,12} \\ \varphi_{1,21} & \varphi_{1,22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \varphi_{2,11} & \varphi_{2,12} \\ \varphi_{2,21} & \varphi_{2,22} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ x_{t-2} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

$\varphi_{1,12} = \varphi_{2,12} = 0 \Rightarrow$ No G-causality from x to y.

$\varphi_{1,21} = \varphi_{2,21} = 0 \Rightarrow$ No G-causality from y to x.

General findings within a stationary framework

(i) create a spurious feedback loop from a unidirectional relation

(ii) erase a feedback loop and create a unidirectional relation

(iii) erase the Granger-causal link altogether.

(iv) The distortions magnify when differencing is used after temporal aggregation to induce stationarity.

Pagan’s (1989) criticism of G-causality testing

What's the solution?

Look for more disaggregated data? Impractical.

Use temporally aggregated data?

Temporal aggregation removes lag effects and increases contemporaneous correlation.
Therefore, need a method that focuses on contemporaneous correlation.

Hoover's (2001) method: (A promising method)

Causal inference based on intervention analysis (wars, strikes etc)

Examine conditional and marginal distributions

Problems:

- Estimated models are dynamic, and subject to distortions of temp aggr
- Situations where intervention history not present?

Swanson and Granger's (1997) graph-theoretic approach:

Assign a causal ordering to contemporaneous links in an SVAR model

Problems:

- Demiralp and Hoover (2003) show subjective info is still needed
- Sign and magnitude distortions of contemporaneous correlations
- Problems with the 2-step estimation method used in SVAR models
- Even if contemporaneous links are correctly captured, impulse responses are still subject to distortions of temp aggregation

Our method:

Use cointegration for causal inference.

Cointegration is invariant to temporal aggregation and implies G-causality. But the direction is unknown.

As temporal aggregation increases, a stationary VAR(p) tends towards VAR(0) leaving behind only the contemporaneous relations.

But a cointegrated VAR(p) process cannot shrink below VAR(1) because of the presence of unit roots.

As a result some adjustment coefficients of the error correction model have to remain non-zero regardless the level of temporal aggregation.

Weak exogeneity and G-causality under cointegration are connected.

Analytical Tools

Vector ECM

$$\underset{(n\times 1)}{\Delta y_t} = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \ldots + \Gamma_{p-1}\Delta y_{t-(p-1)} + \phi D_t + e_t$$

$Var(e_t) = \Sigma$, non-diagonal

Cointegrating relation $u_t = \beta' y_t$ invariant to temporal aggregation.

But α and Γ's are subject to distortions.

Assume data are sufficiently temporally aggregated such that a VAR(1) would become the best fitting model. A higher order VAR is not a problem.

$$\Delta y_t = \alpha\beta' y_{t-1} + e_t$$

Since β is invariant to temp aggregation, assume β known and focus on α.

$$\begin{pmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \begin{pmatrix} \beta_1' & \beta_2' \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

y_{1t} is $(n_1 \times 1)$ and y_{2t} is $(n_2 \times 1)$, $n_1 + n_2 = n$.

$\alpha_2 = 0$, y_{2t} is weakly exogenous for β and α_1 (Johansen, 1995)

$\alpha_2 = 0$ also implies the presence of Granger causality from y_{2t} to y_{1t}

Lagged feedback effects may be present but disappeared into contemporaneous links

Important distinction, Hendry and Mizon (1998):

Causality associated with α: Causality in levels

Causality associated with Γ's: Causality in difference

The sign of the adjustment coefficient

We have worked out only the case of r=1 (one-cointegrating vector)
Still working on the case of r>1.

Cointegration requires α's have the "correct sign".

Correct sign: If $\beta_i > 0$, $\alpha_i < 0$; If $\beta_i < 0$, $\alpha_i > 0$ (see the paper)

Wrong Sign: May still yield cointegration but subject to a condition

Note: $u_t = \rho u_{t-1} + \varepsilon_t$

$\rho = 1 + \alpha'\beta$

Co-integration requires $|\rho| < 1$ which implies $-2 < \alpha'\beta < 0$

ρ measures the degree of co-integration:

$|\rho| \to 0$ higher degree of co-integration, $|\rho| \to 1$ lower degree

Let b_1 ($n_1 \times 1$) be positive and b_2 ($n_2 \times 1$) be negative.
Corresponding a vectors are a_1 and a_2 respectively.
Assume a_1<0 is correctly signed and a_2<0 is wrongly signed.
Given the inequality $-2 < a_1' b_1 + a_2' b_2 < 0$, if $|a_1' b_1| > |a_2' b_2|$ we get $|r| < 1$ even with the wrong sign.
In other words, if the adjustment towards equilibrium is dominated by the adjustment coefficients with the correct sign co-integration continues to hold.
However, the wrong sign lowers the degree of co-integration (increases the absolute value of r).

Bivariate Case

$$\Delta y_{1t} = \alpha_1 u_{t-1} + e_{1t}$$
$$\Delta y_{2t} = \alpha_2 u_{t-1} + e_{2t}$$
where $u_t = \beta_1 y_{1t} + \beta_2 y_{2t}$.

