ENFV

Cash flows of one dollar invested in the productive capital of the economy whose rate of return is either 5% or 1% with equal probabilities.

Time horizon t	Cash flow			r_t^F
	$r = 5\%$	$r = 1\%$	Ee^{rt}	
1 year	1.05	1.01	1.03	3.00 %
10 years	1.63	1.10	1.37	3.17 %
100 years	131.50	2.70	67.10	4.30 %
200 years	17 292.58	7.32	8 649.95	4.64 %

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ENPV

Loan that can be obtained with a repayment of one dollar in *t* years when the interest rate is either 5% or 1% with equal probabilities

Time horizon t	Loan			r_t^P
	$r=5\%$	$r=1\%$	Ee^{-rt}	
1 year	0.95	0.99	0.97	2.96 %
10 years	0.61	0.91	0.76	2.79 %
100 years	$7.60\ 10^{-3}$	0.37	0.19	1.68 %
200 years	$5.78\ 10^{-5}$	0.14	0.07	1.35 %

First-order stochastic correlations

Definitions

- $c_t = c_0 + x_1 + x_2$.
- $G(x) = \Pr\left[x_1 \le x\right]$ *and* $F(x|x_1) = \Pr\left[x_2 \le x|x_1\right]$.
- (x_1, x_2) *are "positively first-order stochastically correlated" (FSC) if F is non-increasing in* x_1 *for all* x_2.

- Example: AR(1): $x_2|x_1 = \phi x_1 + \varepsilon$ *with* $\phi > 0$.
- *Let* y_2 *denote the independent r.v. whose CDF is H, with* $H(y) = \int F(y|x_1)dx_1$.

Isolating the effect of FSC

- We compare

to

$$r_t = \delta - \frac{1}{t} \ln \frac{Eu'(c_0 + x_1 + x_2)}{u'(c_0)}$$

- $$\widehat{r}_t = \delta - \frac{1}{t} \ln \frac{Eu'(c_0 + x_1 + y_2)}{u'(c_0)}.$$

$$r_t \le \widehat{r}_t \quad \Leftrightarrow \quad Eu'(c_0 + x_1 + x_2) \ge Eu'(c_0 + x_1 + y_2)$$

Two results

- Lemma 1: Consider any positive FSC pair (x_1,x_2).

$$Eh(x_1, x_2) \geq Eh(x_1, y_2) \Leftrightarrow h \text{ is supermodular: } \frac{\partial^2 h}{\partial x_1 \partial x_2} \geq 0.$$

- Proposition 1: *The presence of a positive FSC correlation in changes in consumption reduces the long-term efficient discount rate if and only if the representative agent is prudent (u' convex).*

- *Corollary: CRRA+FSC implies decreasing term structure*

Intuition

- The positive FSC correlation in Δc raises the risk of the distant future compared to the i.i.d. case.

$$E\left(c_0 + x_1 + x_2\right) = E\left(c_0 + x_1 + y_2\right)$$

$$E\left(c_0 + x_1 + x_2\right)^2 \geq E\left(c_0 + x_1 + y_2\right)^2$$

- Under prudence, it is efficient to make more efforts for that distant future. This is done by reducing the long-term discount rate.

Remark: De-correlation of growth *rates*

- We compare

$$r_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_0 e^{x_1+x_2})}{u'(c_0)} \quad to \quad \widehat{r}_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_0 e^{x_1+y_2})}{u'(c_0)}$$

- Proposition 2: *The presence of a positive FSC correlation in changes in log consumption reduces the long-term efficient discount rate if and only if –u'''/u''>1.*

- *Notice that* $Ec_0 e^{x_1+x_2} \geq Ec_0 e^{x_1+y_2}$.

Second-order stochastic correlations

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Definitions

- (x_1,x_2) are positive SSC if an increase in x_1 raises the riskiness of the $x_2|x_1$.
- Example: $x_2 \left| x_1 = \mu + \varepsilon \sqrt{x_1} \right.$
- Lemma 2: Consider any positive SSC pair (x_1,x_2).

$$Eh(x_1, x_2) \geq Eh(x_1, y_2) \Leftrightarrow -\frac{\partial h}{\partial x_2} \text{ is supermodular: } \frac{\partial^3 h}{\partial x_1 \partial x_2^2} \leq 0.$$

Results

- Proposition 3: *The presence of a positive SSC correlation in changes in consumption raises the long-term efficient discount rate if and only if u'''' < 0.*

- Intuition: Increased skewness reduces $Eu'(c_t)$ if $u'''' < 0$.

Results

- Proposition 4: *The presence of a positive SSC correlation in changes in log consumption raises the long-term efficient discount rate if and only if* $f(c) = u''(c) + 3cu'''(c) + c^2u''''(c) \leq 0.$

- Intuition: increased skewness reduces $Eu'(c_t)$ if $u'''' < 0$.