Shortfall risk measure for general semimartingales

Takuji Arai

Department of Economics, Keio University

In this talk, we focus on a convex risk measure which is induced by the shortfall risk, which is defined as the weighted expectation by some loss function of the positive part of the difference between a claim and a portfolio value at the maturity of our market. The convex risk measure concerned in this talk expresses the least cost to suppress its shortfall risk below the threshold being determined beforehand.

Consider an incomplete financial market being composed of one riskless asset and d risky assets. The fluctuation of the risky assets is described by an RCLL semimartingale S, which is possibly non-locally bounded. Let l be a loss function, that is, an $\mathbf{R}_+$-valued continuous non-decreasing convex function defined on $\mathbf{R}$. We assume that $l(x) = 0$ if $x \leq 0$, and $l(x) > 0$ if $x > 0$. Let Θ be a predictably convex subset of S-integrable predictable processes which means the set of all admissible strategies, and $G_T(\vartheta) := \int_0^T \vartheta_s dS_s$. Recall that, when the price of a claim H and the hedging strategy are given by $x \in \mathbf{R}$ and $\vartheta \in \Theta$ respectively, the shortfall risk with a loss function l for sellers is expressed by $E[l(-x - G_T(\vartheta) + H)]$. If we denote by $\mathcal{X}$ the collection of all claims, the acceptance set $\mathcal{A}^0$ on $\mathcal{X}$ with a level $\delta > 0$ should be defined as

$$\mathcal{A}^0 := \{X \in \mathcal{X} | E[l(-X)] \leq \delta\}.$$

The level δ is the threshold to decide whether a claim is acceptable or not. That is, if there exists a $\vartheta \in \Theta$ such that $x + G_T(\vartheta) - H \in \mathcal{A}^0$, then $x \in \mathbf{R}$ would be regarded as an acceptable price for sellers. Now, we define a functional ρ_l on L^0 as

$$\begin{aligned} \rho_l(X) \quad := \quad & \inf\{x \in \mathbf{R} | \text{ there exists a } \vartheta \in \Theta \\ & \qquad \text{such that } x + G_T(\vartheta) + X \in \mathcal{A}^0\}. \end{aligned}$$

Thus, ρ_l gives the least price of a claim $-X$ which is acceptable for sellers with a loss function l and a threshold δ.

A representation of ρ_l when S is locally bounded and Θ is linear, has been given. Thus, we try to, in this talk, extend this representation result to the case where S is given by a possibly non-locally bounded process and Θ forms a predictable convex set.