Assume

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim iidN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}\right).$$

Note: $u_t = \rho u_{t-1} + \varepsilon_t$, where $\rho = 1 + \alpha_1\beta_1 + \alpha_2\beta_2$ and $\varepsilon_t = \beta_1 e_{1t} + \beta_2 e_{2t}$ with zero mean and variance $\sigma_\varepsilon^2 = \beta_1^2\sigma_1^2 + \beta_2^2\sigma_2^2$.

Let $w_{1t} = \Delta y_{1t}$ and $w_{2t} = \Delta y_{2t}$.

For i=1,2, the variances and covariances of the above non-aggregate process can be written as

$$\gamma_u(k) = E(u_t u_{t-k}) = E(u_t u_{t+k}) = \rho^k \sigma_\varepsilon^2 /(1-\rho^2) \quad \forall k \tag{6}$$

$$E(e_{it} u_{t+k}) = \begin{cases} 0 & if \quad k < 0 \\ \rho^k \beta_i \sigma_i^2 & if \quad k \geq 0 \end{cases} \tag{7}$$

$$\gamma_{ii}^w(k) = E(w_{it} w_{it-k}) = \begin{cases} \alpha_i^2 \gamma_u(k) + \sigma_i^2 & if \quad k = 0 \\ \alpha_i^2 \gamma_u(k) + \alpha_i \rho^{k-1} \beta_i \sigma_i^2 & if \quad k > 0 \end{cases} \tag{8}$$

$$\gamma_{ii}^w(-k) = \gamma_{ii}^w(k)$$

$$\gamma_{iu}(k) = E(w_{it}u_{t-k}) = \begin{cases} \alpha_i \dfrac{\rho^{k-1}}{1-\rho}\sigma_\varepsilon^2 & \forall k > 0 \\ \alpha_i \dfrac{\rho^{-k+1}}{1-\rho}\sigma_\varepsilon^2 + \rho^{-k}\beta_i\sigma_i^2 & \forall k \le 0. \end{cases} \tag{9}$$

Let $Y_{1\tau}$ and $Y_{2\tau}$ $(\tau = 1,2,..,N; T = mN)$ be the m-period non-overlapping aggregates of y_{1t} and y_{2t} respectively and let $W_{1\tau} = \Delta Y_{1\tau}$ and $W_{2\tau} = \Delta Y_{2\tau}$. We now consider estimating the following aggregated process:

$$W_{1\tau} = \alpha_1^* U_{\tau-1} + E_{1\tau} \tag{10}$$

$$W_{2\tau} = \alpha_2^* U_{\tau-1} + E_{2\tau} \tag{10$'$}$$

where $U_\tau = \sum_{j=m(\tau-1)}^{m\tau-1} u_j$ and $E_{i\tau}$ represent non-overlapping sums of the error process.

The OLS estimates $\hat{\alpha}_i^*$, $p\lim \hat{\alpha}_i^*$ and the t statistics are given by:

$$\hat{\alpha}_i^* = \frac{\sum W_{i\tau}U_{\tau-1}}{\sum U_{\tau-1}^2}, \quad p\lim \hat{\alpha}_i^* = \frac{\gamma_{iU}(1)}{\gamma_U(0)} \tag{11}$$

$$\hat{t}_i^* = \frac{\hat{\alpha}_i^*}{[\mathrm{var}(\hat{\alpha}_i^*)]^{1/2}} \tag{12}$$

where

$$\operatorname{var}(\hat{\alpha}_i^*) = \frac{\sigma_{E_i}^2}{\sum U_\tau^2} = \frac{\sigma_{E_i}^2}{N\gamma_U(0)},\ \sigma_{E_i}^2 = \operatorname{var}(E_i) = \gamma_{ii}^W(0) + \hat{\alpha}_i^{*2}\gamma_U(0) - 2\hat{\alpha}_i^*\gamma_{iU}(1). \tag{13}$$

Using Proposition A.1 in Appendix that establishes the relationship between covariances of the aggregated and the non-aggregate processes we get the following relations, again for i=1,2:

$$\gamma_U(k) = (1 + L + \ldots + L^{m-1})^2 \gamma_u(mk + (m-1)) \quad \forall k\,. \tag{14}$$

$$\gamma_{iU}(k) = (1 + L + \ldots + L^{m-1})^3 \gamma_{iu}(mk + (m-1)) \quad \forall k \tag{15}$$

$$\gamma_{ii}^W(k) = (1 + L + \ldots + L^{m-1})^4 \gamma_{ii}^w(mk + 2(m-1))\,. \tag{16}$$

These expressions provide the link between the parameter estimates and the t-statistics of the aggregated process and the parameters of the non-aggregate process in order to derive a quantitative evaluation of the impact of temporal aggregation.

Distortions

Three cases of Granger causality in the non-aggregate process:

(i) no causality (No problem)

(ii) unidirectional causality

(iii) mutual causality or feedback

Unidirectional causality in the non-aggregate process

Table 1: Unidirectional Causality: $p\lim \hat{\alpha}_1^*$
when $\alpha_2 = 0$**,** $\beta_1 = 1$ **and** $\rho = 1 + \alpha_1$

a_1 across/ b_2 down	**-1.95**	**-1.75**	**-1.5**	**-1.25**	**-1.0**	**-0.75**	**-0.5**	**-0.25**	**-0.05**
m=3									
-20	-2.06	-2.31	-2.43	-2.29	-2.00	-1.63	-1.20	-0.67	-0.15
-10	-2.05	-2.31	-2.43	-2.28	-1.99	-1.63	-1.19	-0.67	-0.15
-8	-2.05	-2.30	-2.42	-2.27	-1.98	-1.62	-1.19	-0.66	-0.15
-6	-2.05	-2.29	-2.40	-2.26	-1.97	-1.61	-1.19	-0.66	-0.15
-4	-2.04	-2.26	-2.36	-2.22	-1.94	-1.59	-1.17	-0.65	-0.14
-2	-2.02	-2.14	-2.19	-2.05	-1.80	-1.49	-1.10	-0.62	-0.14
-1	-1.96	-1.88	-1.81	-1.68	-1.50	-1.26	-0.96	-0.55	-0.12
m=12									
-20	-7.76	-9.18	-8.88	-7.81	-6.49	-5.05	-3.56	-2.01	-0.53
-10	-7.71	-9.12	-8.82	-7.76	-6.45	-5.02	-3.54	-2.00	-0.53
-8	-7.67	-9.08	-8.78	-7.72	-6.42	-5.00	-3.52	-2.00	-0.53
-6	-7.59	-8.99	-8.69	-7.64	-6.35	-4.95	-3.49	-1.98	-0.53
-4	-7.38	-8.73	-8.44	-7.43	-6.18	-4.82	-3.41	-1.94	-0.52
-2	-6.44	-7.58	-7.33	-6.47	-5.40	-4.24	-3.04	-1.77	-0.49
-1	-4.44	-5.15	-4.98	-4.42	-3.75	-3.02	-2.25	-1.41	-0.43

These values are the same for $b_2 > 0$.

Table 2: Unidirectional Causality: $t(\hat{a}_1^*)$

when $a_2 = 0$, $b_1 = 1$ and $r = 1 + a_1$

a_1 across/ b_2 down	**-1.95**	**-1.75**	**-1.5**	**-1.25**	**-1.0**	**-0.75**	**-0.5**	**-0.25**	**-0.05**
Panel 1: T=150, m=3, N=50									
-20	-13.6	-8.0	-7.9	-8.4	-9.2	-10.5	-12.9	-18.2	-31.2
-10	-13.6	-8.0	-7.9	-8.4	-9.2	-10.5	-12.7	-17.5	-20.5
-8	-13.6	-8.0	-7.9	-8.4	-9.2	-10.5	-12.6	-17.1	-17.2
-6	-13.7	-8.0	-7.8	-8.4	-9.2	-10.4	-12.5	-16.2	-13.4
-4	-13.8	-8.0	-7.8	-8.3	-9.1	-10.2	-12.0	-14.3	-9.3
-2	-14.4	-7.9	-7.5	-7.9	-8.6	-9.4	-10.3	-9.9	-4.9
-1	-16.4	-8.1	-7.1	-7.2	-7.7	-8.1	-7.9	-6.3	-2.8
Panel 2: T=600, m=12, N=50									
-20	-4.5	-5.4	-5.8	-6.0	-6.3	-6.5	-6.9	-8.2	-16.2
-10	-4.4	-5.3	-5.8	-6.0	-6.2	-6.4	-6.8	-8.1	-15.6
-8	-4.4	-5.3	-5.8	-6.0	-6.2	-6.4	-6.8	-8.1	-15.1
-6	-4.4	-5.3	-5.7	-6.0	-6.1	-6.4	-6.8	-8.0	-14.4
-4	-4.4	-5.1	-5.6	-5.8	-6.0	-6.3	-6.7	-7.9	-12.7
-2	-4.0	-4.7	-5.1	-5.3	-5.5	-5.8	-6.2	-7.4	-8.7
-1	-3.4	-3.8	-4.1	-4.3	-4.5	-4.8	-5.3	-6.5	-5.4
Panel 3: T=600, m=3, N=200									
-20	-27.2	-16.0	-15.8	-16.9	-18.4	-21.0	-25.7	-36.3	-62.4
-10	-27.3	-16.0	-15.8	-16.9	-18.4	-21.0	-25.5	-35.0	-41.0
-8	-27.3	-16.0	-15.8	-16.8	-18.4	-20.9	-25.3	-34.1	-34.3
-6	-27.4	-16.0	-15.7	-16.8	-18.3	-20.8	-24.9	-32.3	-26.8
-4	-27.6	-16.0	-15.5	-16.5	-18.1	-20.3	-24.0	-28.5	-18.6
-2	-28.8	-15.9	-15.0	-15.8	-17.1	-18.9	-20.7	-19.8	-9.8
-1	-32.8	-16.2	-14.2	-14.5	-15.3	-16.1	-15.8	-12.5	-5.6

These values are the same for $b_2 > 0$.

Table 3: Unidirectional Causality: $p\lim \hat{\alpha}_2^*$
when $a_2 = 0$, $b_1 = 1$ and $r = 1 + a_1$

a_1 across/ b_2 down	**-1.95**	**-1.75**	**-1.5**	**-1.25**	**-1.0**	**-0.75**	**-0.5**	**-0.25**	**-0.05**
m=3									
-20	-0.01	-0.04	-0.06	-0.06	-0.05	-0.04	-0.02	-0.01	-0.00
-10	-0.02	-0.09	-0.12	-0.12	-0.10	-0.07	-0.05	-0.02	-0.00
-8	-0.02	-0.11	-0.15	-0.15	-0.12	-0.09	-0.06	-0.03	-0.01
-6	-0.03	-0.14	-0.20	-0.20	-0.16	-0.12	-0.08	-0.04	-0.01
-4	-0.05	-0.21	-0.29	-0.29	-0.24	-0.17	-0.11	-0.05	-0.01
-2	-0.08	-0.35	-0.50	-0.49	-0.40	-0.30	-0.19	-0.09	-0.02
-1	-0.10	-0.44	-0.63	-0.61	-0.50	-0.37	-0.24	-0.12	-0.02
1	0.10	0.44	0.63	0.61	0.50	0.37	0.24	0.12	0.02
2	0.08	0.35	0.50	0.49	0.40	0.30	0.19	0.09	0.02
4	0.05	0.21	0.29	0.29	0.24	0.17	0.11	0.05	0.01
6	0.03	0.14	0.20	0.20	0.16	0.12	0.08	0.04	0.01
8	0.02	0.11	0.15	0.15	0.12	0.09	0.06	0.03	0.01
10	0.02	0.09	0.12	0.12	0.10	0.07	0.05	0.02	0.00
20	0.01	0.04	0.06	0.06	0.05	0.04	0.02	0.01	0.00
m=12									
-20	-0.33	-0.40	-0.39	-0.34	-0.27	-0.20	-0.13	-0.06	-0.01
-10	-0.66	-0.80	-0.78	-0.67	-0.54	-0.40	-0.26	-0.12	-0.02
-8	-0.82	-1.00	-0.97	-0.84	-0.68	-0.50	-0.32	-0.15	-0.03
-6	-1.08	-1.31	-1.27	-1.10	-0.89	-0.66	-0.43	-0.19	-0.03
-4	-1.57	-1.91	-1.85	-1.60	-1.29	-0.96	-0.62	-0.28	-0.05
-2	-2.67	-3.24	-3.14	-2.72	-2.20	-1.63	-1.05	-0.48	-0.08
-1	-3.34	-4.05	-3.93	-3.40	-2.75	-2.04	-1.31	-0.60	-0.10
1	3.34	4.05	3.93	3.40	2.75	2.04	1.31	0.60	0.10
2	2.67	3.24	3.14	2.72	2.20	1.63	1.05	0.48	0.08
4	1.57	1.91	1.85	1.60	1.29	0.96	0.62	0.28	0.05
6	1.08	1.31	1.27	1.10	0.89	0.66	0.43	0.19	0.03
8	0.82	1.00	0.97	0.84	0.68	0.50	0.32	0.15	0.03
10	0.66	0.80	0.78	0.67	0.54	0.40	0.26	0.12	0.02
20	0.33	0.40	0.39	0.34	0.27	0.20	0.13	0.06	0.01

Table 4: Unidirectional Causality: $t(\hat{\alpha}_2^*)$
when $a_2 = 0$, $b_1 = 1$ and $r = 1 + a_1$

a_1 across/ b_2 down	**-1.95**	**-1.75**	**-1.5**	**-1.25**	**-1.0**	**-0.75**	**-0.5**	**-0.25**	**-0.05**
Panel 1: T=150, m=3, N=50									
-20	-1.1	-2.4	-3.1	-3.2	-3.1	-2.7	-2.2	-1.6	-0.7
-10	-1.1	-2.4	-3.1	-3.2	-3.0	-2.7	-2.2	-1.5	-0.7
-8	-1.1	-2.4	-3.1	-3.2	-3.0	-2.7	-2.2	-1.5	-0.7
-6	-1.0	-2.3	-3.1	-3.2	-3.0	-2.6	-2.2	-1.5	-0.7
-4	-1.0	-2.3	-3.0	-3.1	-3.0	-2.6	-2.1	-1.5	-0.7
-2	-1.0	-2.1	-2.7	-2.9	-2.7	-2.3	-2.0	-1.4	-0.6
-1	-0.7	-1.7	-2.1	-2.2	-2.1	-1.8	-1.5	-1.1	-0.5
1	0.7	1.7	2.1	2.2	2.1	1.8	1.5	1.1	0.5
2	1.0	2.1	2.7	2.9	2.7	2.3	2.0	1.4	0.6
4	1.0	2.3	3.0	3.1	3.0	2.6	2.1	1.5	0.7
6	1.0	2.3	3.1	3.2	3.0	2.6	2.2	1.5	0.7
8	1.1	2.4	3.1	3.2	3.0	2.7	2.2	1.5	0.7
10	1.1	2.4	3.1	3.2	3.0	2.7	2.2	1.5	0.7
20	1.1	2.4	3.1	3.2	3.1	2.7	2.2	1.6	0.7
Panel 2: T=600, m=12, N=50									
-40	-3.7	-4.5	-4.8	-4.9	-4.8	-4.6	-4.1	-3.2	-1.6
-20	-3.7	-4.5	-4.8	-4.9	-4.8	-4.6	-4.1	-3.2	-1.6
-10	-3.6	-4.4	-4.8	-4.8	-4.8	-4.5	-4.1	-3.2	-1.6
-8	-3.6	-4.4	-4.8	-4.8	-4.8	-4.5	-4.1	-3.2	-1.5
-6	-3.6	-4.4	-4.7	-4.8	-4.7	-4.5	-4.1	-3.2	-1.5
-4	-3.5	-4.2	-4.6	-4.6	-4.6	-4.4	-4.0	-3.1	-1.5
-2	-3.2	-3.9	-4.1	-4.1	-4.1	-3.9	-3.6	-2.9	-1.4
-1	-2.4	-2.9	-3.1	-3.1	-3.1	-2.9	-2.7	-2.2	-1.1
1	2.4	2.9	3.1	3.1	3.1	2.9	2.7	2.2	1.1
2	3.2	3.9	4.1	4.1	4.1	3.9	3.6	2.9	1.4
4	3.5	4.2	4.6	4.6	4.6	4.4	4.0	3.1	1.5
6	3.6	4.4	4.7	4.8	4.7	4.5	4.1	3.2	1.5
8	3.6	4.4	4.8	4.8	4.8	4.5	4.1	3.2	1.5
10	3.6	4.4	4.8	4.8	4.8	4.5	4.1	3.2	1.6
20	3.7	4.5	4.8	4.9	4.8	4.6	4.1	3.2	1.6
Panel 3: T=600, m=3, N=200									
-20	-2.1	-4.8	-6.3	-6.5	-6.1	-5.4	-4.4	-3.1	-1.3
-10	-2.1	-4.8	-6.3	-6.5	-6.0	-5.4	-4.4	-3.0	-1.3
-8	-2.1	-4.8	-6.3	-6.5	-6.0	-5.4	-4.4	-3.0	-1.3
-6	-2.0	-4.7	-6.1	-6.4	-6.0	-5.3	-4.4	-3.0	-1.3
-4	-2.0	-4.7	-6.0	-6.3	-5.9	-5.1	-4.2	-3.0	-1.3
-2	-1.9	-4.2	-5.5	-5.7	-5.4	-4.7	-3.9	-2.8	-1.2
-1	-1.5	-3.3	-4.2	-4.4	-4.1	-3.7	-3.0	-2.2	-1.0
1	1.5	3.3	4.2	4.4	4.1	3.7	3.0	2.2	1.0
2	1.9	4.2	5.5	5.7	5.4	4.7	3.9	2.8	1.2
4	2.0	4.7	6.0	6.3	5.9	5.1	4.2	3.0	1.3
6	2.0	4.7	6.1	6.4	6.0	5.3	4.4	3.0	1.3
8	2.1	4.8	6.3	6.5	6.0	5.4	4.4	3.0	1.3
10	2.1	4.8	6.3	6.5	6.0	5.4	4.4	3.0	1.3
20	2.1	4.8	6.3	6.5	6.1	5.4	4.4	3.1	1.3

Table 5: Mutual Causality: $t(\hat{\alpha}_2^*)$
when $b_1 = 1$, $r = 0$ and $a_2 = (r - 1 - a_1)/b_2$

a_1 across/ b_2 down	**-0.95**	**-0.85**	**-0.75**	**-0.65**	**-0.55**	**-0.45**	**-0.35**	**-0.25**	**-0.15**	**-0.05**
Panel 1: T=150, m=3, N=50										
-20	-2.9	-2.3	-1.8	-1.2	-0.4	0.4	1.5	2.8	4.2	6.0
-10	-2.9	-2.3	-1.8	-1.1	-0.4	0.5	1.6	2.8	4.3	6.0
-8	-2.8	-2.3	-1.8	-1.1	-0.3	0.5	1.6	2.9	4.3	6.1
-6	-2.8	-2.3	-1.7	-1.1	-0.3	0.6	1.6	2.9	4.4	6.1
-4	-2.7	-2.2	-1.6	-1.0	-0.2	0.7	1.8	3.1	4.5	6.1
-2	-2.4	-1.8	-1.2	-0.4	0.4	1.5	2.5	3.7	5.1	6.4
-1	-1.7	-0.9	0.0	1.0	2.1	3.2	4.3	5.4	6.4	7.3
Panel 2: T=600, m=12, N=50										
-20	-4.7	-4.5	-4.4	-4.1	-3.7	-3.3	-2.6	-1.6	0.1	3.6
-10	-4.7	-4.5	-4.3	-4.0	-3.7	-3.2	-2.5	-1.5	0.3	3.3
-8	-4.6	-4.5	-4.2	-4.0	-3.6	-3.1	-2.4	-1.3	0.3	3.2
-6	-4.6	-4.4	-4.2	-3.9	-3.5	-3.0	-2.2	-1.2	0.5	3.0
-4	-4.5	-4.2	-4.0	-3.7	-3.2	-2.6	-1.8	-0.7	0.9	2.9
-2	-4.0	-3.6	-3.2	-2.7	-2.1	-1.2	-0.3	0.8	2.0	3.1
-1	-2.8	-2.2	-1.5	-0.7	0.2	1.2	2.1	2.9	3.6	4.2
Panel 3: T=600, m=3, N=200										
-20	-5.7	-4.7	-3.6	-2.3	-0.8	0.9	3.0	5.6	8.5	12.1
-10	-5.7	-4.7	-3.6	-2.2	-0.8	1.0	3.1	5.6	8.6	12.1
-8	-5.6	-4.7	-3.6	-2.2	-0.7	1.0	3.1	5.7	8.6	12.2
-6	-5.6	-4.6	-3.5	-2.1	-0.7	1.1	3.2	5.8	8.7	12.2
-4	-5.5	-4.5	-3.2	-1.9	-0.3	1.5	3.6	6.1	8.9	12.3
-2	-4.8	-3.7	-2.3	-0.8	0.9	2.9	5.0	7.5	10.2	12.9
-1	-3.4	-1.8	0.0	2.0	4.1	6.3	8.6	10.8	12.8	14.5

For b_2 >0 the table entries are the same with the opposite sign.

How to Test for Granger causality with temporally aggregated data?

Sign rule and causal inference:

First, determine the expected sign of the adjustment coefficients from the estimated co-integrating vector.

If the estimated adjustment coefficient appears with the correct sign and is statistically significant then it reflects the underlying causal direction in the non-aggregate form.

If the coefficient appears with the wrong sign then a causal distortion may have occurred and if such a conclusion is supported by non-sample information then we may treat it as resulting from a zero or near zero coefficient in the non-aggregate form.

Monte Carlo results on higher order VAR processes lead to the same conclusions.

Applications

1. Exchange rate, direct vs. cross (Case of Unidirectional Relation)

log(Yen/DM) – log(US$/DM) + log(US$/Yen) is a cointegrating relation

$$\beta: \ (1, \quad -1, \quad 1)$$

Expected sign of α: $(-, \quad +, \quad -)$

Expected value of α: $(-1, \quad 0, \quad 0)$

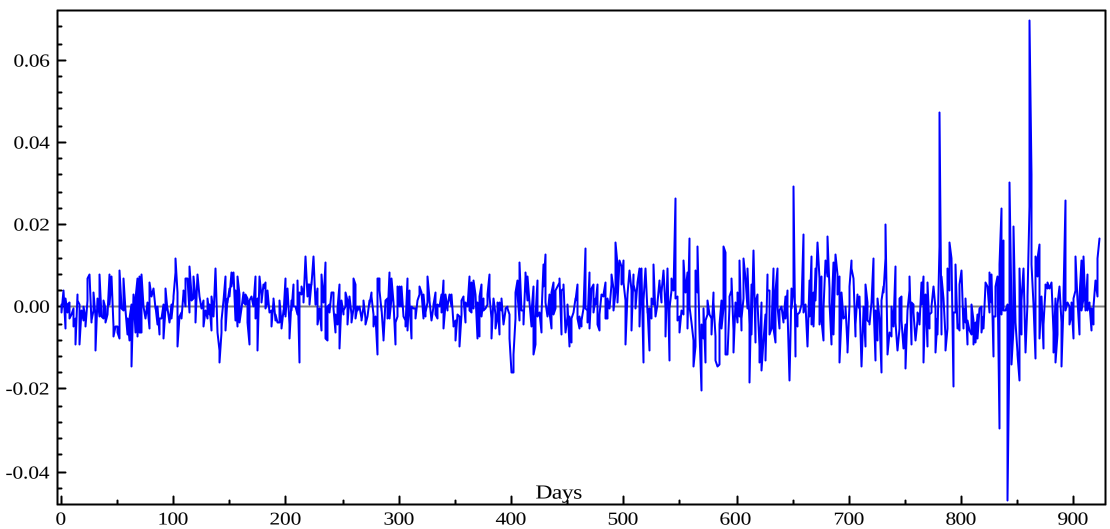


Figure 1. Deviations of logarithms of daily Yen/DM direct rate from the cross rate
(July 3, 1995 – Dec 31, 1998)

Table 8. Estimated adjustment coefficients
Cointegrating relation: log(Yen/DM)-log(US$/DM)+log(US$/Yen)

Adjustment coefficients	Daily rates	Weekly rates End of period	Weekly rates Average
a_1	-0.917* (0.015)	-0.889* (0.236)	-1.940* (0.202)
a_2	-0.046 (0.025)	-0.116 (0.152)	-0.326 (0.182)
a_3	0.050 (0.037)	-0.254 (0.258)	0.666* (0.301)
Sample size	922	184	184

Note: Numbers in parentheses are standard errors. * indicates the absolute values bigger than 2SE. If a_2 and a_3 are restricted to zero the estimates of a_1 in columns 3 and 4 move closer to minus unity.

2. Stock Market and Car Quota Premium in Singapore (skip)

3. Tax Revenue and Government Expenditure in the US

Barro's (1979) tax-sm0othing hypothesis:

G exogenous

T endogenous but follows a random walk

US Data: Annual 1946 - 2002

T = real federal government receipts (deflated by the GNP deflator, P)

G = real federal government expenditure net of interest payments (deflated by P)

Y = real GNP

$y = \ln Y$

Δy =GNP growth rate (%)

π = $(\Delta \ln P)100$ = inflation rate

τ = (T/Y)100 = income tax rate

g = (G/Y)100 = spending rate

Data plot and ADF tests support the assumption that τ, g, and π are I(1) processes.

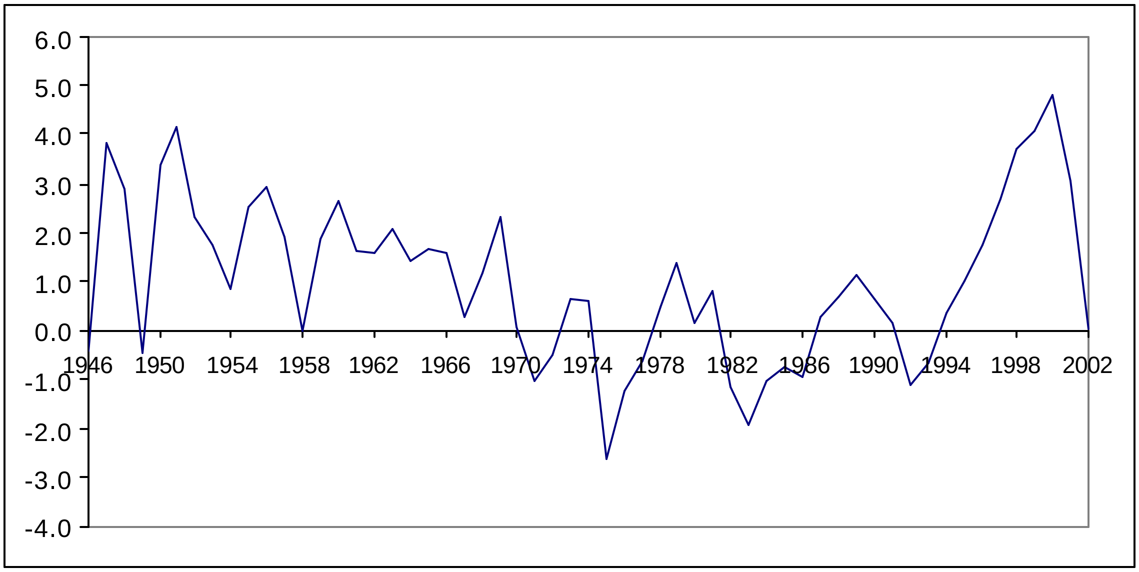


Figure 2. Budget surplus as a ratio of GNP (%)

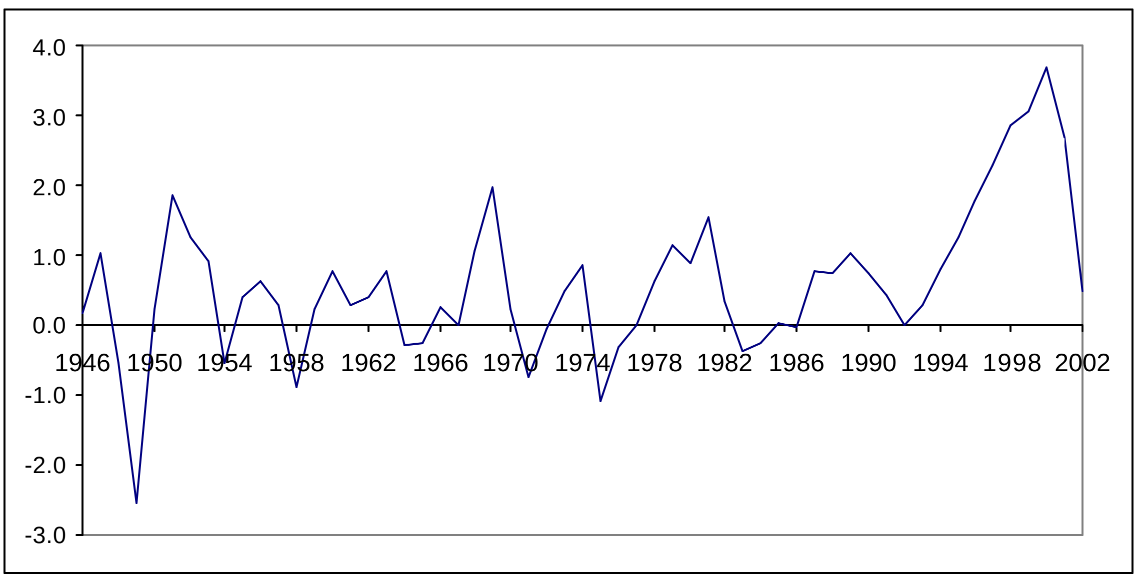


Figure 3. Cointegrating relation between tax rate and spending rate

Cointegrating relation: $z_t = \tau_t - 0.25 g_t - 13$

Cointegrating relation: $z_t = \tau_t - 0.25 g_t - 13$

ECM:

$$\Delta\tau_t = \delta_0 + \delta_1 \Delta\tau_{t-1} + \delta_2 \Delta g_{t-1} + \delta_3 \Delta y_t + \delta_4 \Delta\pi_t + \alpha_1 z_{t-1} + \varepsilon_{1t}$$

$$\Delta g_t = \lambda_0 + \lambda_1 \Delta\tau_{t-1} + \lambda_2 \Delta g_{t-1} + \lambda_3 \Delta y_t + \alpha_2 z_{t-1} + \varepsilon_{2t}$$

Expected signs of adjustment coefficients: $\alpha_1 < 0$, $\alpha_2 > 0$.

FIML estimates very similar to OLS estimates

Estimated ECM (1947 – 1994)

$$\Delta\tau_t = -0.06 + 0.42 \Delta\tau_{t-1} + 0.16 \Delta g_{t-1} + 0.09 \Delta y_t + 0.12 \Delta\pi_t - 0.65 z_{t-1}$$

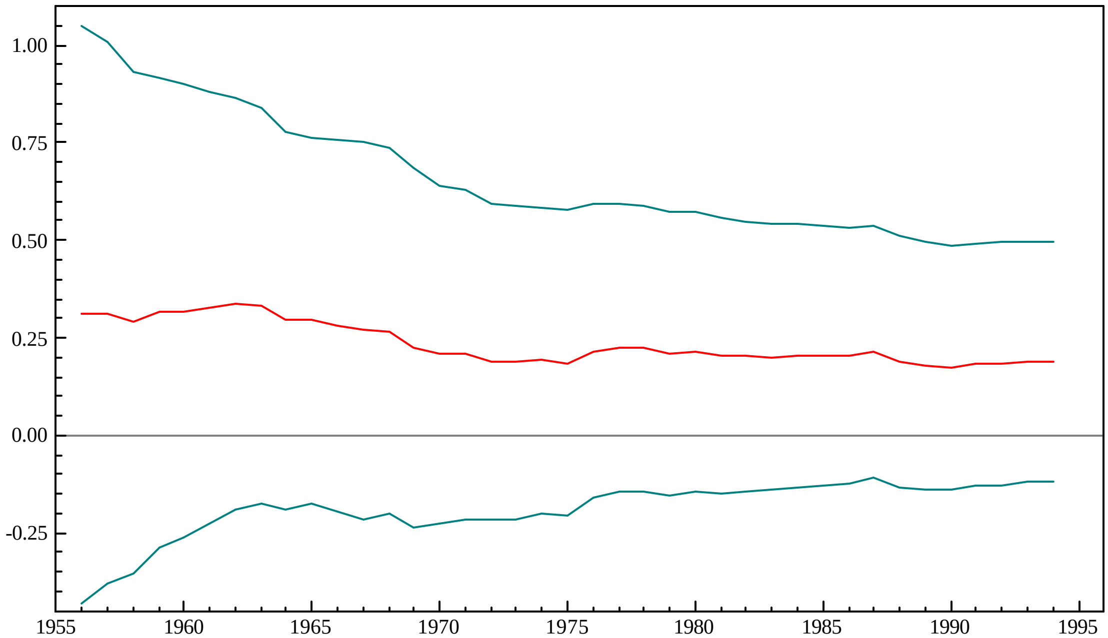
(-0.37) (3.51) (2.28) (2.44) (2.77) (-4.34)

$R^2 = 0.71$, $\hat{\sigma}_1 = 0.50$

$$\Delta g_t = 0.52 - 0.14 \Delta y_t + 0.19 z_{t-1}$$

(2.37) (-2.97) (1.22)

$R^2 = 0.32$, $\hat{\sigma}_2 = 0.68$



Recursive estimates of Alpha2 with 2SE

$$u_t = \sum_{i=1}^{n} b_i y_{it}$$

The long run equilibrium implies that $u_{t-1} = 0$ which gives:

$$y_{it-1} = \frac{-1}{b_i}(b_1 y_{1t-1} + b_2 y_{2t-1} + ... + b_{i-1} y_{i-1t-1} + b_{i+1} y_{i+1t-1} + ... + b_n y_{nt-1}) .$$

If the system is in disequilibrium at date $t-1$ then either $u_{t-1} > 0$ or $u_{t-1} < 0$.

Case 1: $b_i > 0$

If $u_{t-1} > 0$, then $y_{it-1} > \frac{-1}{b_i}(b_1 y_{1t-1} + b_2 y_{2t-1} + ... + b_{i-1} y_{i-1t-1} + b_{i+1} y_{i+1t-1} + ... + b_n y_{nt-1})$ and we expect $a_i u_{t-1} < 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} > 0$, a_i has to be negative ($a_i < 0$).

If $u_{t-1} < 0$, then $y_{it-1} < \frac{-1}{b_i}(b_1 y_{1t-1} + b_2 y_{2t-1} + ... + b_{i-1} y_{i-1t-1} + b_{i+1} y_{i+1t-1} + ... + b_n y_{nt-1})$ and we expect $a_i u_{t-1} > 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} < 0$, a_i has to be negative ($a_i < 0$).

Thus, if $b_i > 0$, then $a_i < 0$ regardless of the sign of the disequilibrium term u_{t-1}.

Case 2: $b_i < 0$

If $u_{t-1} > 0$, then $y_{it-1} < \frac{-1}{b_i}(b_1 y_{1t-1} + b_2 y_{2t-1} + ... + b_{i-1} y_{i-1t-1} + b_{i+1} y_{i+1t-1} + ... + b_n y_{nt-1})$ and we expect $a_i u_{t-1} > 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} > 0$, a_i has to be positive ($a_i > 0$).

If $u_{t-1} < 0$, then $y_{it-1} > \frac{-1}{b_i}(b_1 y_{1t-1} + b_2 y_{2t-1} + ... + b_{i-1} y_{i-1t-1} + b_{i+1} y_{i+1t-1} + ... + b_n y_{nt-1})$ and we expect $a_i u_{t-1} < 0$ in (4) in order to achieve the equilibrium. Since $u_{t-1} < 0$, a_i has to be positive ($a_i > 0$).

Thus, if $b_i < 0$, then $a_i > 0$ regardless of the sign of the disequilibrium term, u_{t-1}.